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On Some Continuous and Irresolute Maps In Ideal Topological Spaces

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Abstract: *In this paper we introduce some continuous and irresolute maps called $\hat{\delta}$-continuity, $\hat{\delta}$-irresolute, $\hat{\delta}_s$-continuity and $\hat{\delta}_s$-irresolute maps in ideal topological spaces and study some of their properties.*
Keywords: $\hat{\delta}$-continuity, $\hat{\delta}$-irresolute, $\hat{\delta}_s$- continuity, $\hat{\delta}_s$-irresolute.

I. Introduction

Ideals in topological space (X, τ) is a non-empty collection of subsets of X which satisfies the properties (i) A∈I and B⊂A ⇒ B∈I (ii) A∈I and B∈I ⇒ A∪B∈I. An ideal topological space is denoted by the triplet (X, τ, I). In an ideal space (X, τ, I), if P(X) is the collection of all subsets of X, a set operator (.)* : P(X) → P(X) called a local function [4] with respect to the topology τ and ideal I is defined as follows: for A⊆X, A* = {x∈X / U∩A∉I, for every open set U containing x}. A Kuratowski closure operator cl*(.) of a subset A of X is defined by cl*(A) = A∪A* [12]. Yuksel, Acikgoz and Noiri [14] introduced the concept of δ-I-closed sets in ideal topological space. M. Navaneethakrishnan, P.Periyasamy, S.Pious missier introduced the concept of $\hat{\delta}$-closed set [9] and $\hat{\delta}_s$-closed set [8] in ideal topological spaces. K. Balachandran, P. Sundaram and H.Maki [1], B.M. Munshi and D.S. Bassan [7], T.Noiri [10], Julian Dontchev and Maximilian Ganster [3], N.Levine [5] introduced the concept of g-continuity, supercontinuity, δ-continuity, δg-continuity, w-continuity, respectively. The purpose of this paper is to introduce the concept of $\hat{\delta}$-continuity, $\hat{\delta}$-irresolute, $\hat{\delta}_s$-continuity, $\hat{\delta}_s$-irresolute maps. Also, we study some of the characterization and basic properties of these maps.

II. Preliminaries

Definition 2.1 A subset A of a topological space (X, τ) is called a
(i) Semi-open set [5] if A ⊆ cl(int(A))
(ii) g-closed set [6] if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ)
(iii) w-closed set [11] cl(A) ⊆ U whenever A ⊆ U and U is semi-open.
(iv) δ-closed set [13] if δcl(A) = A, where δcl(A) = {x∈X : int(cl(U))∩A≠ϕ, for each U∈τ(x)}.
(v) δg-closed set [3] if δcl(A) ⊆ U whenever A⊆U and U is open.

The complement of semi-open (resp. g-closed, w-closed, δ-closed, δg-closed) set is called semi-closed (resp. g-open, w-open, δ-open, δg-open) set.

Definition 2.2 [14] Let (X, τ, I) be an ideal topological space, A a subset of X and x a point of X.
(i) x is called a δ-I-cluster point of A if A∩int cl*(U) ≠ ϕ for each open neighbourhood of x.
(ii) The family of all δ-I-cluster points of A is called the δ-I-closure of A and is denoted by $[A]_{\delta\text{-}I}$ and
(iii) A subset A is said to be δ-I-closed if $[A]_{\delta\text{-}I}$ = A. The complement of δ-I-closed set of X is said to be δ-I-open.

Remark 2.3 [9] From the Definition 2.2 it is clear that $[A]_{\delta\text{-}I}$ = {x∈X: int(cl*(U)) ∩ A ≠ ϕ, for each U∈τ(x)}.
Notation 2.4 [9]**.** Throughout this paper $[A]_{\delta\text{-}I}$ is denotd by σcl(A).
Definition 2.5 A subset A of an ideal topological space (X, τ, I) is called
(i) Ig-closed set [2] if A* ⊆ U whenever A ⊆ U and U is open.
(ii) $\hat{\delta}$-closed set [9] if σcl(A) ⊆ U whenever A ⊆ U and U is open.
(iii) $\hat{\delta}_s$-closed set [8] if σcl(A) ⊆ U whenever A ⊆ U and U is semi-open.

The complement of Ig-closed (resp. $\hat{\delta}$-closed, $\hat{\delta}_s$-closed) set is called Ig-open (resp. $\hat{\delta}$-open; $\hat{\delta}_s$–open) set.

Definition 2.6 Let A be a subset of an ideal space (X, τ, I) then the $\hat{\delta}_s$–closure of A is defined to be the intersection of all $\hat{\delta}_s$–closed sets containing A and is denoted by $\hat{\delta}_s cl(A)$. That is $\hat{\delta}_s cl(A) = \cap\{F: A\subseteq F$ and F is $\hat{\delta}_s$–closed$\}$.

Definition 2.7 A map f : (X, τ) → (Y, σ) is said to be

[1]. g-continuous [1] if $f^{-1}(F)$ is g-closed in X for every closed set F of Y.
[2]. g-irresolute [1] if $f^{-1}(F)$ is g-closed in X for every g-closed set F of Y.
[3]. w-continuous [5] if $f^{-1}(F)$ is w-closed in X for every closed set F of Y.
[4]. w-irresolute [5] if $f^{-1}(F)$ is w-closed in X for every w-closed set F of Y.
[5]. δg-continuous [3] if $f^{-1}(F)$ is δg-closed in X for every closed set F of Y.
[6]. δg-irresolute [3] if $f^{-1}(F)$ is δg-closed in X for every δg-closed set F of Y.
[7]. δ-continuous [10] if $f^{-1}(U)$ is δ-open in X for every δ-open set U in Y.
[8]. Supercontinuous [7] if $f^{-1}(U)$ is δ-open in X for every open set U in Y.

Definition 2.8 A map f: (X, τ, I_1) → (Y, σ, I_2) is said to be

(i) Ig-continuous if $f^{-1}(F)$ is Ig-closed in X for every closed set F of Y.
(ii) Ig-irresolute if $f^{-1}(F)$ is Ig-closed in X for every Ig-closed set F of Y.

Definition 2.9 A topological space (X, τ) is called

(i) $T_{½}$-space [6] if for every g-closed subset of X is closed.
(ii) $T_{¾}$-space [3] if for every δg-closed subset of X is δ-closed.

Definition 2.10 [2] An ideal space (X, τ, I) is called T_I-space if for every Ig-closed subset is *-closed.

III. $\hat{\delta}$-Continuous And $\hat{\delta}$-Irresolute Maps

Definition 3.1 A function f from an ideal space (X, τ, I_1) into an ideal space (Y, σ, I_2) is called $\hat{\delta}$-continuous if $f^{-1}(F)$ is $\hat{\delta}$-closed in (X, τ, I_1) for every closed set F of (Y, σ, I_2).

Definition 3.2 A function f from an ideal space (X, τ, I_1) into an ideal space (Y, σ, I_2) is called $\hat{\delta}$-irresolute if $f^{-1}(F)$ is $\hat{\delta}$-closed in (X, τ, I_1) for every $\hat{\delta}$-closed set F in (Y, σ, I_2).

Theorem 3.3 If a map f: (X, τ, I_1) → (Y, σ, I_2) from an ideal space (X, τ, I_1) into an ideal space (Y, σ, I_2) is $\hat{\delta}$-continuous, then it is g-continuous.

Proof. Let f: (X, τ, I_1) → (Y, σ, I_2) be $\hat{\delta}$-continuous and F be any closed set in Y. Then the inverse image $f^{-1}(F)$ is $\hat{\delta}$-closed in (X, τ, I_1). Since every $\hat{\delta}$-closed set is g–closed, $f^{-1}(F)$ is g-closed in (X, τ, I_1). Therefore f is g-continuous.

Remark 3.4 The converse of Theorem 3.3 need not be true as seen in the following Example.

Example 3.5 Let X=Y={a,b,c,d}with topologies τ = {X, ϕ,{b},{a,b},{b,c},{a,b,c}, {a,b,d}}, σ = {Y, ϕ,{b}, {c,d}, {b,c,d} and ideals I_1={ϕ,{c}, {d}, {c,d}}, I_2= {ϕ,{b}, {c},{b,c}}. Let f: (X, τ, I_1) → (Y, σ, I_2) be a map defined by f(a)=b, f(b)=c, f(c)=d and f(d)=a, then f is g–continuous but not $\hat{\delta}$- continuous, since for the closed set F={a,b} in (Y, σ, I_2), $f^{-1}(F)$ = {a,d} is not $\hat{\delta}$- closed set in (X, τ, I_1).

Theorem 3.6 If a map f: (X, τ, I_1) → (Y, σ, I_2) from an ideal space (X, τ, I_1) into an ideal space (Y, σ, I_2) is $\hat{\delta}$-continuous, then it is I_g-continuous.

Proof. Let f: (X, τ, I_1) → (Y, σ, I_2) be $\hat{\delta}$-continuous and F be any closed set in (Y, σ, I_2). Then the inverse image $f^{-1}(F)$ is $\hat{\delta}$-closed in (X, τ, I_1). Since every $\hat{\delta}$-closed set is I_g –closed, $f^{-1}(F)$ is Ig-closed. Therefore f is I_g-continuous.

Remark 3.7 The converse of the above Theorem is not always true as shown in the following Example.

Example 3.8 Let X=Y={a,b,c,d}with topologies τ = {X, ϕ,{b},{d},{b,d},{b,c,d}}, σ = {Y, ϕ,{b}, {a,b}, {b,c},{a,b,c}, {a,b,d}} and ideals I_1={ϕ,{d}}, I_2= {ϕ,{c}, {d},{c,d}}. Let f: (X, τ, I_1) → (Y, σ, I_2) be a map defined by f(a)=d, f(b)=a, f(c)=b and f(d)=c, then f is I_g –continuous but not $\hat{\delta}$- continuous, since for the closed set F={c} in (Y, σ, I_2), $f^{-1}(F)$ = {d} is not $\hat{\delta}$- closed set in (X,τ,I_1).

Theorem 3.9 A map f: (X, τ, I_1) → (Y, σ, I_2) from an ideal space (X, τ, I_1) into an ideal space (Y, σ, I_2) is supercontinuous, then f is $\hat{\delta}$- continuous.

Proof. Let f : (X, τ, I_1) → (Y, σ, I_2) is supercontinuous and U be an open set in (Y, σ, I_2). Then $f^{-1}(U)$ is δ-open in (X, τ, I_1). Since $f^{-1}(U^c) = [f^{-1}(U)]^c$, $f^{-1}(U^c)$ is δ-closed in (X, τ, I_1) for every closed set U^c in (Y, σ, I_2). Also, since every δ-closed set is $\hat{\delta}$-closed $f^{-1}(U^c)$ is $\hat{\delta}$-closed for every closed set U^c in (Y, σ, I_2). Hence f is $\hat{\delta}$-continuous.

Remark 3.10 The converse of the above Theorem is not always true as shown in the following Example.

Example 3.11 Let X = Y = {a, b, c, d} with topologies τ = {X, ϕ, {b}, {c, d}, {b, c, d}}, σ = {Y, ϕ, {c}, {a, d}, {a, c, d}} and ideals I_1 = {ϕ, {c}}, I_2 = {ϕ, {a}, {d}, {a, d}}. Let f: (X, τ, I_1) $\to$ (Y, σ, I_2) be a map defined by f(a) = b, f(b) = a, f(c) = c and f(d) = d, then f is $\hat{\delta}$-continuous but not supercontinuous because, for the open set U = {a, d} in (Y, σ, I_2), f^{-1}(U) = {b, d} is not δ-open in (X, τ, I_1).

Theorem 3.12 A function f: (X, τ, I_1) $\to$ (Y, σ, I_2) is $\hat{\delta}$-continuous if and only if f^{-1} (U) is $\hat{\delta}$- open in (X,τ,I_1) for every open set U in (Y,σ,I_2).

Proof. Necessity - Let f: (X, τ, I_1) $\to$ (Y, σ, I_2) be $\hat{\delta}$-continuous and U any open set in (Y, σ, I_2). Then $f^{-1}(U^c)$ is $\hat{\delta}$-closed in (X, τ, I_1). But $f^{-1}(U^c) = [f^{-1}(U)]^c$ and so f^{-1}(U) is $\hat{\delta}$-open in (X, τ, I_1).

Sufficiency - Suppose f^{-1}(U) is $\hat{\delta}$-open in (X, τ, I_1) for every open set U in (Y,σ,I_2). Again since $f^{-1}(U^c) = [f^{-1}(U)]^c$, $f^{-1}(U^c)$ is $\hat{\delta}$-closed in X, for every closed set U^c in (Y,σ,I_2). Therefore f is $\hat{\delta}$-**c**ontinuous.

Definition 3.13 A map f: (X,τ,I_1)$\to$(Y,σ,I_2) is called δ-I-closed if the image of δ-I-closed set under f is δ-I-closed.

Theorem 3.14 Let f: (X, τ, I_1) $\to$ (Y, σ, I_2) be continuous and δ-I-closed, then for every $\hat{\delta}$-closed subset A of (X, τ, I_1), f(A) is $\hat{\delta}$-closed in (Y, σ, I_2).

Proof. Let A be $\hat{\delta}$-closed in (X, τ, I_1). Let f(A) $\subseteq$ U where U is open in (Y, σ, I_2). Since A $\subseteq$ f^{-1} (U) and A is $\hat{\delta}$-closed and since f^{-1}(U) is open in (X,τ,I_1), then σcl(A)$\subseteq f^{-1}$(U). Thus f(σcl(A))$\subseteq$U. Hence σcl(f(A))$\subseteq\sigma$cl(f(σcl(A)))= f(σcl(A))$\subseteq$U. Since f is δ-I-closed. Hence f(A) is $\hat{\delta}$-closed in (Y, σ, I_2).

Remark 3.15 Let f: (X, τ, I_1) $\to$ (Y, σ, I_2) be $\hat{\delta}$-continuous function then it is clear that f(σcl(A)) $\subset$ cl(f(A)) for every δ-I-closed subset A of X. But the converse is not true. For instant, let X = Y = {a, b, c, d}, with topologies τ = {X, ϕ, {b}, {c, d}, {b, c, d}}, σ = {Y, ϕ, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d}} and ideals I_1 = {ϕ, {c}}, I_2 = {ϕ, {a}}. Let f: (X, τ, I_1) $\to$ (Y, σ, I_2) be identity map. Then f(σcl(A)) $\subset$ cl(f(A)) for every δ-I-closed subset A of X. But for the closed set {b} of Y, f^{-1}({b}) = {b} is not $\hat{\delta}$-closed set in X. Therefore f is not $\hat{\delta}$-continuous.

Remark 3.16 Let f: (X, τ, I_1) $\to$ (Y, σ, I_2) be a continuous function then it is clear that, f(σcl(A))$\subset$cl(f(A)) for every δ-I-closed subset A of X. But the converse is not true. For instant, let X, Y, τ, σ, I_1, I_2, f be as Example given in Remark 3.15. Then f(σcl(A)) $\subset$ cl(f(A)) for every subset A of X. But for the closed set B = {b} in Y, f^{-1}(B) = {b} is not closed in X. Hence f is not continuous.

Theorem 3.17 A map f: (X, τ, I_1) $\to$ (Y, σ, I_2) from an ideal space (X, τ, I_1) into an ideal space (Y, σ, I_2) is $\hat{\delta}$-irresolute if and only if the inverse image of every $\hat{\delta}$-open set in (Y, σ, I_2) is $\hat{\delta}$-open in (X, τ, I_1).

Proof. Necessity - Assume that f is $\hat{\delta}$-irresolute. Let U be any $\hat{\delta}$-open set in (Y, σ, I_2). Then X–U is $\hat{\delta}$-closed in (Y, σ, I_2). Since f is $\hat{\delta}$-irresolute f^{-1}(X–U) is $\hat{\delta}$-closed in (X, τ, I_1). But $f^{-1}(U^c) = [f^{-1}(U)]^c$ and so f^{-1}(U) is $\hat{\delta}$-open in X. Hence the inverse image of every $\hat{\delta}$-open set in (Y, σ, I_2) is $\hat{\delta}$-open in X.

Sufficiency - Assume that the inverse image of every $\hat{\delta}$-open set in (Y, σ, I_2) is $\hat{\delta}$-open in (X, τ, I_1). Let V be any $\hat{\delta}$-closed set in (Y, σ, I_2). Then X–V is $\hat{\delta}$-open in (Y, σ, I_2). By assumption, f^{-1}(X–V) is $\hat{\delta}$-open in (X, τ, I_1). But $f^{-1}(V^c) = [f^{-1}(V)]^c$ and so f^{-1}(V) is $\hat{\delta}$-closed in (X, τ, I_1). Therefore f is $\hat{\delta}$-irresolute.

Theorem 3.18 Let every $\hat{\delta}$-closed set is δ-closed in (X, τ, I_1) and f: (X, τ, I_1) $\to$ (Y, σ, I_2) be $\hat{\delta}$-irresolute. Then f is δ-continuous.

Proof. Let F be a δ-closed subset of (Y,σ,I_2). By Theorem 3.3 [9], F is $\hat{\delta}$-closed. Since f is $\hat{\delta}$-irresolute, f^{-1}(F) is $\hat{\delta}$-closed in (X, τ, I_1). By hypothesis f^{-1}(F) is δ-closed. Then f is δ-continuous.

Remark 3.19 The converse of Theorem 3.18 is need not be true as shown in the following Example.

Example 3.20 Let X = Y = {a, b, c, d} with topologies τ = {X, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}, {a, b, c}}, σ = (Y, ϕ, {c}, {c, d}, {a, c, d}} and ideals I_1 = {ϕ, {a}, {b}, {a, b}}, I_2 = {ϕ, {b}, {d}, {b, d}}. Here every $\hat{\delta}$-closed set is δ-closed in (X, τ, I_1). Let f: (X, τ, I_1) $\to$ (Y, σ, I_2) be identity map. Then f is δ-continuous, but not $\hat{\delta}$-irresolute because for the $\hat{\delta}$-closed set A = {a, b} in (Y, σ, I_2), f^{-1}(A) = {a, b} is not $\hat{\delta}$-closed in (X, τ, I_1).

Theorem 3.21 If f : (X, τ, I_1) $\to$ (Y, σ, I_2) is bijective, open and $\hat{\delta}$-continuous then f is $\hat{\delta}$-irresolute.

Proof. Let F be any $\hat{\delta}$-closed set in Y and f^{-1}(F)$\subseteq$U, where U$\in\tau$. Then it is clear that σcl(F)$\subseteq$f(U) and therefore f^{-1}(σcl(F))$\subseteq$U. Since f is $\hat{\delta}$-continuous and σcl(F) is a closed subset of (Y, σ, I_2), σcl(f^{-1}(σcl(F)))$\subseteq$U and hence σcl(f^{-1}(F)) $\subseteq$ U. Thus f^{-1}(F) is $\hat{\delta}$-closed in (X, τ, I_1). This shows that f is $\hat{\delta}$-irresolute.

Theorem 3.22 Let f: $(X, \tau, I_1) \rightarrow (Y, \sigma, I_2)$ be surjective, $\hat{\delta}$-irresolute and δ-closed. If every $\hat{\delta}$-closed set is δ-closed in (X, τ, I_1) then the same in (Y, σ, I_2).

Proof. Let F be a $\hat{\delta}$-closed set in (Y, σ, I_2). Since f is $\hat{\delta}$-irresolute, $f^{-1}(F)$ is $\hat{\delta}$-closed in (X, τ, I_1). Then by hypothesis, $f^{-1}(F)$ is δ-closed in (X, τ, I_1). Since f is surjective and F is δ-closed in (Y, σ, I_2).

Theorem 3.23 Let f: $(X, \tau, I_1) \rightarrow (Y, \sigma, I_2)$ and g: $(Y, \sigma, I_2) \rightarrow (Z, \eta, I_3)$ be any two functions. Then the following hold.

(i) gof is $\hat{\delta}$-continuous if f is $\hat{\delta}$-irresolute and g is $\hat{\delta}$-continuous.

(ii) gof is $\hat{\delta}$-irresolute if f is $\hat{\delta}$-irresolute and g is $\hat{\delta}$-irresolute.

(iii) gof is g-continuous if f is g-irresolute and g is $\hat{\delta}$-continuous.

(iv) gof is I_g-continuous if f is Ig-irresolute and g is $\hat{\delta}$-continuous

Proof. (i) Let F be a closed set in (Z, η, I_3). Since g is $\hat{\delta}$-continuous, $g^{-1}(F)$ is $\hat{\delta}$-closed in (Y, σ, I_2). Since f is $\hat{\delta}$-irresolute, $f^{-1}(g^{-1}(F))$ is $\hat{\delta}$-closed in (X,τ, I_1). Thus $(gof)^{-1}(F) = f^{-1}(g^{-1}(F))$ is a $\hat{\delta}$-closed set in (X, τ, I_1) and hence gof is $\hat{\delta}$-continuous.

(ii) Let V be a $\hat{\delta}$-closed set in (Z, η, I_3). Since g is $\hat{\delta}$-irresolute, $g^{-1}(V)$ is $\hat{\delta}$-closed in (Y, σ, I_2). Also, since f is $\hat{\delta}$-irresolute, $f^{-1}(g^{-1}(V))$ is $\hat{\delta}$-closed in (X, τ, I_1). Thus $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is a $\hat{\delta}$-closed set in (X, τ, I_1) and hence gof is $\hat{\delta}$-irresolute.

(iii) Let F be a closed set in (Z, η, I_3). Since g is $\hat{\delta}$-continuous, $g^{-1}(F)$ is $\hat{\delta}$-closed in (Y, σ, I_2). Since every $\hat{\delta}$-closed set is g-closed $g^{-1}(F)$ is g-closed in (Y, σ, I_2). Since f is g-irresolute, $f^{-1}(g^{-1}(F))$ is g-closed in (X, τ, I_1). Thus $(gof)^{-1}(F) = f^{-1}(g^{-1}(F))$ is g-closed in (X, τ, I_1) and hence gof is g-continuous.

(iv) Let F be a closed set in (Z, η, I_3). Since g is $\hat{\delta}$-continuous $g^{-1}(F)$ is $\hat{\delta}$-closed in (Y, σ, I_2). Since every $\hat{\delta}$-closed set is Ig-closed $g^{-1}(F)$ is Ig-closed in (Y, σ, I_2). Since f is Ig-irresolute, $f^{-1}(g^{-1}(F))$ is Ig-closed in (X, τ, I_1). Thus $(gof)^{-1}(F) = f^{-1}(g^{-1}(F))$ is Ig-closed in (X, τ, I_1) and hence gof is Ig-continuous.

Remark 3.24 Composition of two $\hat{\delta}$-continuous functions need not be $\hat{\delta}$-continuous as shown in the following Example.

Example 3.25 Let X = Y = Z = {a, b, c, d} with topologies τ = {X, ϕ, {a}, {c, d}, {a, c, d}, {b, c, d}}, σ = {Y, ϕ, {a, b, c}}, η = {Z, ϕ, {b}, {c, d}, {b, c, d}} and ideals I_1 = {ϕ, {a}}, I_2 = {ϕ, {a}} and I_3 = {ϕ, {d}}. Let f: $(X, \tau, I_1) \rightarrow (Y, \sigma, I_2)$ and g: $(Y, \sigma, I_2) \rightarrow (Z,\eta,I_3)$ defined by f(a) = g(a) = d, f(b) = g(b) = b, f(c) = g(c) = c, and f(d) = g(d) = a. Then f and g are $\hat{\delta}$-continuous but their composition gof: $(X, \tau, I_1) \rightarrow (Z,\eta,I_3)$ is not $\hat{\delta}$-continuous, because for the closed set A = {a, c, d} in (Z,η,I_3), $(gof)^{-1}(A)$ = {a, c, d} is not $\hat{\delta}$-closed in (X, τ, I_1).

IV. $\hat{\delta}_s$-Continuous And $\hat{\delta}_s$–Irresolute Maps

Definition 4.1 A function f from an ideal space (X, τ, I_1) into an ideal space (Y,σ,I_2) is called $\hat{\delta}_s$–continuous if $f^{-1}(F)$ is $\hat{\delta}_s$ – closed in (X,τ, I_1) for every closed set F of (Y,σ,I_2).

Definition 4.2 A function f from an ideal space (X,τ, I_1) into an ideal space (Y,σ,I_2) is called $\hat{\delta}_s$–irresolute if $f^{-1}(F)$ is $\hat{\delta}_s$ – closed in (X,τ, I_1) for every $\hat{\delta}_s$-closed set F of (Y,σ,I_2).

Theorem 4.3 If a map f: $(X,\tau, I_1) \rightarrow (Y,\sigma,I_2)$ from an ideal space (X,τ, I_1) into an ideal space (Y,σ,I_2) is $\hat{\delta}_s$ –continuous then it is $\hat{\delta}$–continuous.

Proof : Let f: $(X,\tau, I_1) \rightarrow (Y,\sigma,I_2)$ be $\hat{\delta}_s$ –continuous and F be any closed set in (Y,σ,I_2). Then the inverse image $f^{-1}(F)$ is $\hat{\delta}_s$-closed. Since every $\hat{\delta}_s$ – closed set is $\hat{\delta}$-closed, $f^{-1}(F)$ is $\hat{\delta}$-closed in (X,τ, I_1). Therefore f is $\hat{\delta}$-continuous.

Remark 4.4 The converse of Theorem 4.3 is not always true as shown in the following Example.

Example 4.5 Let X=Y={a,b,c,d} with topologies τ={X, ϕ,{c}, {c,d}, {a,c,d}}, σ={Y, ϕ, {a}, {c,d}, {a,c,d}, {b,c,d}} and ideals I_1={ϕ,{b},{d},{b,d}}, I_2={ϕ,{a}}. Let f: $(X,\tau, I_1) \rightarrow (Y,\sigma,I_2)$ be an identity map, then F is $\hat{\delta}$-continuous but not $\hat{\delta}_s$–continuous since for the closed set F={a,b} in (Y,σ,I_2), f^{-1} (F)={a,b} is not $\hat{\delta}$–closed in (X,τ, I_1).

Theorem 4.6 If a map f: $(X,\tau, I_1) \rightarrow (Y,\sigma,I_2)$ from an ideal space (X,τ, I_1) into an ideal space (Y,σ,I_2) is $\hat{\delta}_s$ –continuous then it is g-continuous.

Proof : Let f: $(X,\tau, I_1) \to (Y,\sigma,I_2)$ be $\hat{\delta}_s$ –continuous and F be any closed set in (Y,σ,I_2) . Then the inverse image $f^{-1}(F)$ is $\hat{\delta}_s$ – closed in (X,τ, I_1). Since every $\hat{\delta}_s$ – closed set is g-closed, $f^{-1}(F)$ is g-closed in (X,τ, I_1). Therefore f is g-continuous.

Remark 4.7 The reversible implication of Theorem 4.6 is not true as shown in the following Example.

Example 4.8 Let X=Y={a,b,c,d} with topologies τ ={X, ϕ, {a}, {b}, {a,b}}, σ = {X, ϕ, {b},{a,b},{b,c},{a,b,c,},{a,b,d}} and ideals I_1={ϕ{d}}, I_2 = {ϕ, {a}, {b}, {a,b}}. Let f: $(X,\tau, I_1) \to (Y,\sigma,I_2)$ be an identify map, then f is g- continuous but not $\hat{\delta}_s$–continuous, since for the closed set F={c} in (Y,σ,I_2), f^{-1}(F)={c} is not $\hat{\delta}_s$ –closed in (X,τ, I_1) .

Theorem 4.9 If a map f: $(X,\tau, I_1) \to (Y,\sigma,I_2)$ from an ideal space (X,τ, I_1) into an ideal space (Y,σ,I_2) is $\hat{\delta}_s$ – continuous then it is w-continuous.

Proof : Let f: $(X,\tau, I_1) \to (Y,\sigma,I_2)$ be $\hat{\delta}_s$ –continuous and F be any closed set in (Y,σ,I_2). Then the inverse image $f^{-1}(F)$ is $\hat{\delta}_s$ –closed in (X,τ, I_1). Since every $\hat{\delta}_s$ –closed set is w-closed $f^{-1}(F)$ is w-closed in X. Therefore F is w-continuous.

Remark 4.10 The converse of Theorem 4.9 need not be true as seen in the following Example.

Example 4.11 Let X=Y={a,b,c,d} with topologies τ={ X, ϕ, {c}, {a,c},{b,c}, {a,b,c,},{a,c,d}}, σ = {Y,ϕ,{b},{b,c}} and ideals I_1={ϕ,{c}}, I_2={ϕ}. Let f: $(X,\tau, I_1) \to (Y,\sigma,I_2)$ be a map defined by f(a)=c, f(b)=a, f(c)=b and f(d)=d, then f is w-continuous but not $\hat{\delta}_s$ –continuous, because, for the closed set F={a,d} in (Y,σ,I_2), f^{-1}(F)={b,d} is not $\hat{\delta}_s$–closed in (X,τ, I_1).

Theorem 4.12 If a map f: $(X,\tau, I_1) \to (Y,\sigma,I_2)$ from an ideal space (X,τ, I_1) into an ideal space (Y,σ,I_2) is $\hat{\delta}_s$ – continuous, then it is I_g-continuous.

Proof : Let f: $(X,\tau, I_1) \to (Y,\sigma,I_2)$ be $\hat{\delta}_s$ –continuous and F be any closed set in (Y,σ,I_2). Then the inverse image $f^{-1}(F)$ is $\hat{\delta}_s$ –closed. Since every $\hat{\delta}_s$ –closed set is I_g-closed, $f^{-1}(F)$ is I_g-closed in (X,τ, I_1). Therefore f is I_g-continuous.

Remark 4.13 The reversible direction of Theorem 4.12 is not always true as shown in the following Example.

Example 4.14 Let X=Y={a,b,c,d} with topologies τ = {X,ϕ,{b},{d},{b,d},{b,c,d}}, σ = {Y,ϕ,{b}, {a,d}, {a,b,d},{a,c,d}} and ideals I_1={ϕ,{d}}, I_2={ϕ,{b}}. Let f:$(X,\tau, I_1) \to (Y,\sigma,I_2)$ be a map defined by f(a)=c, f(b)=a, f(c)=d, and f(d)=b, then f is I_g-continuous, but not $\hat{\delta}_s$ –continuous, because for the closed set F={b,c} in (Y,σ,I_2), f^{-1}(F)={a,d} is not $\hat{\delta}_s$ –closed in (X,τ, I_1)

Theorem 4.15 If f: $(X,\tau, I_1) \to (Y,\sigma,I_2)$ is supercontinuous then f is $\hat{\delta}_s$ –continuous.

Proof: Let f: $(X,\tau, I_1) \to (Y,\sigma,I_2)$ is supercontinuous and U be any open set in (Y,σ,I_2). Then $f^{-1}(U)$ is δ-open in (X,τ, I_1). Since $f^{-1}(U^c)=[f^{-1}(U)]^c$, $f^{-1}(U^c)$ is δ-closed, in (X,τ, I_1) for every closed Set U^c in (Y,σ,I_2). Also since every δ-closed set is $\hat{\delta}_s$ –closed $f^{-1}(U^c)$ is $\hat{\delta}_s$ –closed for every closed U^c in (Y,σ,I_2). Hence, f is $\hat{\delta}_s$ – continuous.

Remark 4.16 The following Example shows that the converse of Theorem 4.15 is not true.

Example 4.17 Let X,Y,Z,σ,I_1,I_2 and f be as in Example 3.11. Then f is $\hat{\delta}_s$–continuous but it is not supercontinuous because, for the open set U={a,d} in (Y,σ,I_2), f^{-1}(U) ={b,d} is not δ–open in (X,τ, I_1).

Theorem 4.18 Let f:$(X,\tau,I_1) \to (Y,\sigma,I_2)$ be a map from an ideal space (X,τ,I_1) into an ideal space (Y,σ,I_2), then the following are equivalent.

(i) f is $\hat{\delta}_s$ - continuous

(ii) The inverse image of each open set in Y is $\hat{\delta}_s$ open in X.

Proof: (i) $\Rightarrow$ (ii) Assume that f:$(X,\tau,I_1) \to (Y,\sigma,I_2)$ be a $\hat{\delta}_s$ – continuous. Let U be open in (Y,σ,I_2). Then U^c is closed in (Y,σ,I_2). Since f is $\hat{\delta}_s$ – continuous, $f^{-1}(U^c)$ is a $\hat{\delta}_s$ – closed in (X,τ,I_1) But $f^{-1}(U^c) = [f^{-1}(U)]^c$. Thus $[f^{-1}(U)]^c$ is $\hat{\delta}_s$ –closed in (X,τ,I_1) and so $f^{-1}(U)$ is $\hat{\delta}_s$ –open in (X,τ,I_1)

(ii) $\Rightarrow$ (i) Assume that the inverse image of each open set is $\hat{\delta}_s$ – open in (X,τ,I_1). Let F be any closed set in (Y,σ,I_2). Then F^c is open in (Y,σ,I_2). By assumption, $f^{-1}(F^c)$ is $\hat{\delta}_s$ – open in (X,τ,I_1). But $f^{-1}(F^c) = [f^{-1}(F)]^c$. Thus $[f^{-1}(F)]^c$ is $\hat{\delta}_s$ – open in (X,τ,I_1) and so $f^{-1}(F)$ is $\hat{\delta}_s$ – closed in (X,τ,I_1). Therefore f is $\hat{\delta}_s$ – continuous.

Theorem 4.19 Let f:$(X,\tau,I_1)\to(Y,\sigma,I_2)$ be a $\hat{\delta}_s$-continuous function. Then $f(\hat{\delta}_s cl(A)) \subset cl(f(A))$ for every subset A of X.

Proof: Since $f(A) \subseteq cl(f(A))$, we have $A \subseteq f^{-1}(cl(f(A)))$. Also since $cl(f(A))$ is a closed set in (Y,σ,I_2) and hence $f^{-1}(cl(f(A))$ is a $\hat{\delta}_s$-closed set containing A. Consequently $\hat{\delta}_s cl(A) \subseteq f^{-1}(cl(f(A)))$. Therefore $f(\hat{\delta}_s cl(A)) \subseteq f(f^{-1}(cl(f(A)))) \subseteq cl(f(A))$.

Remark 4.20 The following Example shows that the converse of Theorem 4.19 is not true.

Example 4.21 Let X,Y,τ, σ,I_1,I_2 and f be f as Example given in Remark 3.15. Then $f(\hat{\delta}_s cl(A)) \subset cl(f(A))$ for every subset A of X. But for the closed set A={b}, $f^{-1}\{A\}=\{b\}$ is not $\hat{\delta}_s$ – closed in X. Hence f is not $\hat{\delta}_s$– continuous.

Remark 4.22 Let $f:(X,\tau,I_1) \rightarrow (Y,\sigma,I_2)$ be a $\hat{\delta}_s$–continuous function then it is clear that $f(\sigma cl(A)) \subset cl(f(A))$ for every δ-I-closed subset A of X. The following Example shows that the converse is not true. Let $X,Y,\tau, \sigma,I_1,I_2,$ and f be as Example given in Remark 3.15. Then $f(\sigma cl(A)) \subset cl(f(A))$ for every δ-I-closed subset A of X. But for the closed set B={b}, $f^{-1}\{B\}=\{b\}$ is not $\hat{\delta}_s$ – closed in X. Therefore f is not $\hat{\delta}_s$–continuous.

Theorem 4.23 Let $:(X,\tau,I_1) \rightarrow (Y,\sigma,I_2)$ and g: $(Y,\sigma,I_2) \rightarrow (Z,\eta,I_3)$ be any two functions. Then the following hold.

(i) gof is $\hat{\delta}_s$ – continuous if f is $\hat{\delta}_s$ – continuous and g is continuous.

(ii) gof is g – continuous if f is g – irresolute and g is $\hat{\delta}_s$ – continuous.

(iii) gof is I_g – continuous if f is I_g – irresolute and g is $\hat{\delta}_s$ – continuous.

(iv) gof is w – continuous if f is w – irresolute and g is $\hat{\delta}_s$ – continuous.

(v) Let (Y,σ,I_2) be $T_{3/4}$ - space. Then gof is $\hat{\delta}_s$ – continuous if f is $\hat{\delta}_s$ – continuous and g is δg – continuous.

(vi) Let every $\hat{\delta}_s$ – closed set is δ-I-closed in (Y,σ,I_2). Then gof is $\hat{\delta}_s$ – continuous if both f and g are $\hat{\delta}_s$ – continuous.

(vii) Let (Y,σ,I_2) be $T_{1/2}$ – Space. Then gof is $\hat{\delta}_s$ – continuous if f is $\hat{\delta}_s$ – continuous and g is g – continuous

(viii) Let every Ig-closed set is closed in (Y,σ,I_2). Then gof is $\hat{\delta}_s$-continuous if f is $\hat{\delta}_s$ – continuous and g is I_g – continuous

(ix) Let (Y, σ, I_2) be $T_{1/2}$ – Space. Then gof is g- irresolute if f is $\hat{\delta}_s$ – continuous and g is g– irresolute

(x) Let every Ig-closed set is closed in (Y,σ,I_2). Then gof is I_g –irresolute if f is $\hat{\delta}_s$ – continuous and g is I_g – irresolute

Proof: (i) Let F be a closed set in (Z,η,I_3). Since g is continuous $g^{-1}(F)$ is also closed in (Y, σ, I_2). Since f is $\hat{\delta}_s$-continuous $f^{-1}(g^{-1}(F))$ is $\hat{\delta}_s$ closed in (X,τ,I_1). Thus $(gof)^{-1} = f^{-1}(g^{-1}(F))$ is $\hat{\delta}_s$ closed in (X,τ,I_1). Therefore gof is $\hat{\delta}_s$-continuous.

(ii) Let F be a closed set in (Z, η, I_3). Since g is $\hat{\delta}_s$ –continuous, $g^{-1}(F)$ is $\hat{\delta}_s$ closed in (Y, σ, I_2). Since every $\hat{\delta}_s$-closed set is g–closed, $g^{-1}(F)$ is g - closed in (Y,σ,I_2). Also, since f is g-irresolute $f^{-1}(g^{-1}(F))$ is g- closed in (X,τ,I_1) Thus $(gof)^{-1} = f^{-1}(g^{-1}(F))$ is g-closed in (X,τ,I_1). Therefore gof is g-continuous.

(iii). Since g is $\hat{\delta}_s$-continuous, for any closed set F in (Z,η,I_3), $g^{-1}(F)$ is $\hat{\delta}_s$ - closed in (Y, σ, I_2). Since every $\hat{\delta}_s$ - closed set is I_g-closed and f is I_g-irresolute, $f^{-1}(g^{-1}(F))$ is I_g – closed in (X,τ,I_1). Hence gof is I_g-continuous.

(iv) Since g is $\hat{\delta}_s$-continuous for any closed set F in (Z, η, I_3), $g^{-1}(F)$ is $\hat{\delta}_s$ - closed in (Y, σ, I_2). Since every $\hat{\delta}_s$-closed set is w–closed and f is w–irresolute , $f^{-1}(g^{-1}(F))$ is w-closed in (X, τ, I_1). Hence gof is w-continuous.

(v) Since g is δg – continuous, for every closed set F in (Z, η, I_3), $g^{-1}(F)$ is δg-closed in (Y,σ,I_2). Since by hypothesis and f is $\hat{\delta}_s$-continuous, $f^{-1}(g^{-1}(F))$ is $\hat{\delta}_s$-closed in (X, τ, I_1). Hence gof is $\hat{\delta}_s$-continuous.

(vi) Since g is $\hat{\delta}_s$-continuous, for every closed set F in (Z, η, I_3), $g^{-1}(F)$ is $\hat{\delta}_s$ - closed in (Y,σ,I_2). By hypothesis $g^{-1}(F)$ is δ-I-closed. Since every δ-I –closed set is closed, $g^{-1}(F)$ is closed in (Y, σ,I_2). Also since f is $\hat{\delta}_s$-continuous $f^{-1}(g^{-1}(F)$ is $\hat{\delta}_s$-closed in (X, τ, I_1). Therefore gof is $\hat{\delta}_s$-continuous.

(vii) Since g is g-continuous and by the assumption, for every closed set F in (Z, η, I_3), $g^{-1}(F)$ is closed. Also since f is $\hat{\delta}_s$–continuous. $f^{-1}(g^{-1}(F))$ is $\hat{\delta}_s$-closed in (X, τ,I_1). Therefore gof is $\hat{\delta}_s$–continuous.

(viii) Since g is I_g-continuous and by the assumption, for every closed set F in (Z, η, I_3), $g^{-1}(F)$ is *-closed in (Y, σ, I_2). Also, since f is $\hat{\delta}_s$-continuous, $f^{-1}(g^{-1}(F))$ is $\hat{\delta}_s$-closed in (X, τ, I_1) and hence gof is $\hat{\delta}_s$-continuous.

(ix) Since g is g-irresolute and by the assumption, for every g-closed set in (Z, η, I_3), $g^{-1}(F)$ is closed in (Y,σ,I_2). Also, since f is $\hat{\delta}_s$-continuous and every $\hat{\delta}_s$-closed set is g–closed, $f^{-1}(g^{-1}(F))$ is g–closed in (X,τ,I_1). Therefore gof is g- irresolute.

(x) Let F be an I_g-closed set in (Z, η, I_3), Since g is Ig-irresolute and by the assumption, g^{-1} (F) is closed in (Y, σ, I_2). Again, since f is $\hat{\delta}_s$ –continuous and every $\hat{\delta}_s$-closed set is I_g-closed, f^{-1} (g^{-1} (F)) is I_g-closed in (X,τ,I_1). Therefore gof is I_g- irresolute.

Remark 4.24 Composition of two $\hat{\delta}_s$-continuous functions need not be $\hat{\delta}_s$-continuous as shown in the following Example.

Example 4.25 Let X,Y,Z, τ, σ, η, I_1, I_2, I_3, f, g and A be as in Example 3.25. Then f and g are $\hat{\delta}_s$-continuous but their composition gof is not $\hat{\delta}_s$– continuous because, $(gof)^{-1}$(A) ={a,c,d} is not $\hat{\delta}_s$– closed in (X, τ, I_1).

Definition 4.26 Let (X, τ, I_1) be an ideal space and $\tau\hat{\delta}_s$ ={U⊆X : $\hat{\delta}_s$cl(X–U) = X–U}, $\tau\hat{\delta}_s$ is a topology in (X, τ, I_1).

Theorem 4.27 Let f : (X, τ, I_1) → (Y, σ, I_2) be a function from an ideal space (X, τ, I_1) into an ideal space (Y, σ, I_2) such that $\tau\hat{\delta}_s$ is a topology on (X, τ, I_1). Then the following are equivalent.

(i) For every subset A of X, f($\hat{\delta}_s$ cl(A)) ⊆ cl (f(A)) holds.

(ii) f : (X, $\tau\hat{\delta}_s$) → (Y,σ) is continuous.

Proof: (i)⇒ (ii) Let A be a closed subset in Y. By hypothesis f ($\hat{\delta}_s$ cl (f^{-1}(A))) ⊆ cl (f(f^{-1} (A)) ⊆ cl(A) = A. Therefore $\hat{\delta}_s$ cl (f^{-1}(A)) ⊆ f^{-1}(A). Also f^{-1} (A) ⊆ $\hat{\delta}_s$ cl (f^{-1}(A)). Hence $\hat{\delta}_s$ cl (f^{-1}(A)) = f^{-1}(A). Thus f^{-1}(A) is closed in (X,$\tau\hat{\delta}_s$) and so f is continuous.

(ii) ⇒ (i) Let A⊆X, then cl(f(A)) is closed in (Y, σ, I_2). Since f: (X,$\tau\hat{\delta}_s$, I_1) → (Y, σ, I_2) is continuous, f^{-1} (cl(f(A))) is closed in (X, $\tau\hat{\delta}_s$, I_1) and hence $\hat{\delta}_s$cl (f^{-1} (cl(f(A))))= f^{-1} (cl(f(A))). Since A ⊆ f^{-1}(f(A)) ⊆ f^{-1} (cl(f(A))), $\hat{\delta}_s$ cl(A) ⊆ $\hat{\delta}_s$ cl (f^{-1} (cl(f(A))) = f^{-1} (cl(f(A))). Therefore f($\hat{\delta}_s$ cl(A)) ⊆ cl (f(A)).

Theorem 4.28 Let f : (X, τ, I_1) → (Y,σ, I_2) be a function from an ideal space (X, τ, I_1) into an ideal space (Y, σ, I_2). Then the following are equivalent.

(i) For each point x in X and each open set V in Y with f(*x*)∈V, there is a $\hat{\delta}_s$-open set U in X such that x∈U and f(U)⊆V.

(ii) For each subset A of X, f ($\hat{\delta}_s$cl (A)) ⊆ cl(f(A))

(iii) For each subset G of Y, $\hat{\delta}_s$cl (f^{-1}(G) ⊆ f^{-1}(cl (G)

(iv) For each subset G of Y, f^{-1} (int(G) ⊆ $\hat{\delta}_s$int (f^{-1}(G))

Proof : (i) ⇒(ii) Let y∈f ($\hat{\delta}_s$cl (A)) and V be any open set of Y containing y. Since y ∈ f ($\hat{\delta}_s$cl (A)), there exists x∈ $\hat{\delta}_s$cl(A) such that f(*x*)=*y*. Since f(x)∈V, then by hypothesis there exists a $\hat{\delta}_s$- open set U in X such that x∈U and f(U)⊆V. Since x∈ $\hat{\delta}_s$cl(A), then by Theorem 5.7 [8] U∩A ≠ϕ. Then ϕ=f(U∩A)⊆f(U)∩f(A) ⊆ V∩f(A) and hence V∩f(A)≠ϕ. Therefore we have y=f(x)∈cl(f(A).

(ii) ⇒(i) Let x ∈ X and V be any open set in Y containing f(x). Let A = $f^{-1}(V^c)$. Then x∉A. Since f($\hat{\delta}_s$ cl(A))⊆cl(f(A)⊆V^c, $\hat{\delta}_s$ cl(A)⊆$f^{-1}(V^c)$=A. Since x∉A, x∉ $\hat{\delta}_s$ cl (A). By Theorem 5.7 [8] there exists a $\hat{\delta}_s$-open set U containing x such that U∩A = ϕ, and so U⊆A^c and hence f(U)⊆f(A^c)⊆V.

(ii) ⇒(iii) Let G be any subset of Y. Replacing A by f^{-1}(G) in (ii), we get f($\hat{\delta}_s$cl (f^{-1}(G))) ⊆ cl(f(f^{-1}(G))) ⊆ cl (G).

(iii) ⇒(ii) Put G=f(A) in (iii) we get, $\hat{\delta}_s$cl (f^{-1}(f(A))) ⊆ f^{-1}(cl(f(A))) and hence f($\hat{\delta}_s$cl(A)) ⊆ cl (f(A)).

(iii) ⇒(iv) Let G be any subset in Y. Then Y–G⊆Y. By (iii), $\hat{\delta}_s$cl(f^{-1}(Y–G)) ⊆ f^{-1} (cl(Y–G)). Therefore X–$\hat{\delta}_s$ int (f^{-1}(G)) ⊆ X – f^{-1} (int (G)) and so f^{-1} (int (G)) ⊆ $\hat{\delta}_s$ int (f^{-1}(G).

iv) ⇒(iii) Let G be any subset in Y. Then Y–G⊆Y. By (iv), f^{-1} (int (Y–G))⊆ $\hat{\delta}_s$ int(f^{-1}(Y–G)). Therefore X–f^{-1} (cl(G)) ⊆ X–$\hat{\delta}_s$ cl (f^{-1}(G)) and hence $\hat{\delta}_s$cl(f^{-1} (G)) ⊆ f^{-1}(cl(G)).

Theorem 4.29 A map f: (X, τ, I_1) → (Y, σ, I_2) is $\hat{\delta}_s$ – irresolute if and only if the inverse image of every $\hat{\delta}_s$-open set in (Y, σ, I_2) is $\hat{\delta}_s$ –open in (X, τ, I_1).

Proof : Necessity - Assume that f is $\hat{\delta}_s$ - irresolute. Let U be any $\hat{\delta}_s$ - open set in (Y,σ, I_2). Then U^c is $\hat{\delta}_s$ - closed in (Y, σ, I_2). Since f is $\hat{\delta}_s$ – irresolute, $f^{-1}(U^c)$ is $\hat{\delta}_s$ – closed in (X, τ, I_1). But $f^{-1}(U^c)$=$[f^{-1}(U)]^c$ and so f^{-1} (U) is $\hat{\delta}_s$-open in (X, τ, I_1). Hence the inverse image of every $\hat{\delta}_s$-open set in (Y, σ, I_2) is $\hat{\delta}_s$-open in (X, τ, I_1).

Sufficiency – Assume that the inverse image of every $\hat{\delta}_s$ - open set in (Y,σ, I_2) is $\hat{\delta}_s$-open in (X, τ, I_1). Let V be any $\hat{\delta}_s$- closed set in (Y, σ, I_2). Then V^c is $\hat{\delta}_s$-open in (Y,σ, I_2). By assumption, $f^{-1}(V^c)$ is $\hat{\delta}_s$-open in (X, τ, I_1). But $f^{-1}(V^c)$=$[f^{-1}(V)]^c$ and so f^{-1}(V) is $\hat{\delta}_s$-closed in (X, τ, I_1). Therefore f is $\hat{\delta}_s$-irresolute.

Theorem 4.30 Let every $\hat{\delta}_s$ - closed set is δ-closed in (X, τ, I_1). If f: (X, τ, I_1) → (Y, σ, I_2) be $\hat{\delta}_s$-irresolute. Then f is δ-continuous.

Proof : Let F be a δ–closed subset of (Y, σ, I_2). By Theorem 3.2 [8], F is $\hat{\delta}_s$-closed. Since f is $\hat{\delta}_s$-irresolute, f^{-1}(F) is $\hat{\delta}_s$-closed in (X, τ, I_1). By hypothesis f^{-1}(F) is δ–closed. Then f is δ–continuous.

Theorem 4.31 Let f: (X, τ, I_1) → (Y, σ, I_2) and g: (Y, σ, I_2) → (Z, η, I_3) be any two functions. Then the following hold.

(i) gof is $\hat{\delta}_s$ – continuous if f is $\hat{\delta}_s$ – irresolute and g is $\hat{\delta}_s$ – continuous.

(ii) gof is $\hat{\delta}_s$ – irresolute if f is $\hat{\delta}_s$ – irresolute and g is $\hat{\delta}_s$ – irresolute.

(iii) hof is g – continuous if f is g – irresolute and h is $\hat{\delta}_s$ – continuous.

(iv) gof is w – continuous if f is w – irresolute and g is $\hat{\delta}_s$ – continuous.

(v) gof is I_g- continuous if f is I_g – irresolute and g is $\hat{\delta}_s$ – continuous.

(vi) gof is $\hat{\delta}_s$ – continuous if f is $\hat{\delta}_s$ – irresolute and g is $\hat{\delta}_s$ – continuous.

Proof : (i) Since g is $\hat{\delta}_s$ – continuous, for every closed set F in (Z, η, I_3), g^{-1} (F) is $\hat{\delta}_s$ – closed in (Y, σ, I_2). Since f is $\hat{\delta}_s$ – irresolute, $f^{-1}(g^{-1}$(F) is $\hat{\delta}_s$ – closed in (X, τ, I_1).

(ii) Since g is $\hat{\delta}_s$ – irresolute, for every $\hat{\delta}_s$ – closed set F in (Z, η, I_3), g^{-1} (F) is $\hat{\delta}_s$ – closed in (Y, σ, I_2). Since f is $\hat{\delta}_s$ – irresolute, $f^{-1}(g^{-1}$(F)) is $\hat{\delta}_s$ – closed in (X, τ, I_1).

(iii) Since h is $\hat{\delta}_s$ –continuous, for every closed set F in (Z, η, I_3), h^{-1}(F) is $\hat{\delta}_s$ – closed set in (Y,σ, I_2). Since f is g-irresolute and every $\hat{\delta}_s$ - closed set is g–closed, $f^{-1}(h^{-1}$(F)) is g – closed in (X, τ, I_1).

(iv) Since g is $\hat{\delta}_s$ – continuous, f is w – irresolute and every $\hat{\delta}_s$ – closed set is w – closed, f^{-1} (g^{-1}(F)) is w – closed in (X, τ, I_1) for every closed set F in (Z,η,I_3).

(v) Since g is $\hat{\delta}_s$ – continuous, f is I_g – irresolute and every $\hat{\delta}_s$ – closed set is I_g – closed, f^{-1} (g^{-1} (F)) is I_g – closed in (X, τ, I_1) for every closed set F in (Z, η, I_3).

(vi) Since g is $\hat{\delta}_s$ – continuous, f is $\hat{\delta}$ – irresolute and every $\hat{\delta}_s$ – closed set is $\hat{\delta}$ – closed, f^{-1} (g^{-1}(F)) is $\hat{\delta}$ – closed in (X, τ, I_1) for every closed set F in (Z, η, I_3).

Theorem 4.32 (i) f:(X, τ, I_1)→ (Y, σ, I_2) is a $\hat{\delta}_s$ – continuous, surjection and X is $\hat{\delta}_s$ – connected then Y is connected.

(ii) If f:(X, τ, I_1)→ (Y, σ, I_2) is $\hat{\delta}_s$ – irresolute, surjection and X is $\hat{\delta}_s$ – connected then Y is $\hat{\delta}_s$ – connected.

Proof: (i) Suppose Y is not connected. Then Y = A∪B where A∩B = ϕ, A≠ϕ, B≠ϕ and A, B are open in Y. Since f is $\hat{\delta}_s$ – continuous and onto X = f^{-1}(A)∪ f^{-1}(B) where f^{-1}(A) and f^{-1}(B) are disjoint non-empty $\hat{\delta}_s$ – open sets in X. This contradicts the fact that X is $\hat{\delta}_s$ – connected. Hence Y is connected.

(ii) The proof is similar to the proof of (i).

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