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Numerical Solution of linear first order Singular Systems Using He's Variational Iteration Method

S. Sekar[1], B. Venkatachalam alias Ravikumar[2]
[1]*(Department of Mathematics, Government Arts College (Autonomous), Salem – 636 007, India)*
[2]*(Department of Mathematics, Bharathiyar College of Engineering and Technology, Karaikal – 609 609, India)*

Abstract : *In this paper, He's Variational Iteration Method (HVIM) is used to study the linear time-invariant and time-varying singular systems. The results obtained using He's Variational Iteration Method and the methods taken from the literature [17] were compared with the exact solutions of the linear time-invariant and time-varying singular systems. It is found that the solution obtained using the He's Variational Iteration Method is closer to the exact solutions of the linear time-invariant and time-varying singular systems. Error graphs for discrete and exact solutions are presented in a graphical form to highlight the efficiency of this method.*

Keywords: *Singular systems, time-invariant, time-varying, He's Variational Iteration Method, Leapfrog Method.*

I. Introduction

Singular systems are being applied to solve a variety of problems involved in various disciplines of science and engineering. They are applied to analyze neurological events and catastrophic behaviour and they also provide a convenient form for the dynamical equations of large scale interconnected systems. Further, singular systems are found in many areas such as constrained mechanical systems, fluid dynamics, chemical reaction kinetics, simulation of electrical networks, electrical circuit theory, power systems, aerospace engineering, robotics, aircraft dynamics, neural networks, neural delay systems, network analysis, time series analysis, system modelling, social systems, economic systems, biological systems etc.

Many physical problems are governed by a system of singular systems, and finding the solution of these equations has been the subject of many investigators in recent years. The variational iteration method which is proposed by He [6–12] is effectively and easily used to solve delay differential equations [7], autonomous ordinary differential system [11], Burger's and coupled Burger's equations [1], Laplace equation [22], integro-differential equations [20], Helmholtz equation [16] and other fields [2,3,14,15,18,19,21,23]. It is well known that singular systems can be difficult to solve when they have an index greater than 1 [4]. In [5,13], an efficient reducing index method has been proposed for linear semi-explicit singular systems.

Sekar *et al* [17] discussed the linear time-invariant and time varying singular systems using Leapfrog method. The aim of this paper is to extend the variational iteration method to find the solution of linear time-invariant and time varying singular systems. In this paper, the same linear time-invariant and time varying singular systems was considered (Sekar *et al* [17]) but present a different approach using He's variational iteration method for finding the numerical solution of first order linear singular systems of time-invariant and time varying cases with more accuracy. Furthermore, we use some examples to demonstrate the efficiency and effectiveness of the proposed method.

II. Linear First Order Linear Singular Systems

In general a first order linear singular system of time-invariant case is represented in the following form

$$K\dot{x}(t) = Ax(t) + Bu(t) \qquad (1)$$

with initial condition $x(0) = x_0$.

where K is an $n \times n$ singular matrix, A and B are $n \times n$ and $n \times p$ constant matrices respectively. $x(t)$ is an n-state vector and $u(t)$ is the p-input control vector. A first order linear singular system of time-varying case is represented in the following form

$$K(t)\dot{x}(t) = A(t)x(t) + B(t)u(t) \qquad (2)$$

with initial condition $x(0) = x_0$.

where $x(t)$ and $u(t)$ are defined as in (1) and $K(t)$ is an $n \times n$ singular matrix, $A(t)$ and $B(t)$ are $n \times n$ and $n \times p$ matrices respectively. The elements (not necessarily all the elements) of the matrices $K(t), A(t)$ and $B(t)$ are time dependent.

III. He's Variational Iteration Method

In this section, we briefly review the main points of the powerful method, known as the He's variational iteration method [6–12]. This method is a modification of a general Lagrange multiplier method proposed by Inokuti [16]. In the variational iteration method, the differential equation

$$L[u(t)] + N[u(t)] = g(t) \tag{3}$$

is considered, where *L* and *N* are linear and nonlinear operators, respectively and *g(t)* is an inhomogeneous term. Using the method, the correction functional

$$u_{n+1}(t) = u_n(t) + \int \lambda [L[u_n(s)] + N[\tilde{u}_n(s)] - g(s)]ds \tag{4}$$

is considered, where λ is a general Lagrange multiplier, u_n is the nth approximate solution and $\tilde{u}_n$ is a restricted variation which means $\delta \tilde{u}_n = 0$ [7–9].

In this method, first we determine the Lagrange multiplier λ that can be identified via variational theory, i.e. the multiplier should be chosen such that the correction functional is stationary,

i.e. $\delta u_{n+1}(u_n(t), t) = 0$. Then the successive approximation $u_n, n \geq |0$ of the solution *u* will be obtained by using any selective initial function u_0 and the calculated Lagrange multiplier λ . Consequently $u = \lim_{n \to \infty} u_n$. It means that, by the correction functional (4) several approximations will be obtained and therefore, the exact solution emerges at the limit of the resulting successive approximations. In the next section, this method is successfully applied for solving the linear time-invariant and time varying singular systems.

IV. Numerical Examples

In this section, the exact solutions and approximated solutions obtained by He's variational iteration method and Leapfrog method. To show the efficiency of the He's variational iteration method, we have considered the following problem taken from [17], with step size $t = 0.1$ along with the exact solutions.

The discrete solutions obtained by the two methods, He's variational iteration method and Leapfrog method; the absolute errors between them are tabulated and are presented in Table 1 - 2. To distinguish the effect of the errors in accordance with the exact solutions, graphical representations are given for selected values of "*t*" and are presented in Fig. 1-5 for the following problem, using three dimensional effects.

Example 4.1

The first order linear singular system of time-invariant case with three variables of the form (1) is given by [17]

$$K = \begin{bmatrix} 0 & 1 & 4 \\ 0 & -2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$, *A*=I (an identify matrix of appropriate dimension) and *B* = 0

with initial condition $x(0) = [1/6 \quad 1 \quad -1/3]^T$,

and the exact solution is

$$x_1 = (\exp(-t/2))/6$$

$$x_2 = \exp(-t/2)$$

$$x_3 = -(\exp(-t/2))/3$$

Example 4.2

The first order linear singular system of time-varying case with two variables of the form (2) is given by [17]

$$K = \begin{bmatrix} 0 & 0 \\ 1 & t \end{bmatrix}, A = \begin{bmatrix} -1 & 1-t \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} e^t & 1 \\ t^2 & 2 \end{bmatrix} \quad \text{and } u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

with initial condition $x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

and the exact solution is

$$x_1 = (1+t)e^t - t^3$$

$$x_2 = t^2 - e^t$$

using He's variational iteration method and Leapfrog method to solve the above problems, the absolute errors are evaluated and are presented in Table 1-2 with various time step size. Error graphs are presented Fig. 1-5 to highlight the efficiency of the method.

t	Example 4.1					
	Leapfrog Error			HVIM Error		
	x_1	x_2	x_3	x_1	x_2	x_3
0.1	1E-11	4E-11	3E-11	1.00E-13	4.00E-13	3.00E-13
0.2	2E-11	6E-11	5E-11	2.00E-13	6.00E-13	5.00E-13
0.3	3E-11	8E-11	7E-11	3.00E-13	8.00E-13	7.00E-13
0.4	4E-11	1E-10	9E-11	4.00E-13	1.00E-12	9.00E-13
0.5	5E-11	1.2E-10	1.1E-10	5.00E-13	1.20E-12	1.10E-12
0.6	6E-11	1.4E-10	1.3E-10	6.00E-13	1.40E-12	1.30E-12
0.7	7E-11	1.6E-10	1.5E-10	7.00E-13	1.60E-12	1.50E-12
0.8	8E-11	1.8E-10	1.7E-10	8.00E-13	1.80E-12	1.70E-12
0.9	9E-11	2E-10	1.9E-10	9.00E-13	2.00E-12	1.90E-12
1.0	1E-10	2.2E-10	2.1E-10	1.00E-12	2.20E-12	2.10E-12

Table 1 Error for linear time-invariant singular systems

t	Example 4.2			
	Leapfrog Error		HVIM Error	
	x_1	x_2	x_1	x_2
0.1	2E-11	7E-11	2.00E-13	7.00E-13
0.2	5E-11	9E-11	5.00E-13	9.00E-13
0.3	8E-11	1.1E-10	8.00E-13	1.10E-12
0.4	1.1E-10	1.3E-10	1.10E-12	1.30E-12
0.5	1.4E-10	1.5E-10	1.40E-12	1.50E-12
0.6	1.7E-10	1.7E-10	1.70E-12	1.70E-12
0.7	2E-10	1.9E-10	2.00E-12	1.90E-12
0.8	2.3E-10	2.1E-10	2.30E-12	2.10E-12
0.9	2.6E-10	2.3E-10	2.60E-12	2.30E-12
1.0	2.9E-10	2.5E-10	2.90E-12	2.50E-12

Table 2 Error for linear time-varying singular systems

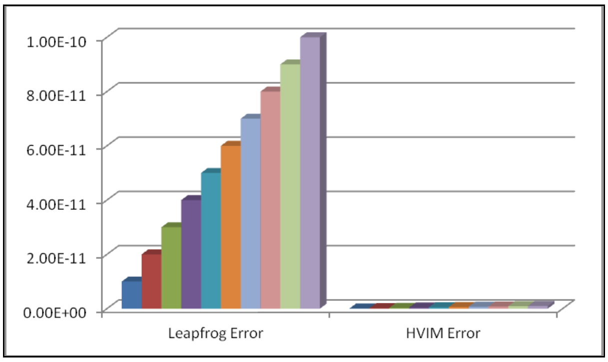

Fig. 1 Error estimation of Example 4.1 at x_1

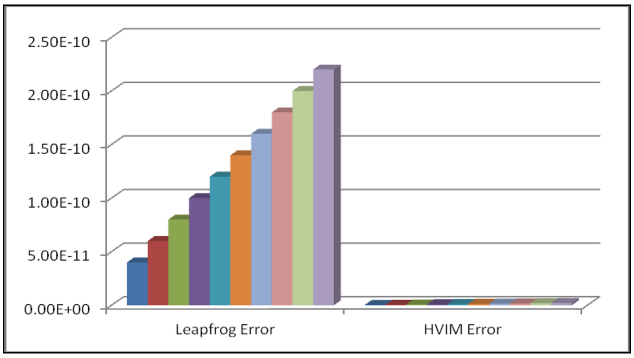

Fig. 2 Error estimation of Example 4.1 at x_2

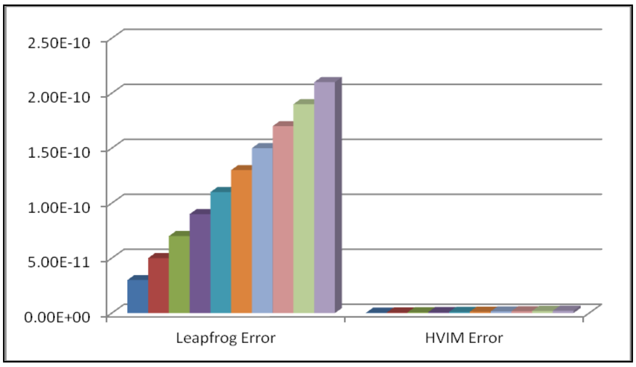

Fig. 3 Error estimation of Example 4.1 at x_3

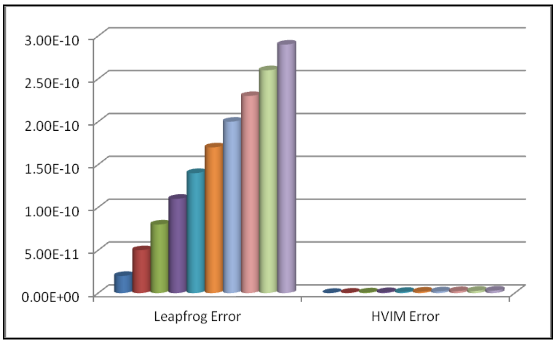

Fig. 4 Error estimation of Example 4.2 at x_1

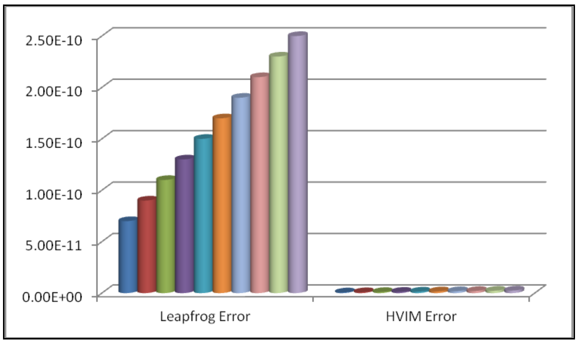

Fig. 6 Error estimation of Example 4.2 at x_2

V. Conclusion

The obtained results (approximate solutions) of the linear time-invariant and time varying singular systems show that the He's variational iteration method works well for finding the solution. From the Table 1–2, it can be observed that for most of the time intervals, the absolute error is less (almost no error) in the He's variational iteration method when compared to the Leapfrog method [5], which yields a small error, along with the exact solutions. From the Fig. 2-3, it can be predicted that the error is very less in He's variational iteration method when compared to the Leapfrog method [5]. Hence the He's variational iteration method is more suitable for studying the linear time-invariant and time varying singular systems.

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