One - Machine Scheduling Problem with No Common Due Dates  
Under Fuzzy Environment

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Abstract: In non-preemptive single machine scheduling problems, processing of a job cannot be temporarily interrupted. In classic scheduling, it was assumed that the machines are always ready to work. So, we need not considered maintenance time for calculations. Single machine, no common due dates, earliness /lateness machine problem and fuzzy environment closely models the situation faced by 'Just in Time' manufacturing. In this paper, we propose more than one jobs to be processed on Single Machine Scheduling Problem (SMSP) involving fuzzy processing time and fuzzy due dates. Result shows that the developed average high ranking method performs well to minimize the total penalty with single machine system.

Keywords: no common due dates, Earliness/Tardiness job, Fuzzy environment, Single machine, etc

I. INTRODUCTION

The problem one-machine scheduling problem with no common due dates under fuzzy environment is one of the most vital issues in service systems and industrial companies. Now a days, production field have started laying emphasis on scheduling products with the objective to maximize the industrial production and minimize the total penalty in non-deterministic time. Since we considered the environment as fuzzy because the environment in modern society is uncertainty. Here Time considered is in three different situations (A, B, C) where A – favorable condition, B- normal condition and C- against conditions. The mean tardiness criterion, there is a standard way of measuring conformance to due dates, although it neglects the consequences of jobs completing early. However, this emphasis has changed with the current interest in ‘Just In Time’ (JIT) production, which espouses the notation that earliness as well as lateness, should be dejected.

Baker and Scudder (1990) [1] studied sequences with earliness and tardiness penalties In a JIT scheduling environment, jobs that complete early must be held in finished goods inventory until their due date, while jobs that complete after their due date may cause a customer to shut down operations. Therefore, an optimal schedule is one in which all jobs finish on their assigned due dates. This can be translated to a scheduling objective in several ways. The most obvious objective is to minimize the deviation of job completion time around these due dates in non deterministic time. The concept of penalizing both earliness and tardiness has spawned a new and rapidly developing line of research in the scheduling field. Because the use of both earliness and tardiness penalties in fuzzy environment give rise to a non-regular performance measure, it has led to new methodological issues in the design of solution procedures. This paper presents a special case of Early/Tardy (E/T) having distinct due dates (DDD) problem, when the earliness and tardiness are penalized at the rates fixed by demand maker for the jobs. The next sections introduce the concept of single machine and the processing time of the jobs in fuzzy environment. The average high ranking and the scheduling of some small systems are determined in the section after. An algorithm based on these arguments is developed here and it is justified by a numerical example.

II. CONCEPT OF ONE MACHINE

In this competitive world FMS machineries are highly expensive to be installed and the technologies are advanced and upgrading as fast as it moves the machineries get outdated in order to satisfy that this concept of single machine scheduling with no common due dates under fuzzy environment by reducing the idle cost, expenditure, timing, maintenance, manpower, resources as a result it increases the rate of production, profit. Contractor wishes to process the work on single machine using an intelligent scheduling system and for the small systems single machine maximizes the profit of whole the project.

III. CONCEPT OF FUZZY ENVIRONMENT TIME

The processing time of a job can vary in many ways, may be due to environmental factor or due to the different work places. We find that when a contractor takes the work from a department, he calculates total expenditure at the time of allotment. But due to many factors like non available of labor, weather not favorable, or sometimes abnormal conditions, cost may vary. Hence due to these reasons work can be completed late and creates due date problem i.e. order can’t be delivered on time, on the other hand if the work completes before...
the due time it arises the inventory problem. So to overcome these factors, the processing time of a job considered here is in three situations- favorable, Normal and worse conditions. In this paper, a new concept of different processing time of each job is considered which helps the contractor to estimate the cost of the work at the time of allotment. In this paper, different due dates for each of the job be considered which meets the demand maker with more satisfaction level. So using the algorithm developed here, contractor can save the penalty cost and can satisfy the demand maker to great extent.

**IV. Measures Taken For Single Machine Scheduling Process:**

The different measures of performance of the single machine scheduling problem with independent jobs are as listed below.

- Minimizing the mean flow time
- Minimizing the maximum lateness
- Minimizing the total tardiness
- Minimizing the number of tardy jobs

**V. Formulation Of Fuzzy Processing Time**

By differing with earlier work in the sense that by using fuzzy environment time of \((A, B, C)\), which is real time situation and is defuzzified by the **Average High Ranking Method**

\[
<AHR> = \frac{3B + (C - A)}{3}
\]


Thus the due date is directly related to the earliness and tardiness penalty in conventional scheduling problems. In this paper, different due dates for each of the jobs is considered. Next jobs are scheduled in increasing order of their slack time.

This paper investigates a different approach to single machine under fuzzy environment with bi-objective criteria. On one side it minimizes the penalty cost of the tardy jobs and on the other side it minimizes the total flow time of all the jobs.

**VI. Assumption And Notation**

The machine scheduling problem studied in this paper requires \(n\) independent jobs \(J_i \ (i= 1,2,3,\ldots, n)\) to be processed on a single machine with the following assumptions:

- i) Only one job can be processed on a given machine at a time
- ii) All jobs are available for processing at time zero.
- iii) One- machine can process at most one job at a time.
- iv) No pre-emption is allowed.

Let \(S\) Schedule for the \(n\) jobs. \(<A, B, C>\) Processing time of job \(i\) on the machine in fuzzy environment.

\(A_i\) Average high ranking of the processing time \(<A, B, C> = \frac{3B + (C - A)}{3}\).

- \(d_i\) Due date for the job \(i\).
- \(c_i\) Completion time of job \(i\)
- \(T_i\) Max. \((0, c_i - d_i)\)
- \(E_i\) Max. \((0, d_i - c_i)\)
- \(S_i\) slack time of job \(i\)
- \(e_i\) penalty per unit time for the earliness of job \(i\).
- \(l_i\) penalty per unit time for the tardiness of job \(i\).
An important special case in the family of E/T problems involves minimizing the sum of absolute deviations of job completion time form a DDD having processing time in fuzzy environment.

In particular, the objective function can be written as

\[ f(s) = \sum |c_i - d_i| = \sum (E_i + T_i) \]

When we write the objective function in this form, it is clear that earliness and tardiness are penalized at the rate \(e_i\) and \(l_i\) for all the jobs. In this paper, processing time of the jobs considered is in triangular fuzzy environment.

**Algorithm**

Step 1. Find average high ranking \(<\text{AHR}>\) of the fuzzy processing time \((A,B,C)\) of all the jobs.

Step 2. Find the slack time of all the jobs \(S = |A_i - d_i|\)

Step 3. Arrange the jobs in increasing order of their slack time. If two jobs have the same slack time then considers the jobs of lowest processing time at the earlier position.

Step 4. Using the sequence obtained in step 3 find the total penalty of all the jobs using earliness \((e_i)\) and lateness \((l_i)\) penalty cost.

**VII. Numerical Example**

Table 8.1 shows a 10-jobs having fuzzy processing time, single machine and distinct due dates. Penalty cost \((e_i)\) for earliness and \((l_i)\) lateness is also given.

<table>
<thead>
<tr>
<th>Job</th>
<th>(P_i)</th>
<th>(&lt;\text{AHR}&gt;)</th>
<th>(d)</th>
<th>(S_i)</th>
<th>(e_i)</th>
<th>(l_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3,4,5</td>
<td>14/3</td>
<td>5</td>
<td>1/3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>8,10,12</td>
<td>34/3</td>
<td>7</td>
<td>13/3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>5,6,8</td>
<td>7</td>
<td>10</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>8,9,11</td>
<td>10</td>
<td>12</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>11,12,15</td>
<td>40/3</td>
<td>8</td>
<td>16/3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>7,9,11</td>
<td>31/3</td>
<td>15</td>
<td>14/3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>5,7,9</td>
<td>25/3</td>
<td>17</td>
<td>26/3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>12,14,15</td>
<td>15</td>
<td>10</td>
<td>5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>5,7,8</td>
<td>8</td>
<td>6</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>6,8,10</td>
<td>28/3</td>
<td>14</td>
<td>14/3</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 8.1

10 jobs having fuzzy processing time \((A,B,C)\) are converted average high ranking by using \(<\text{AHR}> = [3B+ (C-A)]/3\). and as per algorithm mentioned in section 6 the optimal sequence is 1-9-4-3-2-10-6-8-5-7. The table 8.2 shows the total flow time of the system and the total optimized penalty cost due to earliness/tardiness of the jobs.

<table>
<thead>
<tr>
<th>Job</th>
<th>(P_i)</th>
<th>(d)</th>
<th>(S_i)</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-14/3</td>
<td>5</td>
<td>1/3</td>
<td>1/3*2</td>
</tr>
<tr>
<td>9</td>
<td>14-16</td>
<td>6</td>
<td>10</td>
<td>10*3</td>
</tr>
<tr>
<td>4</td>
<td>16-23</td>
<td>12</td>
<td>11</td>
<td>11*3</td>
</tr>
<tr>
<td>3</td>
<td>23-33</td>
<td>10</td>
<td>23</td>
<td>23*3</td>
</tr>
<tr>
<td>2</td>
<td>33-139/3</td>
<td>7</td>
<td>118/3</td>
<td>118/3*3</td>
</tr>
<tr>
<td>10</td>
<td>139/3-170/3</td>
<td>14</td>
<td>129/3</td>
<td>129/3*3</td>
</tr>
<tr>
<td>6</td>
<td>170/3-65</td>
<td>15</td>
<td>50</td>
<td>50*3</td>
</tr>
<tr>
<td>8</td>
<td>65-80</td>
<td>10</td>
<td>70</td>
<td>70*3</td>
</tr>
<tr>
<td>5</td>
<td>80-88</td>
<td>8</td>
<td>80</td>
<td>80*3</td>
</tr>
<tr>
<td>7</td>
<td>88-292/3</td>
<td>17</td>
<td>241/3</td>
<td>241/3*3</td>
</tr>
</tbody>
</table>

Table 8.2
VIII. Conclusion

The objective was to find an optimal scheduling time for the small machineries and also satisfies the demand in the market world.

This paper concludes that after using this one machine with no common due dates in fuzzy environment time which reduces the cost functioning of earliness and lateness with penalty cost. As a result this method gives the optimal solution for the single machine with distinct due dates.

References