Semi-symmetry type Sasakian manifolds

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Abstract: Recently the present author introduced the notion of generalized quasi-conformal curvature tensor which bridges Conformal curvature tensor, Concircular curvature tensor, Projective curvature tensor and Conharmonic curvature tensor. This article attempts to characterize Sasakian manifolds with \(\omega(X,Y)\cdot W = 0\). Based on this curvature conditions, we obtained and tabled the expression for the Ricci tensor for the respective semi-symmetry type Sasakian manifolds.

I. Introduction

In tune with Yano and Sawaki [9], recently the present authors [6] have defined and studied \textit{generalized quasi-conformal curvature tensor} \(W\), in the context of \(N(k,\mu)\)-manifold. The beauty of \textit{generalized quasi-conformal curvature tensor} lies in the fact that it has the flavour of Riemann curvature tensor \(R\), conformal curvature tensor \(C\) [10], conharmonic curvature tensor \(\hat{C}\) [15], concircular curvature tensor \(E\) [8], p.84, projective curvature tensor \(P\) [18], p.84 and \(m\)-projective curvature tensor \(H\) [5] as special cases.

An Sasakian manifold is said to be semi-symmetry type (respectively Ricci semi-symmetry type) if the \textit{generalized quasi-conformal curvature tensor} \(W\) (respectively Ricci tensor \(S\)) admits the condition

\[ \omega(X,Y)\cdot W = 0 \quad \text{(respectively \(W(X,Y)\cdot S = 0\))}, \]

for any \(X, Y\) on \(M\),

\(1\)

where the dot means that \(\omega(X,Y)\) acts on \(W\) (respectively on \(S\)) as derivation. Here \(\omega\) and \(W\) stand for \textit{generalized quasi-conformal curvature tensor} with the associated scalar triples \((\vec{a}, \vec{b}, \vec{c})\) and \((a, b, c)\) respectively. In particular, manifold satisfying the condition \(R(X,Y)\cdot R = 0\) (obtained from 1 by setting \(\vec{a} = \vec{b} = \vec{c} = 0 = a = b = c\)) is said to be semi-symmetric in the sense of Cartan ([4], P- 265 and named by N. S. Sinjukov [12]). A full classification of such space is given by Z. I. Szabó ([16]). This type of the manifolds have been studied by several authors such as Papantoniou [11], Perrone [3] and the references therein.

Our paper is structured as follows. Section 2 is a very brief account of Sasakian manifolds. Definition and some basic results of the \textit{generalized quasi-conformal curvature tensor} are discussed in section 3. In section 4, we investigate Sasakian manifolds with \(\omega(X,Y)\cdot W = 0\). Based on this curvature condition, we obtained and tabled the expressions for the Ricci tensor.

II. Sasakian manifolds

Let \(M^{2n+1}(\phi, \xi, \eta, g)\) be a Sasakian manifold with the structure \((\phi, \xi, \eta, g)\).

Then the following relation hold[2]:

\[ \eta(X) = g(X, \xi), \quad \phi^2 = -I + \eta \circ \xi, \quad (2) \]

\[ \eta(\xi) = 1, \quad \eta \circ \phi = 0, \quad (3) \]

\[ \nabla_X \xi = -\phi X, \quad (4) \]

\[ R(\xi, X)Y = (\nabla_X \phi)(Y) = g(X, Y)\xi - \eta(Y)X, \quad (5) \]

\[ g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad (6) \]
\[(\nabla \phi \eta)(Y) = -g(\phi X, Y), \quad (7)\]

\[R(X, Y)\xi = \eta(Y)X - \eta(X)Y, \quad (8)\]

\[\eta(R(X, Y)Z) = [g(Y, Z)\eta(X) - g(X, Z)\eta(Y)], \quad (9)\]

\[S(\phi X, \phi Y) = S(X, Y) - 2n\eta(X)\eta(Y), \quad (10)\]

\[S(X, \xi) = 2n\eta(X), \quad (11)\]

for any vector fields \(X, Y\) on \(M\), where \(\nabla\) denotes the operator of covariant differentiation with respect to \(g\), \(\phi\) is a skew-symmetric tensor field of type \((1,1)\), \(S\) is the Ricci tensor of type \((0,2)\) and \(R\) is the Riemannian curvature tensor of the manifold.

### III. The generalized quasi-conformal curvature tensor

The generalized quasi-conformal curvature tensor is defined as

\[W(X, Y)Z = \frac{2n-1}{2n+1}[(1+2na-b)C(X, Y)Z + [1-b+2na]\xi(X, Y) + 2n(b-a)P(X, Y)Z + \frac{2n-1}{2n+1}(c-1)\xi(X, Y)Z. \quad (12)\]

for all \(X, Y \in \mathcal{X}(M)\), the set of all vector field of the manifold \(M\), where \(a, b, c\) are real constants. And \(C, E, P\) and \(\hat{C}\) stand for Conformal, Concircular, Projective and Conharmonic curvature tensor respectively. These curvature tensor are defined as follows

\[C(X, Y) = R(X, Y) - \frac{1}{2n-1}[(X \wedge g QY) + (QX \wedge g Y)] + \frac{r}{2n(2n-1)}[X \wedge g Y]. \quad (13)\]

\[E(X, Y) = R(X, Y) - \frac{r}{2n(2n+1)}[X \wedge g Y]; \quad (14)\]

\[P(X, Y) = R(X, Y) - \frac{1}{2n}[X \wedge g QY]; \quad (15)\]

\[\hat{C}(X, Y) = R(X, Y) - \frac{1}{2n-1}[(X \wedge g QY) + (QX \wedge g Y)] \quad (16)\]

for all \(X, Y \in \mathcal{X}(M)\), where \(R, S, Q\) \& \(r\) being Christoffel Riemannian curvature tensor, Ricci tensor, Ricci operator and scalar curvature respectively.

In particular, the generalized quasi-conformal curvature tensor \(W\) induced to be

1. Riemann curvature tensor \(R\), if \(a = b = c = 0\),

2. conformal curvature tensor \(C\), if \(a = b = -\frac{1}{2n-1}, c = 1\),

3. conharmonic curvature tensor \(\hat{C}\), if \(a = b = -\frac{1}{2n-1}, c = 0\),

4. concircular curvature tensor \(E\), if \(a = b = 0\) and \(c = 1\),

5. projective curvature tensor \(P\), if \(a = -\frac{1}{2n}, b = 0, c = 0\) and
(6) \( m \)-projective curvature tensor \( H \), if \( a = b = -\frac{1}{4n}, \ c = 0 \).

The \( m \)-projective curvature tensor is introduced by G. P. Pokhariyal & R. S. Mishra [5]. Which is defined as follows

\[
H(X, Y)Z = R(X, Y) - \frac{1}{4n} [S(Y, Z)X - S(X, Z)Y] \\
+ g(Y, Z)QX - g(X, Z)QY. \tag{17}
\]

Using (13), (14), (15) and (16) in (12), the \textit{generalized quasi-conformal curvature tensor} \( W \) becomes

\[
W(X, Y)Z = R(X, Y)Z + a[S(Y, Z)X - S(X, Z)Y] \\
+ b[g(Y, Z)QX - g(X, Z)QY] \\
- \frac{cr}{2n + 1} \left( \frac{1}{2n} + a + b \right) [g(Y, Z)X - g(X, Z)Y]. \tag{18}
\]

### IV. Sasakian manifolds with semi-symmetry type curvature condition

**Definition 4.1** A \((2n + 1)\)-dimensional \((n > 1)\) Sasakian manifold is said to be semi-symmetry type if the condition \( \omega (X, Y) \cdot W = 0 \) holds, for any vector fields \( X, \ Y \) on the manifold and \( \omega(X, Y) \) acts on \( W \) as derivation, where \( \omega \) and \( W \) stand for generalized quasi-conformal curvature tensor with the associated scalar triples \((\tilde{a}, \tilde{b}, \tilde{c})\) and \((a, b, c)\) respectively.

Now, let us consider a \((2n + 1)\)-dimensional Sasakian manifold \( M \), satisfying the condition

\[
(\omega(X, Y) \cdot W)(Z, U)V = 0. \tag{19}
\]

i.e. \( \omega(X, Y)W(Z, U)V = W(\omega(X, Y)Z, U)V + W(Z, \omega(X, Y)U)V \)

\[
+ W(Z, U)\omega(X, Y)V \tag{20}
\]

which is equivalent to

\[
g(\omega(\xi, X)W(Y, Z)U, \xi) - g(W(\omega(\xi, X)Y, Z)U, \xi) \\
- g(W(Y, \omega(\xi, X)Z)U, \xi) - g(W(Y, Z)\omega(\xi, X)U, \xi) = 0. \tag{21}
\]

Putting \( X = Y = e_i \) in (21) where \( \{e_1, e_2, e_3, ..., e_{2n}, e_{2n+1} = \xi\} \) is an orthonormal basis of the tangent space at each point of the manifold \( M \) and taking the summation over \( i, \ 1 \leq i \leq 2n + 1 \), we get

\[
\sum_{i=1}^{2n+1} [g(\omega(\xi, e_i)W(e_i, Z)U, \xi) - g(W(\omega(\xi, e_i)e_i, Z)U, \xi)] \\
- g(W(e_i, \omega(\xi, e_i)Z)U, \xi) - g(W(e_i, Z)\omega(\xi, e_i)U, \xi)] = 0. \tag{22}
\]

From the equation (18), we can easily bring out the followings

\[
\eta(W(\xi, Z)U) \\
= \left[ \frac{cr}{2n + 1} \left( \frac{1}{2n} + a + b \right) - 2n(a - b) - 1 \right] \eta(Z) \eta(U) \\
+ \left[ 1 + 2nb - \frac{cr}{2n + 1} \left( \frac{1}{2n} + a + b \right) \right] g(Z, U) + aS(Z, U). \tag{23}
\]

\[
\sum_{i=1}^{2n+1} W(e_i, Z, U, e_i) \\
= (1 - b + 2na)S(Z, U) + \left[ br - \frac{2ncr}{2n + 1} \left( \frac{1}{2n} + a + b \right) \right] g(Z, U). \tag{24}
\]
\[
\sum_{i=1}^{2n+1} \eta(W(e_i, Z)e_i) \\
= \left[ -2n(1-a+2nb) - \left\{ a - \frac{2ncr}{2n+1} \left( \frac{1}{2n} + a + b \right) \right\} \right] \eta(Z). \tag{25}
\]

\[
\sum_{i=1}^{2n+1} S(W(e_i, Z)U, e_i) \\
= \left\{ ar + \frac{cr}{2n+1} \left( \frac{1}{2n} + a + b \right) \right\} S(Z, U) - \left( a + b - 1 \right) S^2(Z, U) \\
+ \left\{ b \|Q\|^2 - \frac{cr^2}{2n+1} \left( \frac{1}{2n} + a + b \right) \right\} g(Z, U). \tag{26}
\]

\[
\sum_{i=1}^{2n+1} \eta(e_i) \eta(W(Qe_i, Z)U) \\
= 2n \left[ 1 + 2nb - \frac{cr}{2n+1} \left( \frac{1}{2n} + a + b \right) \right] g(Z, U) + 2naS(Z, U) \\
- 2n \left[ 1 + 2n(a + b) - \frac{cr}{2n+1} \left( \frac{1}{2n} + a + b \right) \right] \eta(Z) \eta(U). \tag{27}
\]

\[
\sum_{i=1}^{2n+1} S(e_i, Z) \eta(W(e_i, \xi)U) \\
= 2n \left[ 1 + 2n(a + b) - \frac{cr}{2n+1} \left( \frac{1}{2n} + a + b \right) \right] \eta(Z) \eta(U) \\
- \left( 1 + 2nb - \frac{cr}{2n+1} \left( \frac{1}{2n} + a + b \right) \right) S(Z, U) - aS^2(Z, U). \tag{28}
\]

Now, \[
\sum_{i=1}^{2n+1} g(\omega(\xi, e_i)W(e_i, Z)U, \xi) \\
= \left[ 1 + 2n \tilde{b} - \frac{\tilde{cr}}{2n+1} \left( \frac{1}{2n} + \tilde{a} + \tilde{b} \right) \right] W(e_i, Z, U, e_i) + \tilde{a}S(W(e_i, Z)U, e_i) \\
+ \left[ \frac{\tilde{cr}}{2n+1} \left( \frac{1}{2n} + \tilde{a} + \tilde{b} \right) - 2n \tilde{b} - 1 - 2n \tilde{a} \right] \eta(W(\xi, U)Z). \tag{29}
\]

In view of (24) & (26), (29) becomes
\[
g(\omega(\xi, e_i)W(e_i, Z)U, \xi) \\
= \left[ 1 + 2n\tilde{b} - \frac{\tilde{cr}}{2n+1} \left( \frac{1}{2n} + \tilde{a} + \tilde{b} \right) \right] (1 + 2na - b) \\
+ \tilde{a} \left\{ a r + \frac{cr}{2n+1} \left( \frac{1}{2n} + a + b \right) \right\} S(Z, U) + \tilde{a}(1 - a - b)S^2(Z, U)
\]
\[ + \left\{ \frac{\bar{c}r}{2n+1}\left( \frac{1}{2n} + \bar{a} + \bar{b} \right) - 2n(\bar{a} + \bar{b}) - 1 \right\} \eta(\mathcal{W}(\xi, U)Z) \]
\[ + \left[ \frac{1 + 2n\bar{b}}{2n + 1} - \frac{\bar{c}r}{2n + 1}\left( \frac{1}{2n} + \bar{a} + \bar{b} \right) \right] \left\{ br - \frac{2n\bar{c}r}{2n + 1}\left( \frac{1}{2n} + a + b \right) \right\} \]
\[ + \frac{\alpha}{2n + 1}\left( \frac{1}{2n} + a + b \right) \right\} g(Z, U). \quad (30) \]

In consequence of (23)-(28), we obtain the followings
\[ \sum_{i=1}^{2n+1} g(\mathcal{W}(\omega(\xi, e_i) Z) U, \xi) \]
\[ = \left\{ 2n \left[ 1 + 2n\bar{b} - \frac{\bar{c}r}{2n + 1}\left( \frac{1}{2n} + \bar{a} + \bar{b} \right) \right] + \alpha r + 2n(\bar{b} - \bar{a}) \right\} \eta(\mathcal{W}(\xi, U)Z) \]
\[ - 2n\bar{b}\left[ 1 + 2n\bar{b} - \frac{\bar{c}r}{2n + 1}\left( \frac{1}{2n} + a + b \right) \right] g(Z, U) - 2n\bar{a}\mathcal{S}(Z, U) \]
\[ + 2n\bar{b}\left[ 1 + 2n(a + b) - \frac{\bar{c}r}{2n + 1}\left( \frac{1}{2n} + a + b \right) \right] \eta(Z) \eta(U). \quad (31) \]
\[ g(\mathcal{W}(e_i, \omega(\xi, e_i) Z) U, \xi) \]
\[ = \left\{ \frac{\bar{c}r}{2n + 1}\left( \frac{1}{2n} + \bar{a} + \bar{b} \right) - 2n\bar{b} - 1 \right\} \eta(\mathcal{W}(\xi, U)Z) \]
\[ + 2n\bar{a}\left[ 1 + 2n(a + b) - \frac{\bar{c}r}{2n + 1}\left( \frac{1}{2n} + a + b \right) \right] \eta(Z) \eta(U) \]
\[ - \bar{a}\left[ 1 + 2n\bar{b} - \frac{\bar{c}r}{2n + 1}\left( \frac{1}{2n} + a + b \right) \right] \mathcal{S}(Z, U) - \bar{a}\mathcal{S}^2(Z, U). \quad (32) \]
\[ \sum_{i=1}^{2n+1} g(\mathcal{W}(e_i, Z) \omega(\xi, e_i) U, \xi) \]
\[ = \left\{ \frac{\bar{c}r}{2n + 1}\left( \frac{1}{2n} + \bar{a} + \bar{b} \right) - 2n\bar{a} - 1 \right\} \eta(\mathcal{W}(e_i, Z)e_i) \eta(U), \quad (33) \]

which is equivalent to
\[ \sum_{i=1}^{2n+1} g(\mathcal{W}(e_i, Z) \omega(\xi, e_i) U, \xi) \]
\[ = -\left\{ \frac{\bar{c}r}{2n + 1}\left( \frac{1}{2n} + \bar{a} + \bar{b} \right) - 2n\bar{a} - 1 \right\} \left[ 2n(1 - \alpha + 2n\bar{b}) \right] \]
\[ + \left\{ \alpha r - \frac{2n\bar{c}r}{2n + 1}\left( \frac{1}{2n} + a + b \right) \right\} \eta(U) \eta(Z). \quad (34) \]
By virtue of (30), (31), (32) & (34), we obtain from (22) that
\[
\left\{1 + 2n\tilde{b} - \frac{\tilde{c}r}{2n + 1}\left(\frac{1}{2n} + a + b\right)\right\}(1-b) + a(1 + 2nb)\right]S(Z,U)
+ \left[2n(1-a + 2nb) + ar - \frac{2ncr}{2n + 1}\left(\frac{1}{2n} + a + b\right)\right] \times 
\left\{\frac{\tilde{c}r}{2n + 1}\left(\frac{1}{2n} + a + b\right) - 2n\tilde{a} - 1\right\} + \left\{1 + 2n(a + b) - \frac{cr}{2n + 1}\left(\frac{1}{2n} + a + b\right)\right\} \times 
\left\{2n\left(1 + 2n\tilde{b} - \frac{\tilde{c}r}{2n + 1}\left(\frac{1}{2n} + a + b\right)\right) + a(2n - 4n^2b)\right\}
+ \tilde{a} \times \left\{\text{O}||\text{f}^3 - r - 2rb\right\}
\right]g(Z,U) + \tilde{a}(1-b)S^2(Z,U) = 0. \tag{35}
\]

**Theorem 4.2** Let \((M^{2n+1},g), \ n > 1\) be an Sasakian manifold. Then for respective semi-symmetry type conditions, the Ricci tensor of the manifold \(M\) takes the respective forms as follows-

<table>
<thead>
<tr>
<th>Curvature condition</th>
<th>Expression for Ricci tensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R(X,Y) \cdot R = 0)</td>
<td>(S = 2ng).</td>
</tr>
<tr>
<td>(R(X,Y) \cdot C = 0)</td>
<td>(S = \left(\frac{r}{2n} - 1\right)g - \left{\frac{r}{2n} - (2n + 1)\right}\eta \otimes \eta.)</td>
</tr>
<tr>
<td>(R(X,Y) \cdot \tilde{C} = 0)</td>
<td>(S = \left(\frac{r}{2n} - 1\right)g - \left{\frac{r}{2n} - (2n + 1)\right}\eta \otimes \eta.)</td>
</tr>
<tr>
<td>(R(X,Y) \cdot E = 0)</td>
<td>(S = 2ng.)</td>
</tr>
<tr>
<td>(R(X,Y) \cdot P = 0)</td>
<td>(S = 2ng - \left(\frac{r}{2n} - 2n - 1\right)\eta \otimes \eta.)</td>
</tr>
<tr>
<td>(R(X,Y) \cdot H = 0)</td>
<td>(S = \left(\frac{r}{2n} - 1\right)g - \left{\frac{r}{2n} - (2n + 1)\right}\eta \otimes \eta.)</td>
</tr>
<tr>
<td>(E(X,Y) \cdot R = 0)</td>
<td>(S = 2ng.)</td>
</tr>
<tr>
<td>(E(X,Y) \cdot C = 0)</td>
<td>(S = \left(\frac{r}{2n} - 1\right)g - \left{\frac{r}{2n} - (2n + 1)\right}\eta \otimes \eta.)</td>
</tr>
<tr>
<td>(E(X,Y) \cdot \tilde{C} = 0)</td>
<td>(S = \left(\frac{r}{2n} - 1\right)g - \left{\frac{r}{2n} - (2n + 1)\right}\eta \otimes \eta.)</td>
</tr>
<tr>
<td>(E(X,Y) \cdot E = 0)</td>
<td>(S = 2ng.)</td>
</tr>
<tr>
<td>(E(X,Y) \cdot P = 0)</td>
<td>(S = 2ng - \left(\frac{r}{2n} - 2\right)\eta \otimes \eta.)</td>
</tr>
<tr>
<td>(E(X,Y) \cdot H = 0)</td>
<td>(S = \left(\frac{r}{2n} - 1\right)g - \left{\frac{r}{2n} - (2n + 1)\right}\eta \otimes \eta.)</td>
</tr>
<tr>
<td>(\tilde{C}(X,Y) \cdot R = 0)</td>
<td>(S = -2ng + \left{r - 2n\right}\eta \otimes \eta + S^2.)</td>
</tr>
</tbody>
</table>
\[
\hat{C}(X, Y) \cdot \hat{C} = 0 \quad \{r-(2n-1)\}S \\
= \left[ \left\{ \frac{r}{2n} - 2n \right\} \{ (r-2n) + r - \|Q\|^2 \} \right] g \\
+ \left[ \left\{ \frac{r}{2n} - 1 \right\} \{ 2n(2n+1) - r \} \right] \eta \otimes \eta.
\]

\[
\hat{C}(X, Y) \cdot C = 0 \quad \{r-(2n-1)\}S \\
= \left[ \left\{ \frac{r}{2n} - 2n \right\} \{ (r-2n) + r - \|Q\|^2 \} \right] g \\
+ \left[ \left\{ \frac{r}{2n} - 1 \right\} \{ 2n(2n+1) - r \} \right] \eta \otimes \eta \\
+ 2n S^2.
\]

\[
\hat{C}(X, Y) \cdot E = 0 \quad 2S = -S^2 + [r+2n]g + (2n-r) \left\{ \frac{r}{2n(2n+1)} - 1 \right\} \eta \otimes \eta.
\]

\[
\hat{C}(X, Y) \cdot P = 0 \quad \left\{ \frac{r}{2n} - 2 \right\} S = -rg + S^2 + \left\{ \frac{r}{2n} - 1 \right\} \times \left\{ (2n+1) - \frac{r}{2n} \right\} \eta \otimes \eta.
\]

\[
\hat{C}(X, Y) \cdot H = 0 \quad \frac{(4n+1)}{4n} S = \left[ \left\{ \frac{2n}{4n} + r \right\} + \frac{1}{4n} \right] ||Q||^2 \right] g \\
- \left( 1 + \frac{1}{4n} \right) S^2 + \left\{ \frac{r}{4n} - \frac{2n+1}{2} \right\} \eta \otimes \eta.
\]

\[
P(X, Y) \cdot R = 0 \quad \left\{ \frac{2n-1}{2n} \right\} S = \left( - \frac{r}{2n} + 2n \right) g + \frac{1}{2n} S^2.
\]

\[
P(X, Y) \cdot \hat{C} = 0 \quad \left\{ \frac{4n^2+1}{2n} \right\} S = S^2 + \left\{ (r-2n) + \frac{1}{2n} \right\} g \\
+ \left\{ (2n+1) - 2n + \left( 1 - \frac{r}{2n} \right) \right\} \eta \otimes \eta.
\]

\[
P(X, Y) \cdot C = 0 \quad \left\{ \frac{4n^2+1}{2n} \right\} S = S^2 + \left\{ (r-2n) + \frac{1}{2n} \right\} g \\
+ \left\{ (2n+1) - \frac{r}{2n} \right\} \left\{ r + 2n \right\} \eta \otimes \eta.
\]

\[
P(X, Y) \cdot E = 0 \quad \left\{ \frac{2n-1}{2n} \right\} S = \left( - \frac{r}{2n} + 2n \right) g + \frac{1}{2n} S^2 \\
+ \frac{1}{2n} \left\{ 2n - \frac{r}{2n} \right\} \left\{ r - (2n+1) \right\} \eta \otimes \eta.
\]

\[
P(X, Y) \cdot P = 0 \quad \left\{ \frac{2n-1}{2n} \right\} S = \frac{1}{2n} S^2 + \left( - \frac{r}{2n} + 2n \right) g.
\]

\[
P(X, Y) \cdot H = 0 \quad S = \frac{1}{2n} \left\{ 1 + \frac{1}{4n} \right\} S^2 + \left\{ n - \frac{1}{8n^2} ||Q||^2 \right\} g.
\]
### Semi-symmetry type Sasakian manifolds

<table>
<thead>
<tr>
<th>Expression</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H(X, Y) \cdot R = 0$</td>
<td>[ S = \left( -\frac{r}{4n} + n \right)g + \left( \frac{r}{4n} - \frac{1}{2n} \right)\eta \otimes \eta + \frac{1}{4n}S^2 ]</td>
</tr>
<tr>
<td>$H(X, Y) \cdot \hat{C} = 0$</td>
<td>[ S = \left( \frac{2n^2 + 1}{4n} \right)g + \left( \frac{1}{2n^2} \right)\eta \otimes \eta ]</td>
</tr>
<tr>
<td>$H(X, Y) \cdot E = 0$</td>
<td>[ S = \left( \frac{2n-1}{4n} \right)g + \left( \frac{r}{2n} \right)\eta \otimes \eta + \frac{1}{4n}S^2 ]</td>
</tr>
<tr>
<td>$H(X, Y) \cdot P = 0$</td>
<td>[ S = \left( \frac{2n-1}{4n} \right)g + \left( \frac{n+1}{2n} \right)\eta \otimes \eta + \frac{1}{4n}S^2 ]</td>
</tr>
<tr>
<td>$H(X, Y) \cdot H = 0$</td>
<td>[ S = \left( -\frac{r}{4n} - \frac{1}{8n^2} + \frac{r}{n+1} \right)g + \left( \frac{1}{2n^2} \right) + \frac{1}{4n}S^2 ]</td>
</tr>
<tr>
<td>$C(X, Y) \cdot R = 0$</td>
<td>[ S = -2ng + \left( \frac{r}{2n} \right)\eta \otimes \eta + S^2. ]</td>
</tr>
<tr>
<td>$C(X, Y) \cdot \hat{C} = 0$</td>
<td>[ \left( r - 2n - 1 \right)g = \left( \frac{r}{2n} - 2n \right) + \left( \frac{r}{2n} \right)(r - 2n - 1) + \left( \frac{r}{2n} \right)(r - 2n) ]</td>
</tr>
<tr>
<td>$C(X, Y) \cdot P = 0$</td>
<td>[ S = \left( \frac{r}{2n} - 2n \right) + \left( \frac{r}{2n} - 1 \right)(2n - 1) + \left( \frac{r}{2n} \right)(r - 2n) ]</td>
</tr>
<tr>
<td>$C(X, Y) \cdot E = 0$</td>
<td>[ S = -2ng + \left( \frac{r}{2n} \right)\eta \otimes \eta + S^2. ]</td>
</tr>
<tr>
<td>$C(X, Y) \cdot C = 0$</td>
<td>[ \left( r - 2n - 1 \right)g = \left( \frac{r}{2n} - 2n \right) + \left( \frac{r}{2n} \right)(r - 2n - 1) + \left( \frac{r}{2n} \right)(r - 2n) ]</td>
</tr>
</tbody>
</table>

### References

[1] B. J. Papanioniou, *Contact Riemannian manifolds satisfying $R(\xi, X) \cdot R = 0$ and $\xi \in (k, \mu)$-nullity distribution*, Yokohama Math. J., 40 (1993), 149-161.


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