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Stochastic Analysis of Prophylactic Treatment of a Diabetic Person with Erlang-2

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Abstract: *Diabetic mellitus is a chronic disease of pancreatic origin which is not fully curable once a person became diabetic. This paper treats the model with prophylactic treatment, considering a person who is exposed to diabetes and organ failure. In this model, time to diabetes is Erlang-2 where as organ failure process is general. In this paper we present the expected time to treatment and expected cure time with numerical examples.*
Keywords: *Chronic diseases, Prophylactic treatment, Preventive maintenance, Erlang-2.*

I. Introduction

In Medical Science and Reliability Theory studies, prevention of either the disease or damage to men or machines are given very much importance as that would prevent loss of life or production failure. Many research scholar [2, 8, 9] have studied Mathematical Reliability models elaborately and intensively in this area. Gopalan and D'Souza[7] treated preventive maintenance models in 2-unit system. Ramanarayanan's [10] study centered on the concept of alertness of worker in the prevention of damages. Several issues on risk factors of diabetes mellitus were analyzed in [1, 3, 4, 5, 6]. Usha and Eswariprem [11] have focused their discussions on the models for time to diabetic and Markovian processes for organs with prophylactic treatment to avoid the disease. Mathematical models or assumptions play a great and distinctive role in this area.

Any study in the prophylactic treatment area will be very beneficial to society, since the expenses incurred for treating a disease is very high and far beyond the reach of a majority. Moreover cure from the disease after treatment is time consuming and above all seldom achieved in many cases. There are many cases who have not recovered fully even after a proper treatment for diabetes and other diseases. In this chapter we concentrate on various situations on prophylactic treatment to prevent the disease, applying the recent advancements in probability and Operations Research. We analyze models in which a person is provided with treatment for diabetes or he is advised to take prophylactic treatment to avoid falling a victim to either to becoming diabetic or damages to organs. Based on the medical reports before admission, quite often the person is treated not only for the diagnosed disease concerned namely diabetic but also for any suspected damages occurred to his external and internal organs.

We consider models wherein cure the damaged organ one by one. We study the model under the assumption that time to prophylactic treatment is a random variable and its treatment span also a random variable. We present the joint Laplace-Stieltjes transform of time to treatment and treatment time distributions, expected time to treatment and treatment time for models with suitable and reliable medical facilities.

II. Model: $F(.)$ Erlang-2, Damage Process General.

2.1 Assumptions

The general assumptions of models studied are given below :

i. A person becomes diabetic at the end of a random length of time U with cdf F(.).
ii. He is exposed to several risk and damage process to his organs before becoming diabetic due to stress and strain or due to his habits. The damages occur to his organs in accordance with a general renewal process with inter-occurrence cdf G(.). When the damage caused to his organs exceeds a threshold Y, he is taken for hospitalization. The threshold Y has a discrete distribution with $P(Y = k) = p_k,\ k > 0$ which is the probability of organ failure on the k^{th}damage and $\sum_1^\infty p_k = 1$. The random variable V is the time at which the threshold is exceeded for the first time.
iii. He is sent for prophylactic treatment to avoid diabetes after a time W which has exponential distribution with parameter α.
iv. Initially at time 0, he is normal and his organs are free from damages. He is sent for hospitalization when either he becomes diabetic or damage caused to his organ exceeds the threshold or a prophylactic treatment is required.

v. During hospitalization he is treated for diabetes if he is so. He is given prophylactic treatment if admitted for the same. All damages to his organs are treated one by one. Treatment time for diabetes, the damages caused and prophylactic treatment are independent with distinct distributions $R_d(.)$, $R(.)$ and $R_p(.)$.

In this model the cdf $F(.)$ is Erlang -2 with parameter λ and $G(.)$ is general.

Hospitalization for the person starts at T, where $T = \min(U, V, W)$. (2.1)
When $T = U$, the person becomes diabetic, when $T = V$, the damage caused at time V has exceeded the threshold level Y of the organ and when $T = W$, the person is sent for prophylactic treatment. Let n be the number of damages occurred during the time before treatment and R be the individual treatment time, R_d and R_p be the treatment times for diabetes and prophylactic treatment respectively. Then total treatment time $\hat{A}$ is
$\hat{A} = R_d + \sum_{i=1}^{n} R_i$ or $\sum_{i=1}^{n} R_i$ or $R_p + \sum_{i=1}^{n} R_i$, (2.2)
when the person is admitted for diabetes or organ damages or prophylactic treatment respectively. The cdf $F_1(t)$ of V is calculated as follows:

$$F_1(t) = P(V \le t) = \sum_{k=1}^{\infty} \int_0^t g_k(u) p_k \, du = \sum_{k=1}^{\infty} p_k G_k(t), \quad (2.3)$$

where g_k is the $k-$fold convolution of g and $G_k(x) = \int_0^t g_k(u) du$.
The joint distribution functions of T and $\hat{A}$ is given by the following equation:

Since U is Erlang- 2 with parameter λ, and W is exponential with parameter α,

$$\begin{aligned} P(T \le x, \hat{A} \le y) = & \int_0^x \lambda^2 u e^{-\lambda u} \sum_{k=0}^{\infty} [G_k(u) - G_{k+1}(u)] P_k e^{-\alpha u} \, du \\ & \times \int_0^y \int_0^w r_d(v) r_k(w-v) dv dw \\ & + \int_0^x \left(e^{-\lambda u} + \lambda u e^{-\lambda u}\right) \sum_{k=1}^{\infty} g_k(u) p_k e^{-\alpha u} \, du \int_0^y r_k(v) dv \\ & + \int_0^x \left(e^{-\lambda u} + \lambda u e^{-\lambda u}\right) \sum_{k=0}^{\infty} [G_k(u) - G_{k+1}(u)] P_k \alpha e^{-\alpha u} \, du \\ & \times \int_0^y \int_0^w r_k(v) r_p(w-v) dv dw, \end{aligned} \quad (2.4)$$

where $P(Y > k) = P_k = 1 - \sum_{i=1}^{k} p_i$, $k = 1, 2, 3, \cdots$, (2.5)
r_k is the $k-$ fold convolution of r with itself.

The first term of the right hand side of equation (2.4) is the probability that the person becomes diabetic before x, k- damages have occurred to organs which survives the damages during $(0, x)$, time to prophylactic treatment is not over before x and the treatment is completed during $(0, y)$, for diabetes and damages. The second term is the probability that the organ failure occurs during $(0, u)$ on the k^{th} damage before he becomes diabetic or sent for prophylactic treatment and the organ damages are treated during $(0, y)$. The third term is the probability that the person is sent for prophylactic treatment during $(0, x)$ before he becomes diabetic or organ failure occurs and he is provided with prophylactic treatment and treatment for organ damages during $(0, y)$.

Let $\phi(s) = \sum_{k=1}^{\infty} p_k s^k$. (2.6)

Then $\Phi(s) = \frac{1-\phi(s)}{1-s}$, (2.7)

where $\Phi(s) = \sum_{k=0}^{\infty} P_k s^k$.
The joint Laplace transform of pdf T and $\hat{A}$ is

$$E\left(e^{-\xi T} e^{-\eta \hat{A}}\right) = \int_0^\infty \int_0^\infty \frac{\partial^2}{\partial x \partial y} P(T \le x, \quad \hat{A} \le y) \, dx dy.$$

From equation (2.5) we get,

$$\begin{aligned} E\left(e^{-\xi T} e^{-\eta \hat{A}}\right) = & -\lambda\{(-g^*(\xi+\lambda+\alpha)) \times \Phi[r^*(\eta) g^*(\xi+\lambda+\alpha)] \times \frac{[\lambda r_d^*(\eta) + \alpha r_p^*(\eta)]}{(\xi+\lambda+\alpha)}\} \\ & -\lambda\{(1-g^*(\xi+\lambda+\alpha)) \times \Phi'[r^*(\eta) g^*(\xi+\lambda+\alpha)] \\ & \times (r^*(\eta) g^{*'}(\xi+\lambda+\alpha)) \times \frac{[\lambda r_d^*(\eta) + \alpha r_p^*(\eta)]}{(\xi+\lambda+\alpha)}\} \\ & + \lambda\{(1-g^*(\xi+\lambda+\alpha)) \times \Phi[r^*(\eta) g^*(\xi+\lambda+\alpha)] \times \frac{[\lambda r_d^*(\eta) + \alpha r_p^*(\eta)]}{(\xi+\lambda+\alpha)^2}\} \\ & + 1 - \{(1 - r^*(\eta) g^*(\xi+\lambda+\alpha)) \times \Phi[r^*(\eta) g^*(\xi+\lambda+\alpha)]\} \\ & -\lambda\{(-\Phi'[r^*(\eta) g^*(\xi+\lambda+\alpha)]) \times (r^*(\eta) g^{*'}(\xi+\lambda+\alpha)) \end{aligned}$$

$$+\Phi[r^*(\eta)g^*(\xi+\lambda+\alpha)]\times(r^*(\eta)g^{*'}(\xi+\lambda+\alpha))$$
$$+(r^*(\eta)g^*(\xi+\lambda+\alpha))\times(\Phi'[r^*(\eta)g^*(\xi+\lambda+\alpha)])\times(r^*(\eta)g^{*'}(\xi+\lambda+\alpha))\}$$
$$+(1-g^*(\xi+\lambda+\alpha))\times\Phi[r^*(\eta)g^*(\xi+\lambda+\alpha)]\times\frac{[\alpha\, r_p^*(\eta)]}{(\xi+\lambda+\alpha)} \tag{2.8}$$

where * denotes Laplace transform.

The Laplace transform of pdf T is given by

$$E(e^{-\xi T})=-\lambda\{(-g^*(\xi+\lambda+\alpha))\times\Phi[g^*(\xi+\lambda+\alpha)]\times\frac{[\lambda+\alpha]}{(\xi+\lambda+\alpha)}\}$$
$$-\lambda\{(1-g^*(\xi+\lambda+\alpha))\times\Phi'[g^*(\xi+\lambda+\alpha)]\times g^{*'}(\xi+\lambda+\alpha)\frac{[\lambda+\alpha]}{(\xi+\lambda+\alpha)}\}$$
$$+\lambda\{(1-g^*(\xi+\lambda+\alpha))\times\Phi[g^*(\xi+\lambda+\alpha)]\frac{[\lambda+\alpha]}{(\xi+\lambda+\alpha)^2}\}$$
$$+1-\{(1-g^*(\xi+\lambda+\alpha))\times\Phi[g^*(\xi+\lambda+\alpha)]\}$$
$$-\lambda\{(-\Phi'[g^*(\xi+\lambda+\alpha)])\times(g^{*'}(\xi+\lambda+\alpha))$$
$$+\Phi[g^*(\xi+\lambda+\alpha)]\times(g^{*'}(\xi+\lambda+\alpha))$$
$$+(g^*(\xi+\lambda+\alpha))\times(\Phi'[g^*(\xi+\lambda+\alpha)])\times(g^{*'}(\xi+\lambda+\alpha))\}$$
$$+(1-g^*(\xi+\lambda+\alpha))\times\Phi[g^*(\xi+\lambda+\alpha)]\times\frac{\alpha}{(\xi+\lambda+\alpha)}.$$

Using (2.7) we get,

$$E(e^{-\xi T})=\{\frac{(1-\Phi[g^*(\xi+\lambda+\alpha)])}{(1-g^*(\xi+\lambda+\alpha))^2}-1\}\times g^*(\xi+\lambda+\alpha)g^{*'}(\xi+\lambda+\alpha)\{\frac{\lambda\xi}{\xi+\lambda+\alpha}\}$$
$$-(1-g^*(\xi+\lambda+\alpha))\frac{\xi(1-\alpha-2\lambda)}{(\xi+\lambda+\alpha)^2}-\Phi[g^*(\xi+\lambda+\alpha)]\times[g^{*'}(\xi+\lambda+\alpha)]^2\times\{\frac{\lambda\xi}{\xi+\lambda+\alpha}\}. \tag{2.9}$$

and

$$E(T)=\frac{\lambda g^*(\lambda+\alpha)g^{*'}(\lambda+\alpha)(1-\phi[g^*(\lambda+\alpha)])}{(1-g^*(\lambda+\alpha))^2(\lambda+\alpha)^2}-g^*(\lambda+\alpha)g^{*'}(\lambda+\alpha)\left\{\frac{\lambda}{\lambda+\alpha}\right\}$$
$$-(1-g^*(\lambda+\alpha))\frac{(1-\alpha-2\lambda)}{(\lambda+\alpha)^2}-\phi'[g^*(\lambda+\alpha)]g^{*'}(\lambda+\alpha)\times\{\frac{\lambda}{\lambda+\alpha}\}. \tag{2.10}$$

Further we get,

$$E(e^{-\eta\hat{A}})=-\lambda\{\left(-g^{*'}(\lambda+\alpha)\right)\times\left(\frac{1-\phi[g^*(\lambda+\alpha)]}{1-g^*(\lambda+\alpha)}\right)\times\frac{[\lambda r_d^*(\eta)+\alpha r_p^*(\eta)]}{\lambda+\alpha}\}$$
$$-\lambda\{(1-g^*(\lambda+\alpha))\times\left(\frac{1-\phi[g^*(\lambda+\alpha)]}{(1-g^*(\lambda+\alpha))^2}-\frac{\phi'[r^*(\eta)g^*(\lambda+\alpha)]}{1-g^*(\lambda+\alpha)}\right)$$
$$\times(r^*(\eta)g^{*'}(\lambda+\alpha))\}$$
$$+\lambda\{(1-g^*(\lambda+\alpha))\times\left(\frac{1-\phi[g^*(\lambda+\alpha)]}{1-g^*(\lambda+\alpha)}\right)\times\frac{[\lambda r_d^*(\eta)+\alpha r_p^*(\eta)]}{(\lambda+\alpha)^2}\}$$
$$+1-\{(1-r^*(\eta)g^*(\lambda+\alpha))\times\left(\frac{1-\phi[g^*(\lambda+\alpha)]}{1-g^*(\lambda+\alpha)}\right)\}$$
$$-\lambda\{\left(\frac{1-\phi[g^*(\lambda+\alpha)]}{(1-g^*(\lambda+\alpha))^2}-\frac{\phi'[g^*(\lambda+\alpha)]}{1-g^*(\lambda+\alpha)}\right)\times(-r^*(\eta)g^{*'}(\lambda+\alpha))$$
$$+\left(\frac{1-\phi[g^*(\lambda+\alpha)]}{1-g^*(\lambda+\alpha)}\right)\times(r^*(\eta)g^{*'}(\lambda+\alpha))$$
$$+(r^*(\eta)g^*(\lambda+\alpha))\times\left(\frac{1-\phi[g^*(\lambda+\alpha)]}{(1-g^*(\lambda+\alpha))^2}-\frac{\phi'[g^*(\lambda+\alpha)]}{1-g^*(\lambda+\alpha)}\right)$$
$$\times(r^*(\eta)g^{*'}(\lambda+\alpha))\}+(1-g^*(\lambda+\alpha))\times\left(\frac{1-\phi[g^*(\lambda+\alpha)]}{1-g^*(\lambda+\alpha)}\right)\times\frac{[\alpha\, r_p^*(\eta)]}{(\lambda+\alpha)}. \tag{2.11}$$

and we have

$$E(\hat{A})=\left(\frac{1-\phi[g^*(\lambda+\alpha)]}{1-g^*(\lambda+\alpha)}\right)\times\{\frac{[\lambda E(R_d)+\alpha E(R_p)]}{(\lambda+\alpha)}\times(\lambda g^{*'}(\lambda+\alpha))\times(1-E(R))\times(\frac{\lambda(1-g^*(\lambda+\alpha))}{(\lambda+\alpha)})$$
$$-\left(1-E(R)g^{*'}(\lambda+\alpha)\right)+\left(1-g^*(\lambda+\alpha)\right)\times\left(\frac{\alpha}{\lambda+\alpha}\right)E(R_p)\}$$
$$+1+\{\lambda E(R)\left(1-g^*(\lambda+\alpha)\right)\times g^{*'}(\lambda+\alpha)\times(\frac{[\lambda E(R_d)+\alpha E(R_p)]}{\lambda+\alpha})$$
$$+\lambda E(R)g^{*'}(\lambda+\alpha)\times(1+E(R)g^*(\lambda+\alpha))\}\times\{\frac{\phi'[g^*(\lambda+\alpha)]}{1-g^*(\lambda+\alpha)}\}. \tag{2.12}$$

III. Numerical Illustrations

We consider a special case for numerical results in which the damage occurs in the organs in accordance with a general renewal process with rate β. Then

$$g^*(t) = \frac{\beta}{\beta+t}, \quad \phi(s) = \sum_{k=1}^{\infty} p_k s^k. \qquad (2.13)$$

where $s = g^*(t), t = \lambda + \alpha.$ (2.14)

Substituting (2.13), (2.14) in (2.10) and (2.12), we get,

$$E(T) = \left\{\frac{\left(1-\phi(s)(-\lambda\beta^2)\right)}{(\beta+t)t^4}\right\} + \left\{\frac{\lambda\beta^2}{t(\beta+t)^3}\right\} - \left\{\frac{1-\alpha-2\lambda}{t(\beta+t)}\right\} + \left\{\frac{\phi'[s]\lambda\beta}{t(\beta+t)^2}\right\}. \qquad (2.15)$$

$$E(\hat{A}) = \left\{\frac{(\beta+t)(1-\phi(s))}{t}\right\}$$
$$\times\left\{\left(\frac{[\lambda E(R_d)+\alpha E(R_p)]}{t}\right)\left[\left(\frac{-\lambda\beta}{(\beta+t)^2}\right)(1-E(R)) + \frac{\lambda t}{t(\beta+t)}\right] - \left(1-\frac{\beta E(R)}{(\beta+t)}\right) + \frac{\alpha E(R_p)}{\beta+t}\right\} + 1$$
$$+\left\{\left(\frac{-\lambda\beta t E(R)}{(\beta+t)^2}\right)\times\left(\frac{[\lambda E(R_d)+\alpha E(R_p)]}{t}\right) + \left(\frac{-\lambda\beta E(R)}{(\beta+t)^2}\right)\times\left(1+\frac{\beta E(R)}{(\beta+t)}\right)\right\}\times\left\{\frac{(\beta+t)\phi'[s]}{t}\right\}. \qquad (2.16)$$

We consider a special case $\phi(s) = 0.5s + 0.3s^2 + 0.2s^3$ and calculate the expected time to damage of the organs and time to treatment based on the assumptions of $E(W) < E(U)$ by using the graph. For various values of λ and the parameters α and β to obtain the values of E(T) and E(Â) graphically.

λ=0.2, β=0.1, $E(R_d)$=0.2, $E(R_p)$=0.2,E(R)=0.1		
α	E(T)	E(Â)
1	0.4513	2.3481
2	1.4999	4.6136
3	2.5327	6.8631
4	3.5641	9.1081
5	4.5971	11.3511
6	5.6330	13.5932
7	6.6722	15.8347
8	7.7149	18.0759
9	8.7614	20.3167
10	9.8116	22.5575

Table 1.1

λ =0.4,β=0.2, $E(R_d)$=0.1, $E(R_p)$=0.4,E(R)=0.1		
α	E(T)	E(Â)
1	1.0404	2.8776
2	2.1898	5.4982
3	3.3470	8.0835
4	4.5293	10.6573
5	5.7406	13.2260
6	6.9824	15.7920
7	8.2552	18.3563
8	9.5595	20.9196
9	10.8953	23.4822
10	12.2629	26.0443

Table 1.2

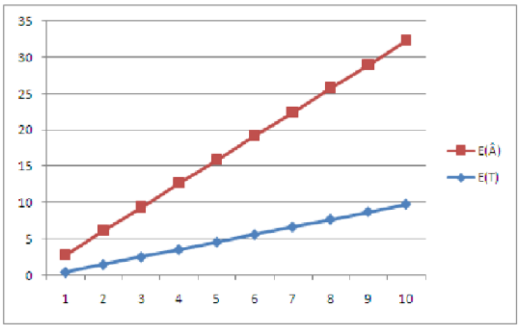

Figure 1.1

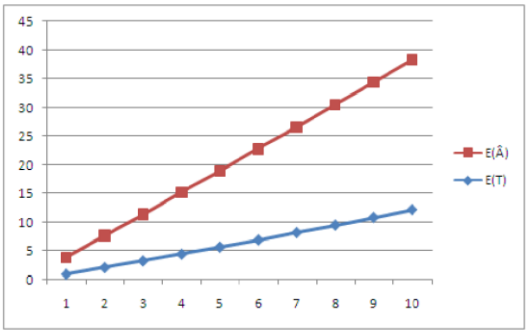

Figure 1.2

λ=0.1, β=0.1, E(R_d)=0.1, E(R_p)=0.2,E(R)=0.2		
α	E(T)	E($\hat{A}$)
1	0.2262	2.3117
2	1.2743	4.5517
3	2.2987	6.7789
4	3.3182	9.0026
5	4.3370	11.2248
6	5.3564	13.4463
7	6.3770	15.6674
8	7.3991	17.8882
9	8.4228	20.1088
10	9.4483	22.3294

Table 1.3

λ =0.2,β=0.2, E(R_d)=0.2, E(R_p)=0.3,E(R)=0.1		
α	E(T)	E($\hat{A}$)
1	0.5190	2.0180
2	1.6348	3.9263
3	2.7257	5.8048
4	3.8215	7.6745
5	4.9287	9.5405
6	6.0496	11.4046
7	7.1852	13.2676
8	8.3359	15.1298
9	9.5019	16.9916
10	10.6836	18.8530

Table 1.4

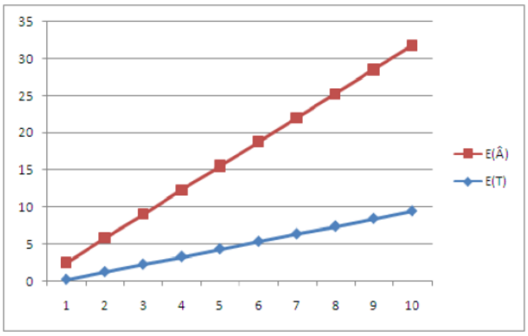

Figure 1.3

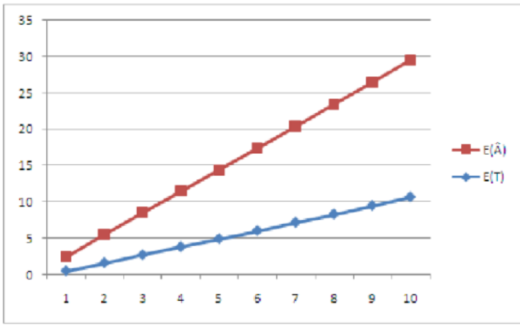

Figure 1.4

λ=0.2,α =2, E(R_d)=1, E(R_p)=2,E(R)=0.4		
β	E(T)	E($\acute{A}$)
.1	1.4999	46.2665
.2	1.6348	26.3810
.3	1.8159	19.8261
.4	2.0560	16.6020
.5	2.3697	14.7089
.6	2.7733	13.7733
.7	3.2853	12.6304
.8	3.9261	12.0168
.9	4.7186	11.5601
1.0	5.6877	11.2129

Table 2.1

λ =0.1, α=1, E(R_d)=2, E(R_p)=4,E(R)=0.2		
β	E(T)	E($\acute{A}$)
.1	0.2262	46.7374
.2	0.2597	26.8330
.3	0.3045	20.2588
.4	0.3617	17.0167
.5	0.4487	15.1032
.6	0.5614	13.8552
.7	0.7119	12.9876
.8	0.9101	12.3560
.9	0.1675	11.8819
1.0	0.4968	11.5177

Table 2.2

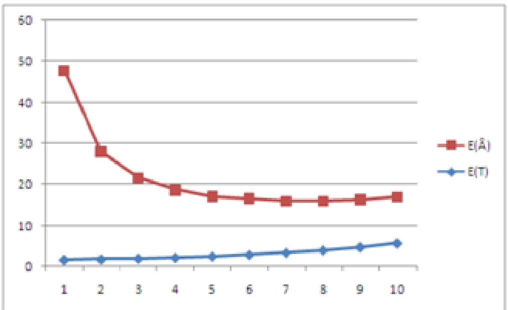

Figure 2.1

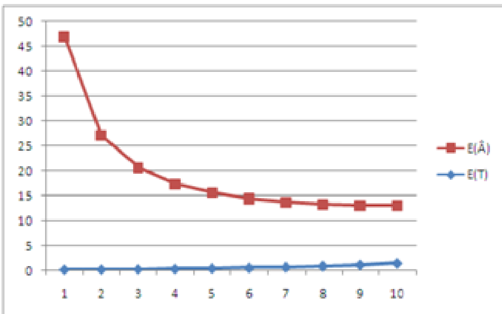

Figure 2.2

λ=0.2,α =2, E(R_d)=0.2, E(R_p)=0.5,E(R)=0.4		
β	E(T)	E($\hat{A}$)
1	1.4999	11.5606
2	1.6348	6.5855
3	1.8159	4.9433
4	2.0560	4.1340
5	2.3697	3.6577
6	2.7733	3.3477
7	3.2853	3.1326
8	3.9261	2.9767
9	4.7186	2.8602
10	5.6877	2.7712

Table 2.3

λ =0.4, α=2, E(R_d)=0.2, E(R_p)=0.5,E(R)=0.4		
B	E(T)	E($\hat{A}$)
1	1.9554	11.8780
2	2.1898	6.9154
3	2.5267	5.2858
4	2.9929	4.4893
5	3.6187	4.0258
6	4.4381	3.7287
7	5.4888	3.5266
8	6.8129	3.3837
9	8.4565	3.2804
10	10.4703	3.2045

Table 2.4

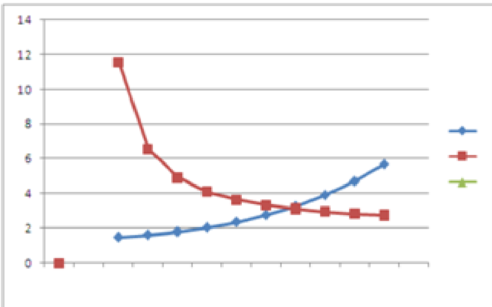

Figure 2.3

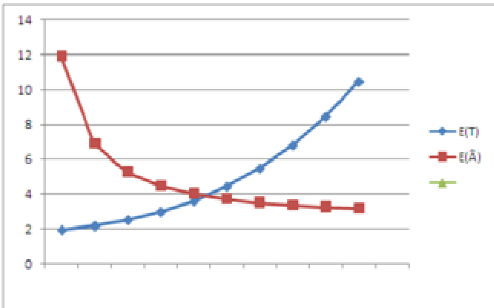

Figure 2.4

From the tables 1.1 to 1.4 we observe the behavior of mean time to damages $E(T)$ and $E(\hat{A})$ for fixed values of λ, β, p_k, $E(R_d)$, and $E(R)$. When the parameter α and $E(R_p)$ increase, both $E(T)$ and $E(\hat{A})$ increases in the tables from 1.1 to 1.4.

From the tables 2.1 to 2.4 we observe the behavior of mean time to damages $E(T)$ and mean curing time $E(\hat{A})$ for fixed values of λ, α, p_k, λ, β, p_k, $E(R_d)$ and $E(R)$. When the parameter β and $E(R_p)$ increase, mean time to damages $E(T)$ increases and $E(\hat{A})$ decreases.

IV. Conclusion

In any study the Mathematical and Stochastic models developed, are done taking into consideration the problems that occur in real life scenarios. Similarly, we have developed Diabetes mellitus models, that track and possibly predict the progression of the disease. In this chapter we model various scenarios involving prophylactic treatment to prevent the disease by applying recent advancements in Probability and Operations Research. We analyze models in which, a person is treated for diabetes or he is advised to take prophylactic treatment to avoid becoming diabetic or developing secondary complications. We used the distribution Erlang-2 and the general process to find the expected time to treatment and expected cure time. From the table 1.1 to1.4, when the parameter α and $E(R_p)$ increases, mean time to damages E(T) increases slowly whereas the mean time to curing increases rapidly as depicted in the figures 1.1 to1.4. From the table 2.1 to2.4, when the parameter β and $E(R_p)$ increases, mean time to damages E(T) increases, whereas the mean time to curing decreases rapidly which as in the figures 2.1 to 2.4.

Diabetic pandemic threatens to become a rapidly expanding burden in the developing countries. The direct and indirect costs involved in the treatment of chronic disease, especially when associated with the complications are enormous.. The results of our models, which use Erlang, Hyper exponential, or Modified Erlang distributions, can also be used to reduce this cost. .

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