

Hall Effects on Unsteady MHD Free Convection Flow Over A Stretching Sheet With Variable Viscosity And Viscous Dissipation

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Abstract: The unsteady magnetohydrodynamics free convection flow of an incompressible viscous fluid past a stretching surface is analyzed by taking into account the variable viscosity and viscous dissipation under the influence of hall currents effect. The problem is governed by coupled nonlinear partial differential equations. The coupled non-linear ordinary differential equations have been solved numerically by the implicit finite difference method. The effect of magnetic parameter M , Hall parameter m , Prandtl number Pr , Eckert number Ec , on velocity and temperature fields are investigated through graphs.

Keywords: MHD, viscous dissipation, implicit finite difference method, Hall parameter.

I. Introduction

The study of the flow of electrically conducting fluid in the presence of magnetic field is important from the technical point of view and such types of problems have received much attention by many researchers. The specific problem selected for study is the flow and heat transfer in an electrically conducting fluid adjacent to the surface. The surface is maintained at a uniform temperature T_w , which may either exceed the ambient temperature T_∞ or may be less than T_∞ . When $T_w > T_\infty$, an upward flow is established along the surface due to free convection, where as for $T_w < T_\infty$, there is a down flow. The interaction of the magnetic field and the moving electric charge carried by the flowing fluid induces a force, which tends to oppose the fluid motion. The velocity is very small so that the magnetic force, which is proportional to the magnitude of the longitudinal velocity and acts in the opposite direction, is also very small. Additionally, a magnetic field of strength acts normal to the surface. Consequently, the influence of the magnetic field on the boundary layer is exerted only through induced forces within the boundary layer itself, with no additional effects arising from the free stream pressure gradient. The stress work effects in laminar flat plate natural convection flow have been studied by Ackroyd [1]. However, the influence and importance of viscous stress work effects in laminar flows have been examined by Gebhart [2] and Gebhart and Mollendorf [3]. In both of the investigations, special flows over semi-infinite flat surfaces parallel to the direction of body force were considered.

In all the above-mentioned studies, the viscosity of the fluid was assumed to be constant. However, it is known that the fluid physical properties may change significantly with temperature changes. To accurately predict the flow behaviour, it is necessary to take into account this variation of viscosity with temperature. Recently, many researchers investigated the effects of variable properties for fluid viscosity and thermal conductivity on flow and heat transfer over a continuously moving surface. Seddeek [4] investigated the effect of variable viscosity on hydro magnetic flow past a continuously moving porous boundary. Seddeek [5] also studied the effect of radiation and variable viscosity on an MHD free convection flow past a semi-infinite flat plate within an aligned magnetic field in the case of unsteady flow. Dandapat et al. [6] analyzed the effects of variable viscosity, variable thermal conducting, and thermocapillarity on the flow and heat transfer in a laminar liquid film on a horizontal stretching sheet. Mukhopadhyay [7] presented solutions for unsteady boundary layer flow and heat transfer over a stretching surface with variable fluid viscosity and thermal diffusivity in presence of wall suction. When the conducting fluid is an ionized gas and the strength of the applied magnetic field is large, the normal conductivity of the magnetic field is reduced to the free spiraling of electrons and ions about the magnetic lines force before suffering collisions and a current is induced in a normal direction to both electric and magnetic field. This phenomenon is called Hall effect. When the medium is a rare field or if a strong magnetic field is present. Abo-Eldahab et al. [8] and Salem and Abd El-Aziz [9] dealt with the effect of Hall current on a steady laminar hydromagnetic boundary layer flow of an electrically conducting and heat generating/absorbing fluid along a stretching sheet. Pal and Mondal [10] investigated the effect of temperature-dependent viscosity on non-Darcy MHD mixed convective heat transfer past a porous medium by taking into account Ohmic dissipation and non-uniform heat source/sink. Abd El-Aziz [11] investigated the effect of Hall currents on the flow and heat transfer of an electrically conducting fluid over an unsteady stretching surface in the presence of a strong magnet. When the strength of magnetic field is strong, one can't neglect the effect of hall current. It is of considerable importance and interest to study how the results of the hydro dynamical

problems get modified by the effect of hall current's. The hall effect is due merely to the sideways magnetic force on the drifting free charges. The electric field has to have a component transverse to the direction of the current density to balance this force. In many works on plasma physics, the hall effect is ignored. But if the strength of magnetic field is high and the number density of electrons is small, the hall effect cannot be disregarded as it has a significant effect on the flow pattern of an ionized gas. Hall effect results in a development of an additional potential difference between opposite surfaces of a conductor for which a current is induced perpendicular to both the electric and magnetic field. This current is termed as hall current. Model studies on the effect of hall current on MHD convection flows have been carried out by many authors due to application of such studies in the problems of MHD generators and hall accelerators as studied by kishan et al [12]. Some of them are Pop [13], Kinyanjui *et al* [14], Aboeldahab [15], Datta *et al* [16], studied the hall effects on MHD flow past an accelerated plate. Halls effect is also important when the fluid is an ionized gas with low density or the applied magnetic field is very strong. Because the electrical conductivity of the fluid will then be a tensor and a current (Hall current) is induced which is likely to be important in many engineering situations. The effect of hall current on the fluid flow with variable concentration has many applications in MHD power generation, in several astrophysical and metrological studies as well as in plasma flow through MHD power generators. In the present investigation, it is proposed to study the effect of magnetohydrodynamics unsteady free convection flow of an incompressible viscous fluid past a stretching surface is analysis by taking into account the hall effects and viscous dissipation. Fluid viscosity is assumed to vary as an exponential function of temperature while the fluid thermal diffusivity is assumed to vary as a linear function of temperature. The governing equations are solve by the using the implicit finite difference scheme using C-programming. Recently, S.Shateyi [17]studied the variable viscosity on magnetic hydrodynamic fluid flow and heat transfer over an unsteady stretching surface with hall effect.

II. Mathematical Analysis

We consider the unsteady flow and heat transfer of a viscous, incompressible, and electrically conducting fluid past a semi-infinite stretching sheet coinciding with the plane $y = 0$, then the fluid is occupied above the sheet $y \geq 0$. The positive x coordinate is measured along the stretching sheet in the direction of motion, and the positive y coordinate is measured normally to the sheet in the outward direction toward the fluid. The leading edge of the stretching sheet is taken as coincident with z -axis. The continuous sheet moves in its own plane with velocity $U_w(x, t)$ and the temperature $T_w(x, t)$ distribution varies both along the sheet and time. A strong uniform magnetic field is applied normally to the surface causing a resistive force in the x -direction. The stretching surface is maintained at a constant temperature and with significant hall currents. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. The effect of Hall current gives rise to a force in the z -direction, which induces a cross flow in that direction, and hence the flow becomes three dimensional. To simplify the problem, we assume that there is no variation of flow quantities in z -direction. This assumption is considered to be valid if the surface is of infinite extent in the z -direction. Finally, we assume that the fluid viscosity is to vary with temperature while other fluid properties are assumed to be constant. Using boundary layer approximations, the governing equations for unsteady laminar boundary layer flows are written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial x} \right) - \frac{\sigma B^2}{\rho(1+m^2)} (u + mw) \tag{2}$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left(\mu \frac{\partial w}{\partial x} \right) + \frac{\sigma B^2}{\rho(1+m^2)} (mu - w) \tag{3}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\rho c_p} \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \mu \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] \tag{4}$$

The boundary conditions are defined as follows;

$$u = U_w(x, t), \quad v = 0, \quad T = T_w(x, t) \quad \text{at } y = 0 \tag{5}$$

$$u \rightarrow 0, \quad w \rightarrow 0, \quad T \rightarrow T_w \quad \text{as } y \text{ tends to } \infty,$$

where u and v are the velocity components along the x - and y -axis, respectively, w is the velocity component in the z direction, ρ is the fluid density, β is the coefficient of thermal expansion, μ is the kinematic viscosity, g is the acceleration due to gravity, c_p is the specific heat at constant pressure, and k is the temperature-dependent thermal conductivity. Following Elbashbeshy and Bazid, we assume that the stretching velocity $U_w(x, t)$ is to be of the following form

$$U_w = \frac{bx}{(1-ct)} \tag{6}$$

where b and c are positive constants with dimension reciprocal time. Here, b is the initial stretching rate, whereas the effective stretching rate $b/(1 - ct)$ is increasing with time. In the context of polymer extrusion, the material properties and in particular the elasticity of the extruded sheet vary with time even though the sheet is being pulled by a constant force. With unsteady stretching, however, c^{-1} becomes the representative time scale of the resulting unsteady boundary layer problem. The surface temperature T_w of the stretching sheet varies with the distance x along the $U_w(x, t)$ sheet and time t in the following form:

$$T_w(x, t) = T_\infty + T_0 \left[\frac{bx^2}{v^*} \right] (1 - ct)^{-3/2} \tag{7}$$

Where T_0 is a (positive or negative; heating or cooling) reference temperature. The governing differential equations (1) – (4) together with the boundary conditions (5) are non-dimensionalised and reduced to a system of ordinary differential equations using the following dimensionless variables:

$$\eta = \left(\frac{b}{v} \right)^{1/2} ((1 - ct))^{-1/2} y, \quad \psi = (vb)^{1/2} ((1 - ct))^{-1/2} x f(\eta), \quad w = bx((1 - ct))^{-1} h(\eta),$$

$$T = T_\infty + T_0 \left[\frac{bx^2}{v} \right] (1 - ct)^{-3/2} \theta(\eta), B^2 = B_0^2 ((1 - ct))^{-1}, Ec = \frac{u^2}{c_w(T_w - T_\infty)} \tag{8}$$

where $\psi(x, y, t)$ is the physical stream function which automatically assures mass conservation (1) and B_0 is constant. We assume the fluid viscosity to vary as an exponential function of temperature in the non dimensional form $\mu = \mu_\infty e^{-\beta_1 \theta}$, where μ_∞ is the constant value of the coefficient of viscosity far away from the sheet, β_1 is the variable viscosity parameter. The variation of thermal diffusivity with the dimensionless temperature is written as $k = k_0(1 + \beta_1 \theta)$ where β_2 is a parameter which depends on the nature of the fluid, k_0 is the value of thermal diffusivity at the temperature T_w . Upon substituting the above transformations into (1)–(4) we obtain the following:

$$f''' - \beta_1 \theta' f'' + e^{\beta_1 \theta} [f f'' - (f')^2 - S(f' + \frac{\eta}{2} f'') - \frac{M^2}{1+m^2} (f' + mh)] = 0 \tag{9}$$

$$h'' - \beta_1 \theta' h' + e^{\beta_1 \theta} [f h' - (h f')] - S(h' + \frac{\eta}{2} h'') + \frac{M^2}{1+m^2} (m f' - h) = 0 \tag{10}$$

$$(1 + \beta_1 \theta) \theta'' + \beta_2 (\theta')^2 + Pr(f \theta' - 2 \theta f') - S(3 \theta + \eta \theta') + Ec[(f')^2 + (f'')^2] = 0 \tag{11}$$

where the primes denote differentiation with respect to η , and the boundary conditions are reduced to

$$f(0) = 0, \quad f'(0) = 1, \quad h(0) = 0, \quad \theta(0) = 1 \tag{12(a)}$$

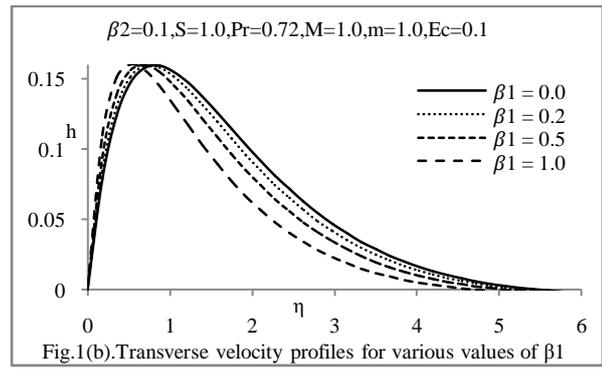
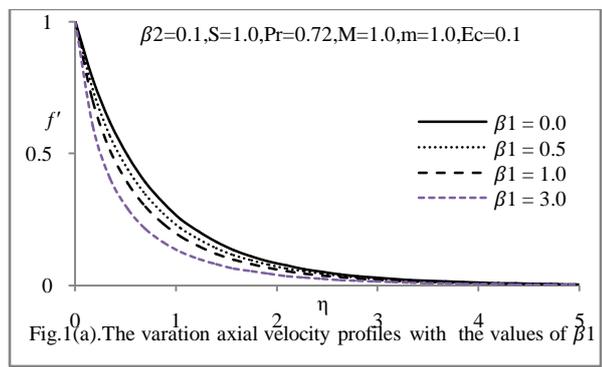
$$h(\infty) = 0, \quad f(\infty) = 0, \quad \theta(\infty) = 0 \tag{12(b)}$$

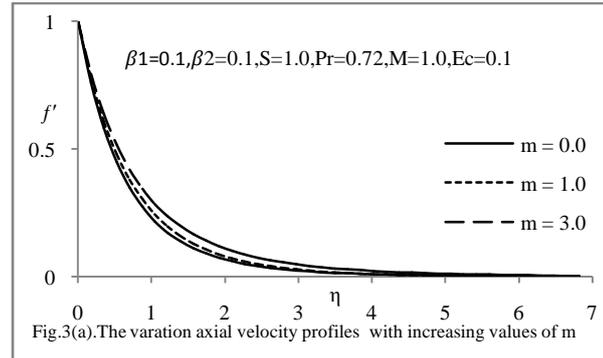
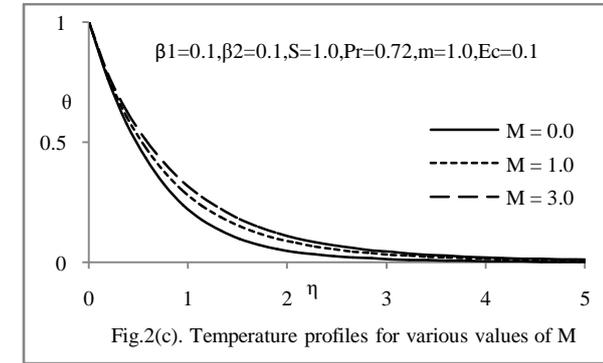
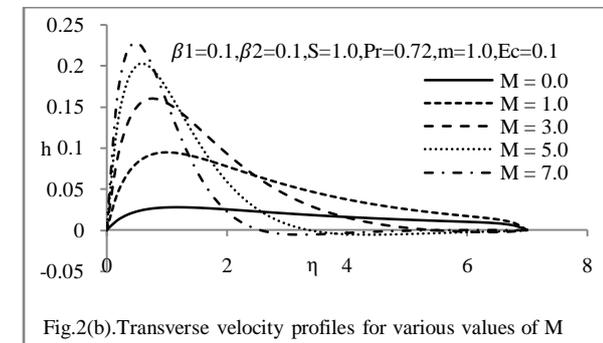
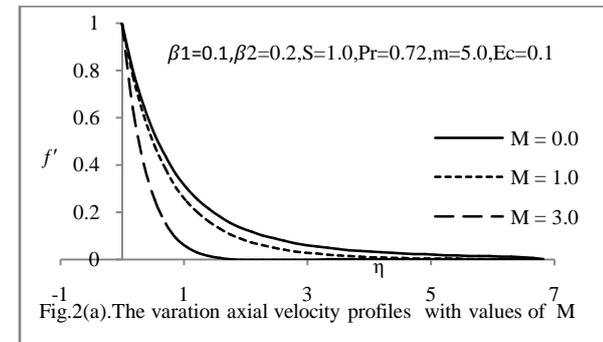
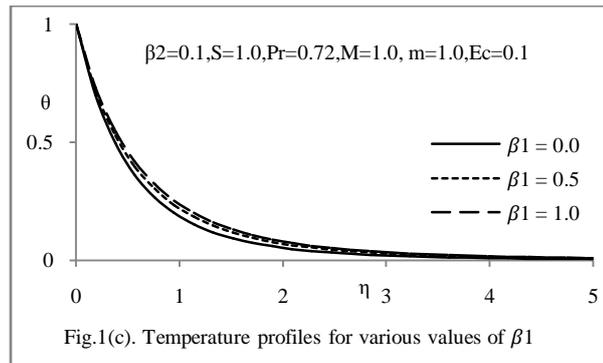
The governing non dimensional equations (9) – (11) along with the boundary conditions 12(a)-12(b) are solve by the using the implicit finite difference scheme using C-programming.

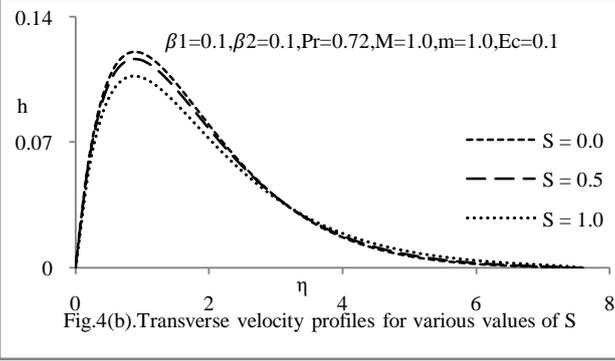
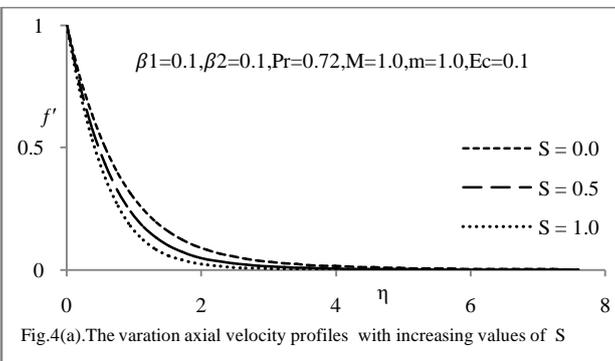
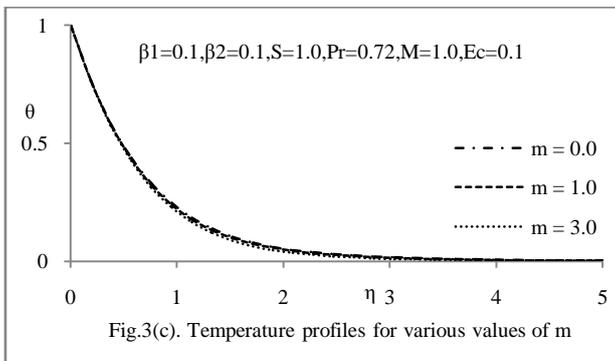
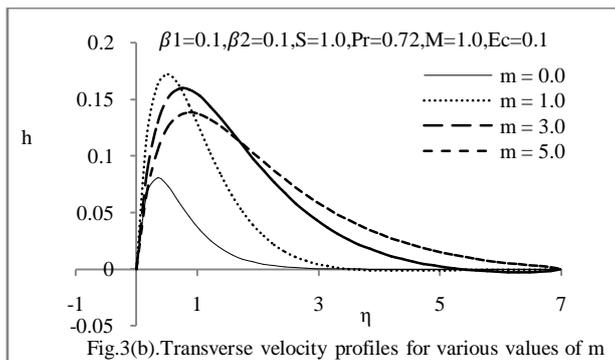
III. Results and discussion

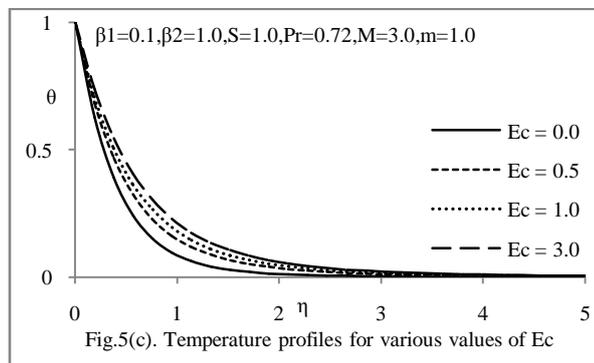
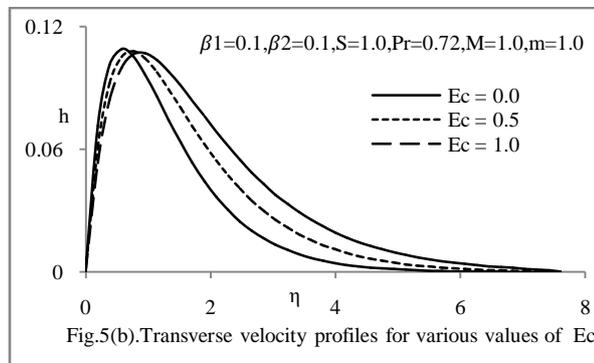
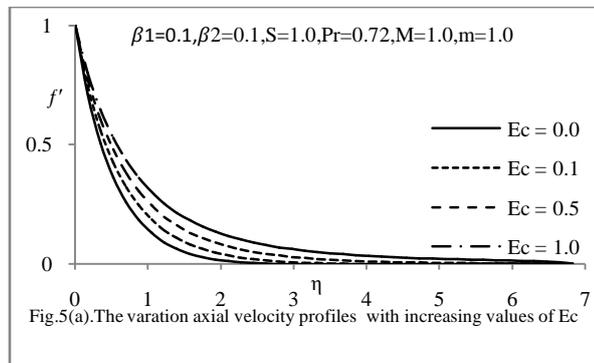
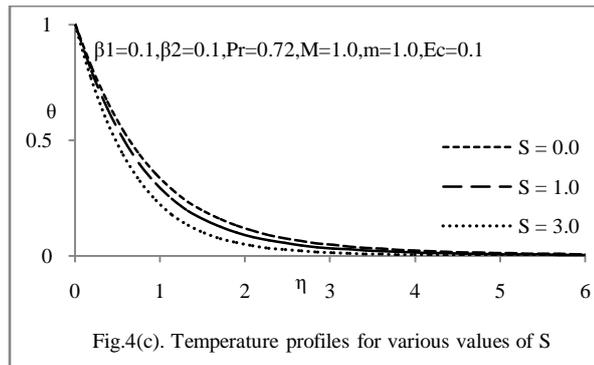
In order to solve the unsteady, non-linear coupled equations 9-11 along with boundary conditions 12 along with boundary conditions 12(a)-12(b) an implicit finite difference scheme of cranck-nicklson type has been employed. The finite difference schemes, the dimensionless governing equations are reduced to try-diagonal system of equations which are solved by Thamos algorithm. The η_{max} chosen as 10 corresponds to $\eta \rightarrow \infty$ after some preliminary investigation so that the last two boundary condations 12 (a), 12(b) are satisfied at $\eta \rightarrow \infty$ with in the tolerance limit of 10^{-5} the mesh size has been fixed as 0.01. The numerical computations are have been carried out for the different governing parameters such as magnetic parameter M , Hall parameter m , Prandtal number Pr , Eckert number Ec , unsteadiness parameter S , viscosity parameter β_1 and diffusivity parameter β_2 only selective figures have been shown here for brevity. In figure 1(a)-1(c) the influence of variable viscosity β_1 on axial velocity, Transverse velocity and temperature profiles respectively are shown. It is observed from the figure 1(a) that the effective of the variable viscosity β_1 is to reduce the axial velocity profiles f' due to the increase of β_1 the boundary layer thickness decreases. It can be seen from the figure 1(b) with the increase of β_1 the transverse velocity increases to a peak value near the boundary wall and then decays rapidly to the relevant free stream velocity. From figure 1(c) it can be conclude that the distribution of temperature $\theta(\eta)$ increases the variable viscosity β_1 increases. Figures 2(a)-2(c) depicts the influence of magnetic field parameter on axial velocity ,transitive velocity and

temperature profiles respectively. From the figure one can find that the axial velocity profiles decreases if the increase of magnetic parameter M , while it can be seen that the temperature profiles increases with the increase of magnetic parameter M . It is obvious that the effect of the magnetic parameter results in a decreasing velocity distribution across the boundary layer. This is due to act that the effect of a transverse magnetic field gives rise to a resistive type force called the Lournitz force. This force has a tendency to flow down the motion of the fluid. From figure 2(b) it can be noticed that the effect of magnetic field increases the transverse velocity filed increases and reaches to a peak value near the vicinity of the boundary layer and approaches to zero. It is also noticed that more influence of magnetic field will reach peak value and reaches to zero near the boundary layer, whereas the less magnetic field influence have less peak value and it will reaches to zero far away from the boundary. Figure 2(c) it can be observed that the effect of magnetic field parameter increases the temperature profiles. The effect of hall current parameter m on axial velocity, transverse velocity and temperature profiles are shown in figure 3(a)-3(c). It is observed that as the hall current parameter increase the transverse velocity profiles increases up to the value of $m=1.5$. the transverse flow in η direction is to increases. However for the values of hall current parameter m greater than 1.5. the transverse velocity profile decreases has these values increases. This is due to fact that for large values of m the term $M/(1+m^2)$ is very small ; and hence the resistive effect of the magnetic field is to diminished. Figures 4(a) to 4(c) illustrates the effect of the unsteadiness parameter S on axial velocity, transverse velocity and temperature profiles respectively. From these figures, it can be seen that the increase of unsteadiness parameter S is to decrease the transverse magnetic field as well as temperature profiles. it also observe from the figure 4(b) that the transverse velocity profiles is to decreases the transverse velocity greatly near the plate and the reverse happen far away from the plate. The effects due to viscous dissipation Ec on axial velocity, transverse velocity and the temperature profiles are shown in plated figures 5(a)-5(c). From these figures reveals that the influence of viscous dissipation effects is to increases the axial velocity profile and temperature profiles. It can be observed from figure 5(b) that the transverse velocity profiles decreases with the increases of Eckert number nearest vicinity of the wall and the reverse phenomenon is observed away from the wall and the effect is high away from the plate. This is due to the fact that the heat energy is stored in the fluid due to the frictional heating. So we can say that the strong frictional heating slow down the cooling processes and in this case the study suggest that the rapidly cooling of the surface can be made possible if the viscous dissipation can be made as small as possible. Figure 6(a) presents typical profile of transverse velocity profiles for the different values of thermal diffusivity parameter β_2 . It can be noticed figure is decreases the transverse velocity profiles. The effect of thermal diffusivity parameter β_2 is to increases the temperature distribution is noticed from figure 6(b). This is due to the ticking of the thermal boundary layer as results of increasing of thermal diffusivity. Figure .7 presents the effects of prandtl number Pr effect on temperature profiles increasing the value of Pr has the tendensive decreases the fluid temperature in the boundary layer as were as the thermal boundary layer thickness.









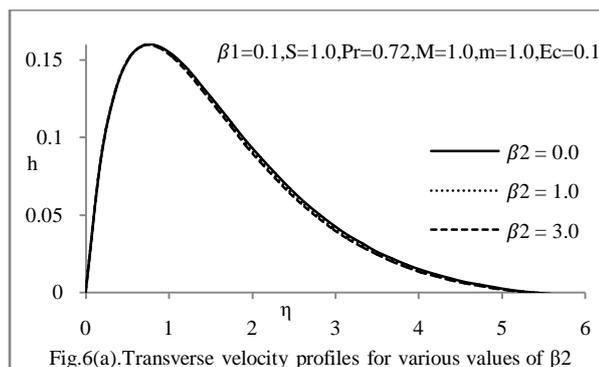
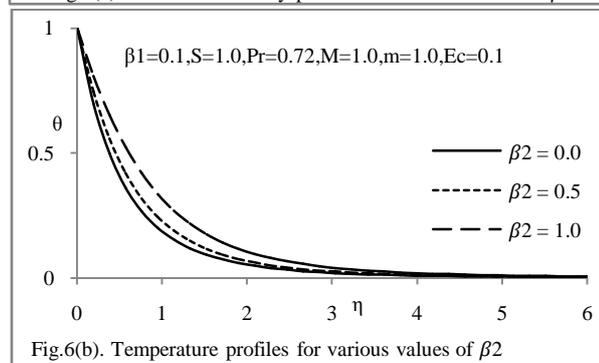
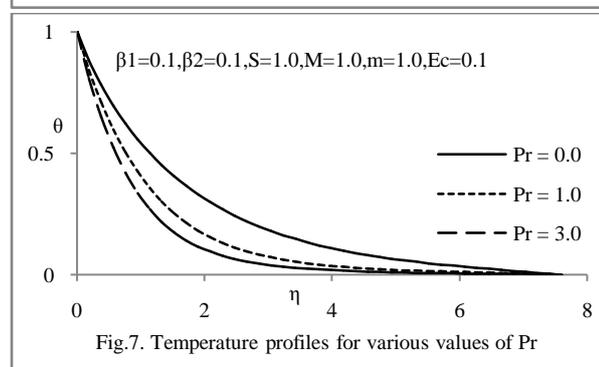
Fig.6(a). Transverse velocity profiles for various values of β_2 Fig.6(b). Temperature profiles for various values of β_2 

Fig.7. Temperature profiles for various values of Pr

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