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Universal Optimality of Circular Neighbor Balanced design under Equal Right and Left Neighbor Effects

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Abstract: *The purpose of this paper is to study the optimality of circular neighbor balanced designs for total effects when equal right and left neighbor effects are present in the model and the observation errors are correlated according to first order circular stationary autoregressive process. Few results pertaining to the optimality conditions under some specified conditions are provided and the efficiencies of circular neighbor balanced designs relative to the optimal continuous block designs are also investigated. The efficiency of the circular neighbor balanced designs is illustrated corresponding to the optimal continuous block designs.*
Key Words: *Auto regressive process, Block Design, Circular, Correlated observation, Equal Neighbor balanced effect, Total effect, Universal optimality*.

One major issue faced in the areas like agricultural trails and horticultural trails is that the treatment applied in one plot shows its impact on the plot to which it was applied as well as on the plot which are neighboring to it. Sometimes only one neighbor plot will undergo such neighboring effect. In few cases, plots that are in the left side as well in the right side of the original plot receive the neighboring effect. For example, in cereal crops or sunflowers, tall varieties may shade the plot on their North side and influence the response of the plot. Sunflowers are traditionally very tall plants. When new, short-stalked varieties were introduced, agricultural research stations wanted to do experiments to compare the new varieties with the old. Similarly, in pesticides or fungicide experiment, part of the treatment may spread to the plot immediately downwind; so may spores from untreated plots. In plants with roots, such as potatoes, varieties which germinate earlier will establish their roots and take nutrients from adjoining plots on both sides if the crop is grown in linear ridges, or on all sides if the crop is grown in a two dimensional area with no gaps. Similar effects are reported on oil seed rape, on field beans, in anti feedants, in forestry, and in horticulture.The detailed discussion can be found in most recent texts on the design of experiments, e.g., in Dey (1986), Pukelsheim(1993), Wu and Hamada (2000) and Box, Hunter and Hunter (2005).

Under the linear models with the neighbor effects, many optimality results of block designs are established for treatment and neighbor effects separately. Hedayat and Afsarinejad (1978), Cheng and Wu (1980), Kunert (1984b) and Kushner (1997) for cross-over designs, Kunert (1984a) and AzaÃ³s, Bailey and Monod(1993), Druilhet (1999) and Filipialk and Markiewicz(2005) were dealt with circular neighbor- balanced designs.

Bailey and Druilhet (2004) pointed out that the effect of most importance is the sum of the direct effect of the treatment and the neighbor effects of the same treatment that is the total effect. Furthermore, they also showed that a circular neighbor-balanced design is universally optimal [in Kiefer's (1975) sense] for total effects under linear models containing the neighbor effects at distance one among the class of all designs with no treatment preceded by itself. Optimality of circular neighbor – balanced designs for total effects with Autoregressive correlated observations was studied by Yun long Yu, MingYao Ai, and Shayuan He (2009).In this paper we study the universal optimality of circular neighbor-balanced designs for total effects, but when the observation errors are correlated according to a first-order circular autoregressive process under the assumption that the left neighbor effects and left neighbor effects are equal.

In this paper, Section 2 deals with some definitions and preliminaries. Section 3 presents the main results that circular neighbor- balanced designs are universally optimal under some conditions for the total effects in linear models which incorporate equal two-sided neighbor effects when the observation errors are correlated according to a first-order circular autoregressive process. In order to discuss the efficiency of circular neighbor-balanced designs among all possible block designs with the same parameters, the optimal continuous block designs are characterized in Section 4. Section 5 presents the efficiency of circular neighbor-balanced designs with blocks of small size, based on the previous structure of optimal equivalence classes of sequences.

I. Model and Definition

In many occasions, it is reasonable to believe that the neighbor effects of each treatment from the left and the right should be the same i.e. $\lambda=\rho$ (Filipiak 2012, Wei Cheng 2014). By assuming this condition, Consider a set of circular block designs $\Omega_{(t,b,k)}$. For a design $d\in\Omega_{(t,b,k)}$, the two-sided Neighbor linear effect additive model with equal right and left neighbor effects can be written in vector form as,

$$Y = 1_{bk}\mu + T_d\tau + (L_d + R_d)\lambda + (I_b \otimes I_k)\beta + \varepsilon \qquad \text{---- (1)}$$

Where,

- $Y = (Y_{11},...,Y_{1k},...,Y_{b1},...,Y_{bk})'$, Y_{ij} is the observation response on plot j of block i,
- μ is the general mean, τ and λare, respectively, the t-dimensional vectors of the direct effects, left-Neighbor effects and right-Neighboreffects of the t treatments,
- T_d, $L_d$and $R_d$are the corresponding incidence matrices,
- β is the b-dimensional vector of the block effects, and
- ε is the vector of random errors,
- 1_n denotesan n-dimensionalvector of ones
- Thesymbol ⊗denotes the Kro- Necker product.

Observations in different blocks are statistically independent while the observations within each block have the following stationary, first-order, autoregressive covariance structure:

$$\operatorname{cov}(Y_{il}, Y_{jl}) = \sigma^2 \frac{\rho^{|i-j|} + \rho^{k-|i-j|}}{1+\rho^k}$$

wherei and j index the positions of the observations in the block and i denotes the block. Because the observations in different blocks are assumed to be independent, C is block-diagonal and so is its inverse. The ij-th element of the inverse of the covariance matrix for any block is given by

$$\frac{1+\rho^k}{\sigma^2(1-\rho^2)(1-\rho^k)}\begin{cases} 1+\rho^2, if i = j \\ \rho, if\ |i-j| = 1 or k-1 \\ 0, otherwise \end{cases} \ldots(2)$$

Also, assume that the errors in each block are correlated according toa first-order circular auto regressive process, denoted by AR(1,C) as in the case ofKunert and Martin (1987), D. Richard Cutler (1990) and Ai M, Yu Y, He S (2009). The AR(1,C) process can be represented in the recursive form $\boldsymbol{\varepsilon_i = \rho\varepsilon_{i-1} + \eta_i}$with $|\rho| < 1$wherethe η_i's are uncorrelated noises with $E(\eta_i)=0$ and Var $(\eta_i)=\sigma^2$, and $E(\varepsilon_0)=0$.Then $E(\varepsilon) = 0$,Cov $(\varepsilon) = \sigma^2 I_b \otimes S$ and hence, we can write the inverse of the variance – covariance matrix as follows(Cutler 1990)

$$S^{-1} = (1+\rho)^2 I_k - \rho(H + H') \qquad \text{----(3)}$$

$$= \begin{bmatrix} 1+\rho^2 & -\rho & 0 & \ldots & 0 & -\rho \\ -\rho & 1+\rho^2 & -\rho & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 1+\rho^2 & -\rho \\ -\rho & 0 & 0 & \ldots & -\rho & 1+\rho^2 \end{bmatrix} \qquad \text{----(4)}$$

Where H is a k x k matrix and is given by,

$$H = \begin{bmatrix} 0 & 0 & 0 & . & 0 & 1 \\ 1 & 0 & 0 & . & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Note that when ρ=0, the structure of errors is reduced to the popular i.i.d. case.

Lemma 1

Let $C_d[\alpha]$ be the information matrix for some effect α based on a design d. Assume that a design $d^* \in D$ has its information matrix completely symmetric,then d* is universally optimal for the effect α over a class D of designs if and onlyif $tr(C_{d*}[\alpha] = \max_{d\in D} tr\{C_d[\alpha]\}$

Let φdenote the total effects of the t treatments in the model(1), that is φ=τ+ρ. Thus, we can obtain the following universal optimality results of CNBD's for the total effects..

Theorem 1

For3≤k≤t,aCNBD (2)in$\Omega_{(t,b,k)}$ is universally optimal for the total effects in the model (1) among all the designs with no treatment Neighbor of itself when0≤ρ<1.,andamongallthedesignswithnotreatment Neighbor ofitself at distance 1or2when−1<ρ<0.

Proof:

For a design, $d \in \Omega_{(t,b,k)}$, the information matrix $C_d[\alpha]$ for the effect α=[τ',λ']' in the model (1) can be expressed as,

$$C_d[\alpha] = (T_d, L_d)'(I_b \otimes S^{-1/2})' pr^{\perp}_{(I_b\otimes S^{-1/2}I_k)}(I_b \otimes S^{-1/2})(T_d, L_d)$$

$$= (C_{d_{ij}})1 \le i, j \le 2,$$

Where the submatrices $(C_{d_{ij}})1 \le i, j \le 2,$ have the forms

$$C_{d_{11}} = T_d'(I_b \otimes \tilde{S})T_d \quad \text{----- (5)}$$

$$C_{d_{12}} = T_d'(I_b \otimes \tilde{S})(L_d + R_d) \quad \text{----- (6)}$$

$$C_{d_{22}} = L_d'(I_b \otimes \tilde{S})L_d \quad \text{----- (7)}$$

And

$$\tilde{S} = (1+\rho^2)I_k - \rho(H+H') - \frac{(1-\rho)^2}{k}1_k1_k'$$

Since S is a cyclic matrix, so $HSH' = H'SH = S$. For a circular design d, $L_{du} = HT_{du}, 1 \le u \le b.$ It implies that $C_{d_{11}} = C_{d_{22}}$.

For a CNBD (2) d*, we have

$$T_{d^*}'(I_b \otimes H)T_{d^*} = T_{d^*}'(I_b \otimes H')T_{d^*} = T_{d^*}'(I_b \otimes H'H')T_{d^*} = \frac{bk}{t(t-1)}(1_t1_t' - I_t)$$

Then,

$$C_{d_{11}^*} = T_{d*}'(I_b \otimes \tilde{S})T_{d^*}$$

$$= (1+\rho^2)T_{d^*}'(I_b \otimes I_k)T_{d^*} - \frac{(1-\rho)^2}{k}T_{d^*}'(I_b \otimes 1_k1_k')T_{d^*} - \rho T_{d^*}'(I_b \otimes H)T_{d^*} - \rho T_{d^*}'(I_b \otimes H')T_{d^*}$$

$$=(1+\rho^2)\frac{bk}{t}I_t - \frac{(1-\rho)^2}{k}[\frac{bk}{t}I_t + \frac{bk(k-1)}{t(t-1)}(1_t1_t' - I_t)] - \frac{2abk}{t(t-1)}(1_t1_t' - I_t)$$

$$= [\frac{(1+\rho^2)bk}{t} - \frac{b(1-\rho)^2(t-k)}{t(t-1)} + \frac{2\rho bk}{t(t-1)}]I_t - [\frac{b(k-1)(1-\rho)^2}{t(t-1)} + \frac{2\rho bk}{t(t-1)}]1_t1_t']$$

Now Consider,

$$C_{d^*_{12}} = T'_{d^*}(I_b \otimes \tilde{S})(L_{d^*} + R_{d^*})$$
$$= T'_{d^*}(I_b \otimes \tilde{S})L_{d^*} + T'_{d^*}(I_b \otimes \tilde{S})R_{d^*}$$
$$= (1+\rho^2)T'_{d^*}(I_b \otimes H)T_{d^*} - \frac{(1-\rho)^2}{k}T'_{d^*}(I_b \otimes 1_k 1'_k)T_{d^*} - \rho T'_{d^*}(I_b \otimes HH)T_{d^*} - \rho T'_{d^*}(I_b \otimes I_k)T_{d^*}$$
$$+ (1+\rho^2)T'_{d^*}(I_b \otimes H')T_{d^*} - \frac{(1+\rho^2)}{k}T'_{d^*}(I_b \otimes 1_k 1'_k)T_{d^*} - \rho T'_{d^*}(I_b \otimes H'H')T_{d^*} - \rho T'_{d^*}(I_b \otimes I_k)T_{d^*}$$
$$= (1+\rho^2)T'_{d^*}(I_b \otimes H)T_{d^*} - 2\frac{(1-\rho)^2}{k}T'_{d^*}(I_b \otimes 1_k 1'_k)T_{d^*} - 2\rho T'_{d^*}(I_b \otimes I_k)T_{d^*} - \rho T'_{d^*}(I_b \otimes HH)T_{d^*}$$
$$- \rho T'_{d^*}(I_b \otimes H'H')T_{d^*} + (1+\rho^2)T'_{d^*}(I_b \otimes H')T_{d^*}$$

Hence all $C_{d^*_{ij}}$ (1≤ i, j≤2) are completely symmetric.

Rewrite φ = K'α with $K = 1_2 \otimes I_t$. It is obvious that K'K=2I$_t$. By Lemma and equation, for any design $d \in \Omega_{(t,b,k)}$,

$$C_d[\phi] \le \frac{1}{4}K'C_d[\alpha]K$$
$$= \frac{1}{4}(C_{d_{11}} + C_{d_{12}} + C_{d_{21}} + C_{d_{22}})$$
$$= \frac{1}{4}[2(1+\rho^2)T'_d(I_b \otimes I_k)T_d - \frac{2(1-\rho)^2}{k}T'_d(I_b \otimes 1_k 1'_k)T_d - 2\rho T'_d(I_b \otimes H)T_d -$$
$$2\rho T'_d(I_b \otimes H')T_d + 2(1+\rho^2)T'_d(I_b \otimes H)T_d - \frac{4(1-\rho)^2}{k}T'_d(I_b \otimes 1_k 1'_k)T_d -$$
$$4\rho T'_d(I_b \otimes I_k)T_d - 2\rho T'_d(I_b \otimes HH)T_d - 2\rho T'_d(I_b \otimes H'H')T_d + 2(1+\rho^2)T'_d(I_b \otimes H')T_d$$
$$= \frac{1}{4}[2(1-\rho)^2 T'_d(I_b \otimes I_k)T_d - \frac{6(1-\rho)^2}{k}T'_d(I_b \otimes 1_k 1'_k)T_d + 2(1+\rho^2-\rho)T'_d(I_b \otimes H)T_d \text{ ---- (8)}$$
$$+ 2(1+\rho^2-\rho)T'_d(I_b \otimes H')T_d - 2\rho T'_d(I_b \otimes HH)T_d - 2\rho T'_d(I_b \otimes H'H')T_d$$

Since $C_{d^*_{ij}}$ (1≤ i, j ≤2) are completely symmetric $C_{d^*_{12}} = C_{d^*_{21}}$. So $C_{d^*}[\alpha]$ commutes with $pr_{(K)} = \frac{1}{2}(1_2 1'_2 \otimes I_t)$. Then

$$C_{d^*}[\phi] = \frac{1}{4}K'C_{d^*}[\alpha]K = \frac{1}{4}(C_{d^*_{11}} + C_{d^*_{12}} + C_{d^*_{21}} + C_{d^*_{11}})$$

And consequently $C_{d^*}[\phi]$ is also completely symmetric. Consider now (8). When -1<ρ<0, for a design d in $\Omega_{t,b,k}$ with no treatment neighbor of itself at distance 1 or 2, the traces of $T'_d(I_b \otimes H)T_d$, $T'_d(I_b \otimes H')T_d$, $T'_d(I_b \otimes HH)T_d$, $T'_d(I_b \otimes H'H')T_d$ are all zero, and tr($T'_d(I_b \otimes I_k)T_d$) is a constant. So tr$\{C_d[\phi]\}$ depends only on tr$T'_d(I_b \otimes 1_k 1'_k)T_d$. Moreover, a CNBD(2) is a balanced block design, so it also minimize tr$T'_d(I_b \otimes 1_k 1'_k)T_d$ among all possible designs of the same size. Therefore tr$\{C_d[\phi]\}$ attains the maximum. When $0 \le \rho < 1$, the traces of both $T'_d(I_b \otimes HH)T_d$, $T'_d(I_b \otimes H'H')T_d$ must be non-negative. However, for a CNBD(2) d*, they are all zero. So for a design with no treatment neighbor of

Therefore $tr\{C_d[\phi]\}$ attains the maximum. When $0 \le \rho \le 1$, the traces of both $T_d^{'}(I_b \otimes HH)T_d$, $T_d^{'}(I_b \otimes H^{'}H^{'})T_d$ must be non-negative. However, for a CNBD(2) d*, they are all zero. So for a design with no treatment neighbor of itself at distance 1, it still holds that $tr\{C_{d^*}[\phi]\} \ge tr\{C_d[\phi]\}$ Hence the theorem follows from the lemma 1.

3. Optimal continuous block designs

In this section we will be discussing the optimality of continuous block designs. The optimal designs among all possible designs with the same parameters are characterized according to the method introduced by Kushner (1997) and Bailey and Druilhet (2004).

For u=1, 2, . . .,b, let T_{du} be the incidence matrix of the direct effects of the treatment in block u, 1≤u≤b.

Then $T_d = (T_{d1},T_{d2},\ldots T_{db})'$ is just the incidence matrix of the direct effects. For each u, define L_{du}=H T_{du}, R_{du}=H'T_{du}. Thus, it is obvious that $L_d = (I_b \otimes H)T_d$ and $R_d = (I_b \otimes H')\,T_d$ are exactly the incidence matrices of the left-Neighbor effects and of the right-Neighboreffects.

Two sequencesoftreatmentsonablockareequivalentifonesequence can be obtained from the other by relabeling the treatments and denote by s the equivalence class of the sequence l on the block u. Because tr(C_{du})are in variant under permutations of treatment labels, so the value tr(C_{du}) remains the same for any sequence in the same equivalence class. Thus, we can define,

$$c(s)=tr\,(C_{du})=\frac{1}{2}\left[2(1-\rho)^2\ k+2(1+\rho^2-\rho)\sum_{i=1}^{t}m_i-4\rho\sum_{i=1}^{t}p_i-\frac{3(1-\rho)^2}{k}\sum_{i=1}^{t}n_i^{\ 2}\right] \quad (8)$$

Where

- n_i is the number of occurrences of treatment i in the sequence l,
- m_i is the number of times treatment i is on the left-hand side of itself in the sequence l
- $p_i$is the number of plots having treatmenti both on the left-hand side and on the right-hand side.

From, Ai M, Yu Y, He S (2009) we have the following propositions.

Proposition 1:

When k = 3 or 4, for any ρ∈(−1,1), a CNBD (2) is universally optimalforthetotaleffectsinthemodel(1)amongallpossibledesignswithequalsize.

Proposition2:

When k ≥ 5, v ≥ 2 and v_1 = 0 or 1 in any optimal sequence.

Proposition 3

When $\rho \in (0.3819,1)$ for any positive integer k ≥ 5, if kisodd, then the optimal sequence has the form of '$a_1a_2a_2a_3a_3\cdots a_{[k/2]}a_{[k/2]}$', while if kiseven, thenthe optimal sequence has the form of '$a_1a_1a_2a_2\cdots a_{[k/2]}a_{[k/2]}$',where$a_1,\ldots,a_{[k/2]}$ are distinct treatments.

PROOF:

If $\sum_{i=1}^{t}p_i$ decreases by one unit, then $\sum_{i=1}^{t}m_i$ decreases definitely by one unit, and correspondingly c(s) will increase by $4\rho-2(\rho^2-\rho+1)$. Also for the value ρ between 0.38191 and 1, the above increment takes the positive value. Thus from the Proposition 3 of Ai M, Yu Y, He S (2009), we have the remainder proof of this theorem.

4. Optimal equivalence classes of sequences:

Using the above propositions now in this section we exhibit the optimal sequences of treatment for some block size.

Let *l*be sequence in an equivalence class. Denote by N_1 and N_2, respectively, the setsoftreatments appearing just once and at least twice in l. Then N=N_1∪ $N_2$is the set of distinct treatments in l. Let v_1 =|N_1|,v_2 = |N_2|andv = |N|, where |N| denotes the cardinality of the set N.

For illustration, under the condition of, $-1 < \rho <$the optimaltreatment sequences for the given parameters $\{v_1,v_2\}$ are listed together with the corresponding tr (C_{du}) for k = 6, 7, 8, …11, respectively. Note that the sequence for aCNBD (2) is also listed in the last row for the convenience of comparison.

Optimal equivalence classes of sequences when k=6

Table 1.Optimal sequencesfor all possible pairs of $\{v,v_1\}$ for k=6

S.No	OPTIMAL SEQUENCE	v	v_1	tr (C_{du})
1	aaabbb	2	0	$1/2(5\rho^2-6\rho+5)$
2	abbbbb	2	1	$1/2(\rho^2-1)$
3	aabbcc	3	0	$1/3(\rho^2-\rho+1)$
4	abbccc	3	0	$1/2(5\rho^2-6\rho+5)$
5	abcdef	6	0	$3/2(\rho^2-2\rho+1)$

Among the above sequences, the sequence "aabbcc" is the optimal sequence by Proposition 3.

The below table represents all the optimal sequences for $6 \le k \le 11$. Also Note that the below table shows the optimal sequence and the last column lists the values tr(C_{du}) of a CNBD (2) d.

Block Size	Optimal sequence	c (s*)	tr(C_{du})
6	aabbcc	$1/3(\rho^2-\rho+1)$	$3/2\ (\rho^2-2\rho+1)$
7	aabbbccc abbccdd	$1/7(27\rho^2-33\rho+27)$ $1/7(26\rho^2-31\rho+26)$	$2(\rho^2-2\rho+1)$
8	aabbccdd	$5\rho^2-6\rho+5$	$5/2(\rho^2-2\rho+1)$
9	aaabbbccc aabbccddd abbccddee	$6\rho^2-9\rho+6$ $1/9(15\rho2-11\rho+15)$ $1/9(51\rho^2-66\rho+51)$	$3(\rho^2-2\rho+1)$
10	aabbbcccddd aabbccddee	$1/10(71\rho^2-102\rho+71)$ $7\rho^2-9\rho+7$	$7/2(\rho^2-2\rho+1)$
11	aabbbcccddd aabbccddeee abbccddeeff	$1/11(99\rho^2-144\rho+99)$ $1/11(89\rho^2-123\rho+89)$ $1/11(84\rho^2-113\rho+84)$	$4(\rho^2-2\rho+1)$

5. Efficiency of CNBD (2) corresponding to optimal continuous designs:

Now let us discuss and calculate the efficiency of CNBD(2) corresponding to the optimal continuous block design for various block size.

The optimal equivalence class of sequence s* is obtained by making use of Kunert and Martin (2000b). i.e the optimal sequence is the one among all possible sequences, which maximizes c(s) in (3). It was shown in Theorem 10 of Bailey and Druilhet (2004) that a design$d*$which has each sequence in $s*$equally often is universally optimal among all possible designs with the same size. Since the values t r(C_{du}) are invariant to any block u for aCNBD (2), so we can define the efficiency of aCNBD (2) d relative to the optimal continuous block design $d*$as

$$\mathrm{Eff(d)} = \frac{\mathrm{tr(C_d)}}{\mathrm{tr(C_{d*})}} = \frac{\mathrm{tr(C_{du})}}{\mathrm{c(s^*)}}$$

The below tables show the calculations of tr(C_{du}) and c(s*).

Efficiency of CNBD (2) when the block size k=6

Table 10 Efficiency of CNBD (2) when k=6

S.No	V	c(S*)	tr(Cdu)	Eff(d)
1	-1	9.00	6.00	0.6667
2	-0.8	7.32	4.86	0.6639
3	-0.6	5.88	3.84	0.6531
4	-0.4	4.68	2.94	0.6282
5	-0.2	3.72	2.16	0.5806
6	0	3.00	1.50	0.5000
7	0.2	2.52	0.96	0.3810
8	0.4	2.28	0.54	0.2368
9	0.6	2.28	0.24	0.1053
10	0.8	2.52	0.06	0.0238
11	1	3.00	0.00	0.0000

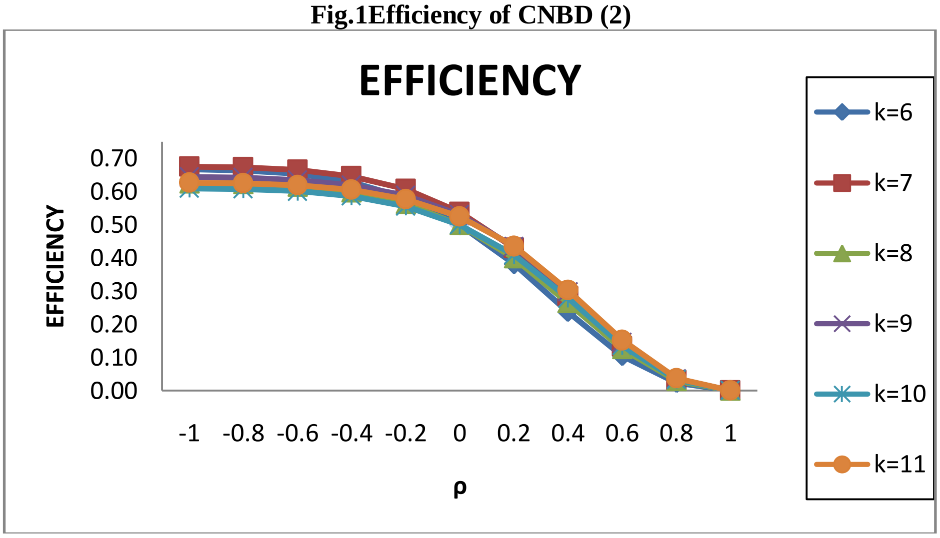
From the above table it is evident that the efficiency of a CNBD (2) approaches to 0 as ρ tends to 1 for k= 6.

The Efficiency of CNBD (2) d for ρbelongs to (-1, 1) are given in the below table for different blocksize.

ρ	Block Size					
	6	**7**	**8**	**9**	**10**	**11**
-1	0.6667	0.6747	0.6250	0.6429	0.6087	0.6263
-0.8	0.6639	0.6726	0.6231	0.6412	0.6071	0.6248
-0.6	0.6531	0.6642	0.6154	0.6344	0.6005	0.6188
-0.4	0.6282	0.6447	0.5976	0.6185	0.5853	0.6046
-0.2	0.5806	0.6065	0.5625	0.5870	0.5551	0.5762
0	0.5000	0.5385	0.5000	0.5294	0.5000	0.5238
0.2	0.3810	0.4299	0.4000	0.4337	0.4088	0.4348
0.4	0.2368	0.2838	0.2647	0.2967	0.2788	0.3032
0.6	0.1053	0.1337	0.1250	0.1452	0.1359	0.1516
0.8	0.0238	0.0314	0.0294	0.0350	0.0327	0.0372
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

The following figure shows the relationship between the efficiency Eff(d) of a CNBD (2) d and v for block size, 6≤ k ≤11. It can be seen that the efficiency of a CNBD (2) approaches to 0 as ρ tends to 1 for any k.

Fig.1Efficiency of CNBD (2)



II. Summary And Conclusion

In this research paper, the optimality and efficiency of circular Neighbor balanced design when the neighbor effects from left side and right side are equal have been investigated.We also have constructed the efficiency of circular neighbor balanced designs among all possible block designs with the same parameters. For different block size, the optimal continuous blocks are derived, and the efficiencies of circular neighbor balanced designs with blocks of small size k ≤ 11 are illustrated. Also the value of the correlation co – efficient

From Fig 1, we could see that the efficiency of CNBD (2) approaches 1 as ρtends to -1 for block sizes k= 6,…,11. or the efficiency is getting decreased and tends to zero as the ρ value increases to one. So we can conclude that the Circular neighbor balanced design is an efficient design.

Thus we can conclude that CNBD (2) is always a good choice when the adjacent observation errors have strong negative correlation when the left and right neighbor effects are equal.

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