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Common Fixed Point Theorems For Occasionally Weakely Compatible Mappings

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Abstract: *Som [11] establishes a common fixed point theorem for R-weakly Commuting mappings in a Fuzzy metric space.The object of this Paper is to prove some fixed point theorems for occasionally Weakly compatible mappings by improving the condition of Som[11].*

Keywords: *Common fixed point ,Fuzzy metric space, Compatible maps, Occasionally weakly compatible mappings*

I. Introduction

Zadeh's [13] introduction of the notion of Fuzzy set in 1965 laid the foundation of Fuzzy mathematics . George and Veeramani [4] modified the concept of Fuzzy metric space introduced by Kramosil and Michalek[8]in 1975.Vasuki [12] and Singh and Chauhan[9] introduced the concept of R-weakly commuting and compatible maps respectively

In Fuzzy metric space.Cho[2,3] introduced the concept of compatible maps Of type (α) and compatible maps of type (β) in Fuzzy metric space. Singh et. Al.[10] proved Fixed point theorems in a Fuzzy metric space. Recently in 2012 Jain et. al. [6]proved various Fixed point theorems usingThe concept of Semi compatible mapping. In this paper we have used theConcept of Occasionally weakly compatible mappings to prove further Results.

II. Preliminaries and Definations

Definition 2.1 [12] Let X be any set .A Fuzzy set A in X is a function with domain X and values in [0,1].

Definition2.2[4] A binary operation *: [0,1]× [0,1]-→[0,1] is called a continuous t- norm if an abelian topological monoid with unit 1 such that a*b≤c*d whenever a≤c and b≤d ,for all a,b.c,d in [0,1].

Definition2.3[4] The triplet (X,M,*) is said to be a Fuzzy metric space if, X is an arbitrary set,* is a continuous t- norm and M is a Fuzzy set on X×X×[0,1)

Satisfying the following conditions ; for all x,y,z in X and s,t>0,

(i) M(x,y,0)=0 ,M(x,y,t)>0;
(ii) M(x,y,t)=1 for all t>0 if and only if x=y,
(iii) M(x,y,t)=M(y,x,t),
(iv) M(x,yt)*M(y,z,t)≤M(x,z,t+s)
(v) M(x,y,·): [0,∞) → [0,1] is left continuous.
(vi) M(x,y,t)=1.

It is important to note that every metric space (X,d) induces a Fuzzy metric space (X,M,*) where a *b = min{a,b} and for all a,b ϵX

We have M(x,y,t)=t/t+d(x,y), for all t>0, and M(x,y,0)=0, so called the Fuzzy metric space induced by the metric d.

Definition2.4 [4] A sequence $\{x_n\}$ in a Fuzzy metric space (X,M,*) is called a Cauchy sequence if , $\lim_{n\to\infty} M(x_{n+p}, x_n, t)=1$ for every t>0 and for each p>0.

A Fuzzy metric space (X,M,*) is Complete if , every Cauchy sequence in X Converges in X.

Definition2.5[4] A sequence $\{x_n\}$ in a fuzzy metric space (X,M,*) is said to be Convergent to x in X if , $\lim_{n\to\infty} M(x_n, x.t)=1$, for each t>0.

Definition2.6 [1] Self mappings A and S of a Fuzzy metric space (X,M,*) are said to be Compatible if and only if $M(ASx_n, SAx_n, t)\to 1$ for all t>0, whenever $\{x_n\}$ is a sequence in X such that $Sx_n, Ax_n \to p$ for some p in X as n→∞.,

Definition2.7[7] Two self maps A and S of a Fuzzy metric space (X,M,*) are Said to be Weakly Commuting if M(ASx,SAx,t) ≥ M(Ax,Sx,t) for every xϵX.

Definition2.8 [7] Two self maps A and S of a Fuzzy metric space are R-Weakly Commuting provided there exist some positive real number R such
That $M(ASx,SAx,t) \geq M(Ax,Sx,t/R)$ for all $x \epsilon X$.

Definition 2.9 [7] Self maps A and S of a Fuzzy metric space (X,M,*) are said to be Weakly Compatible if they commute at their coincidence points,
if, AP=SP for some $p \epsilon X$ then ASp=SAp.

Definition 2.10[7] Self maps A and S of a Fuzzy metric space (X,M,*) is
said to be Occasionally weakly compatible if and only if there is a point x in X which is coincidence point of A and S at which A and S commute.

Lemma 2.1 [5] Let (X,M,*) be a Fuzzy metric space . Then for all $x,y \epsilon X$
M(x,y,.) is a non – decreasing function.

Lemma 2.2 [2] Let (X,M,t) be a Fuzzy metric space . If there exists $k \epsilon (0,1)$ such that for all $x,y \in X$,$M(x,y,kt) \geq M(x,y,t)$, for all t>0 ,then x=y.

Lemma 2.3 [10] Let $\{x_n\}$ be a sequence in a Fuzzy metric space (X,M,*) . If there exists a number $k \epsilon (0,1)$ such that $M(x_{n+2},x_{n+1},kt) \geq M(x_{n+1},x_n,t)$, for all
t>0 , and $n \epsilon N$. Then $\{x_n\}$ is a Cauchy sequence in X.

Using R-weak Commutativily , Som [ ] proved the following results:

Theorem - Let S and T be two continuous self mappings of a complete
Fuzzy metric space (X,M,*) . Let A be a self mapping of X satisfying the
Following condition :

(i) $A(X) \subset S(X) \cap T(X)$
(ii) (A,S) and (A,T) are R- weakly commuting ,
(iii) $M(Ax,Ay,t) \geq r\,(Min\{M(Sx,Ty,T),M(Sx,Ax,t),M(Sx,Ay,t),M(Ty,Ay,t)\}$
For all $x,y \in X$, where $r: [0,1] \to [0,1]$ is a continuous function such that
(iv) $r(t)>t$, for each t<1 and r(t)=1 for t =1.

Let the sequence $\{x_n\}$ and $\{y_n\}$ in X be such that $\{x_n\} \to X$ and $\{y_n\} \to y$, t>0 implices $M(x_n, y_n,t) \to M(x,y,t)$. Then A,S,T have a common fixed point in X.

III. Main Results

Now we state and prove main theorem for occasionally weakly compatible mappings.

Theorem 3.1 : Let A,S,T be self map on a complete Fuzzy metric space (X,M,*) , where * is a continuous t- norm satisfying-

(i) $A(X) \subseteq S(X) \cap T(X)$
(ii) The pair (A,S) and (A,T) are occasionally weakly compatible,
(iii) There exists $k \in (0,1)$ such that , for all $x,y \in X$ and t>0,
$M(Ax,Ay,kt) \geq \Phi\,(Min\{M(Sx,Ty,t), M(Sx,Ax,t),M(Sx,Ay,t), M(Ty,Ay,t)\})$, for all $x,y \in X$ and t>0 , where $\Phi: [0,1] \to [0,1]$
Is a continuous function such that
(iv) $\Phi(t) \geq t$ for each 0<t<1.

Then A,S,T have a common fixed point in X.

Proof: Let $x_0 \in X$ be any arbitrary point **.** Since $A(X) \sqsubseteq S(X)$ and $A(X) \subseteq T(X)$ **,** then there exists a point $x_1, x_2 \in X$ such that

Thus we can construct sequence $\{x_n\}$ and $\{y_n\}$ in X such that
$Y_{2n+1}=Ax_{2n}=Tx_{2n+1}$, $y_{2n+2}=Ax_{2n+1}=Sx_{2n+2}$, for n=0,1,…
Thus ,by inequality (iii) ,

$$M(y_{2n+1}, y_{2n+2}, kt) \geq \Phi(Min\{M(Sx_{2n},Tx_{2n+1},t) , M(Sx_{2n},Ax_{2n},t), M(Sx_{2n},Ax_{2n+1},t), M(Tx_{2n+1},Ax_{2n+1},t)\})$$
$$\geq \Phi(Min\{ M(y_{2n},y_{2n+1},t) , M(y_{2n},y_{2n+1},t), M(y_{2n},y_{2n+2},t), M(y_{2n+1},y_{2n+1},t) \})$$

$M(y_{2n+1},y_{2n+2} , kt) \geq \Phi\, M(y_{2n}, y_{2n+1},t)$

Similarly , $M(y_{2n+2}, y_{2n+3}, kt) \geq \Phi\, M(y_{2n+1},y_{2n+1},,t)$

Now, generally

$M(y_{n+1},y_n,kt) \geq \Phi M\,(y_n,y_{n+!},t)$

Therefore , $M(y_{n+!},y_n ,kt)$ is an increasing sequence of positive real numbers in [0,1] and tends to limit $L \leq 1$.

We claim that L=1 . if L<1 , then
$M(y_{n+!},y_n, kt) > \Phi\,(M(y_n , y_{n-1} , t)$. On letting $n \to \infty$ we get

$\lim_{n\to\infty} M(y_{n+!}, y_n, kt) \geq \Phi(\lim_{n\to\infty} M(y_n, y_{n-1}, t))$

That is $L \geq \Phi(L) > L$

a contradiction . Now for any positive integer m ,

$M(Ax_n, Ax_{n+m}, kt) \geq M(Ax_n, Ax_{n+1}, t/m) * M(Ax_{n+1}, Ax_{n+2}, t/m) * \ldots$
$* M(Ax_{n+m+1}, Ax_{n+p}\ t/m)$
$> (1-\varepsilon) * (1-\varepsilon) * \ldots$ m – times = $1-\varepsilon$

Thus , $M(Ax_n, Ax_{n+m}, kt) > 1-\varepsilon$, for all $t > 0$.

Hence $\{Ax_n\}$ is a Cauchy sequence in X.

Since X is complete $\{Ax_n\} \to z_1 \in X$. Hence the subsequences $\{Sx_n\}$ and $\{Tx_n\}$ of $\{Ax_n\}$ also converges to z_1 in X .

We have also the following subsequence ,

$\{Ax_{2n+1}\} \to z_1$, and $\{Tx_{2n+1}\} \to z_1$.

Since , $A(X) \subseteq S(X)$ then exists a point $p \in X$ such that$Sp=z_1$

Then by (iii) , we have

$M(Ap, Ax_n, kt) \geq \Phi(\text{Min}\{M(Sp, Tx_n, t), M(Sp, Ap, t), M(Sp, Ax_n, t), M(Tx_n, Ax_n, t)\}$

On letting $n\to\infty$, we have

$M(Ap, z_1, kt) \geq \Phi(\text{Min}\{(z_1, z_1, t), M(z_1, Ap, t), M(z_1, z_1, t), M(z_1, z_1, t)\})$
$\geq \Phi(\text{Min}\{1, M(z_1, Ap, t), 1, 1\})$
$> M(z_1, Ap, t)$

Which gives , $Ap=z_1$

Therefore , $Ap= z_1= Sp$

Similarly , since $A(X) \subseteq T(X)$, there must exists a point $q \in X$, such that
$z_1=Tq$

Then by (iii), we have

$Aq= z_1= Tq$

Hence , $Ap=z_1 =Sp= Aq = Tq$.

Since , (A,S) is Occasionally weakly compatible , therefore we have

$ASp = SAp \Rightarrow Az_1 = Sz_1$

Similarly , (A,T) is Occasionally weakly compatible , then we have

$ATq = TAq \Rightarrow Az_1 = Tz_1$

Now ,by (iii) , we have (at $x=z_1$, $y=x_{2n+1}$)

$M(Az_1, Ax_{2n+1}, kt) \geq \Phi(\text{Min}\{M(Sz_1, Tx_{2n+1}, t), M(Sz_1, Az_1, t), M(Sz_1, Ax_{2n+1}, t),$
$M(Tx_{2n+1}, Ax_{2n+1}, t)\})$

$M(Az_1, Ax_{2n+1}, kt) \geq \Phi(\text{Min}\{M(Az_1, Tx_{2n+1}, t), M(Az_1, Az_1, t), M(Az_1, Ax_{2n+1}, t),$
$M(Tx_{2n+1}, Ax_{2n+1}, t)\})$

Taking the limit $n\to\infty$, we have

$M(Az_1, z_1, kt) \geq \Phi(\text{Min}\{M(Az_1, z_1, t), 1, (Az_1, z_1, t), M(z_1, z_1, t)\})$

$M(Az_1, z_1, kt) \geq M(Az_1, z_1, t)$.

Therefore by lemma 2.2 , we have

$Az_1= z_1$ .Since $Az_1=Sz_1$ and $Az_1=Tz_1$,

Thus we have , $z_1=Az_1=Sz_1=Tz_1$.

Hence z_1 is common fixed point of A, S , and T.

Uniqueness – Let z_1 and z_2 be two common fixed points of the maps A ,S , and T .

Then ,

$z_1=Az_1=Sz_1=Tz_1$ and $z_2=Az_2=Sz_2=Tz_2$

Now ,by (iii), we have (at $x=z_1$, $y=z_2$)

$M(Az_1, Az_2, kt) \geq \Phi(\text{Min}\{M(Sz_1, Tz_2, t), M(Sz_1, Az_1, t), M(Sz_1, Az_2, t), M(Tz_2, Az_2, t)\})$

$M(z_1, z_2, kt) \geq \Phi(\text{Min}\{M(z_1, z_2, t), M(z_1, z_1, t), M(z_1, z_2, t), M(z_2, z_2, t)\})$

$M(z_1, z_2, kt) \geq \Phi(\text{Min}\{M(z_1, z_2, t), 1, M(z_1, z_2, t), 1\})$

$M(z_1, z_2, kt) \geq \Phi(\text{Min}\{M(z_1, z_2, t)\})$

Therefore , by lemma 2.2 , we have , $z_1=z_2$.

Hence z is the unique common fixed point of the three self maps A ,S and T.

This completes the proof.

IV. Conclusion

Theorem is a generalization of the result of Som [11] in the sence that condition of R-weakly commuting of the pairs of self maps has been Restricted to Occasionally weakly compatible self maps and the requirement Of continuity is completely removed.

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