IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 1 Ver. I (Jan. - Feb. 2016), PP 36-49
www.iosrjournals.org

Fuzzy Inventory Model Fornon-Instantaneous Deteriorating Items With Stock and Time Dependent Demand with Partial Backlogging and Permissible Delay in Payments

D. Chitra*, Dr. P.Parvathi**
** Assistant Professor, Department of Mathematics, Quiad-E-Millath Government College for Women(Autonomous), Chennai - 600002*
*** Head & Associate Professor, Department of Mathematics, Quiad-E-Millath Government College for Women(Autonomous), Chennai - 600002*

***Abstract**: This paper proposes, a Fuzzy economic order quantity based Inventory model for Non-Instantaneous deteriorating items under trade credit period. This model aids in minimizing the Fuzzy total inventory cost by finding an optimal replenishment policy. In this model, shortages are allowed and partially backlogged. The backlogging rate varies inversely as the waiting time for the next replenishment. All the inventory costs involved here are taken as pentagonal Fuzzy numbers. Graded mean representation method is used to defuzzify the model. The model is illustrated with the help of numerical examples. Sensitivity analysis of the optimal solution with respect to various parameters of the system is carried out and the results are discussed in detail.*
***Keywords and Phrases:** Stock and time dependent demand, Partial backlogging, Non-instantaneous deterioration, pentagonal fuzzy numbers.*

I. Introduction

The Fuzzy set theory in inventory modeling is the closest possible approach to reality. As reality is not exact and can only be calculated to some extent. Same way, Fuzzy theory helps one to incorporate unpredictability in the design of the model, thu bringing it closer to reality.

Many researcher assume that the deterioration of an item in an inventory starts from the instant of their arrival in stock. Infact most goods would have span of maintaining quality or original condition.(e.g. Vegetables, Fruits, Fish, Meat and so on), namely, during that period there is no deterioration is occurring defined as "non-instantaneous deterioration". In the real world, this type of phenomenon exist commonly such as first hand vegetables and fruits have short span of maintaining fresh quality, in which there is almost no spoilage. After words, some of the items will start to decay. For this kind of items, the assumption that the deterioration is starts from the instant of arrival in stock may cause retailer to make in appropriate replenishment policies due to over value the total annual relevant inventory cost. Therefore, in the field of inventory management, it is necessary to consider the inventory problems for Non-instantaneous deteriorating items.

Deterioration plays a significant role in many inventory system. Deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of a commodity that results in decrease usefulness. Most physical good undergo decay or deterioration over time, examples being medicines, volatile liquids, blood banks and so on. So decay or deterioration of physical goods in stock is very realistic factor and there is big need to consider this in inventory modeling.

In recentyears,inventoryproblemsfordeterioratingitemshavebeenwidelystudied after Ghare and Schrader [4] .They presented an EOQ model for an exponentially decaying inventory. Later Covertand Philip [3]formulated the model with variable deterioration rate with two-parameter Weibull distribution. Philip [8] then developed the inventory model with a three-parameter Weibull distribution rate without shortages. Shah[10] extended Philip's model and considered that shortage was allowed. Goyal and Giri [13] provided a detailed review of deteriorating inventory literatures. Sana, Goyal and Chaudhuri[9] developed production inventory model for deteriorating items. In all the above literatures, almost all the inventory models for deteriorating itemsassumethatthe deterioration occursas soonasthe retailerreceives the commodities. However, in real life, most of the goods would have a span of maintaining quality or the original condition, for some period. That is during that period there was no deterioration occurring. We term the phenomenon as "Non-instantaneous deterioration". Recently, Wuet.al. [12]developed an inventory model for Non-instantaneous deteriorating items with stock–dependent demand.

Furthermore, when the shortages occur, it is assumed that it is either completely backlogged or completely lost. But practically some customers are willing to wait for back order and others would turn to buy from other sellers. Researchers such as Park [7], Hollier and Mak[6] and Wee[11] developed inventory models with partial backorders.GoyalandGiri[5]developedproductioninventorymodelwithshortages partially backlogged.

In framing the traditional inventory model it was assumed that payment must be made to the supplier for the items immediately after receiving consignment. However, in practice for encouraging the retailer to buy more, the supplier allows a certail fixed period for settling the account and does not charge any interest from retailer on the amount owed during this period. R.Udhayakumar et.al [2] have developed a model for Non-instantaneous deteriorating inventory system with partial backlogging under trade credit period.

This model is an extension of R. Udhayakumar et.al.[1] by taking stock and time dependent demand with partial backlogging under trade credit period in a fuzzy environment.

All the inventory costs involved here are taken as pentagonal fuzzy numbers. Graded mean representation method is used for defuzzification. An optimization framework is presented to derive optimal replenishment policy where the total cost is to be minimized. Numerical examples are provided to illustrate the optimizationprocedure.Inaddition,thesensitivityanalysisoftheoptimalsolution with respect to parameters of the system is carried out.

II. Fuzzy Preliminaries

Definition 1

Let X denotes a universal set. Then the fuzzy subset $\tilde{A}$ of X is defined by its membership function $\mu_{\tilde{A}}(x) : X \to [0,1]$ which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval [0,1], to each element x∈X where the value of $\mu_{\tilde{A}}(x)$ at x shows the grade of membership of x

Definition 2

A fuzzy set $\tilde{A}$ on R is convex if $\tilde{A}\ (\lambda x_1 + (1-\lambda)x_2) \geq min\left[\tilde{A}(x_1), \tilde{A}(x_2)\right]$ for all $x_1, x_2 \in R$ and $\lambda \in [0,1]$.

Definition 3

A fuzzy set $\tilde{A}$ in the universe of discourse X is called as a fuzzy number in the universe of discourse X.

Definition 4

A pentagonal fuzzy number (PFN)[9] $\tilde{A} = (a, b, c, d, e)$ is represented with membership function $\mu_{\tilde{A}}$

As:

$$\mu_{\tilde{A}}(x) = \begin{cases} L_1(x) = \dfrac{x-a}{b-a}, & a \leq x \leq b \\ L_2(x) = \dfrac{x-b}{c-b}, & b \leq x \leq c \\ 1 \quad , & x = c \\ R_1(x) = \dfrac{d-x}{d-c}, & c \leq x \leq d \\ L_2(x) = \dfrac{e-x}{e-d}, & d \leq x \leq e \\ 0 \quad , & otherwise \end{cases}$$

The a-cut of $\tilde{A} = (a, b, c, d), 0 \leq a \leq 1$ is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$ where

$A_{L_1}(\alpha) = a + (b-a)\alpha = L_1^{-1}(\alpha)$, $A_{L_2}(\alpha) = b + (c-b)\alpha = L_2^{-1}(\alpha)$ and

$A_{R_1}(\alpha) = d - (d-c)\alpha = R_1^{-1}(\alpha)$ $A_{R_2}(\alpha) = e - (e-d)\alpha = R_2^{-1}(\alpha)$

So $L^{-1}(\alpha) = \dfrac{L_1^{-1}(\alpha) + L_2^{-1}(\alpha)}{2} = \dfrac{a + (b-a)\alpha + b + (c-b)\alpha}{2} = \dfrac{a + b + (b - a + c - b)\alpha}{2} = \dfrac{a + b + (c-a)\alpha}{2}$

$$R^{-1}(\alpha) = \frac{R_1^{-1}(\alpha) + R_2^{-1}(\alpha)}{2} = \frac{d - (d-c)\alpha + e - (e-d)\alpha}{2} = \frac{d + e - (d - c + e - d)\alpha}{2} = \frac{d + e - (e-c)\alpha}{2}$$

Definition 5:

If $\tilde{A} = (a, b, c, d, e)$ is a pentagonal fuzzy number then the graded mean integration representation of $\tilde{A}$

Is defined as $P(\tilde{A}) = \dfrac{\int_0^{W_A} h\left(\dfrac{L^{-1}(h) + R^{-1}(h)}{2}\right)dh}{\int_0^{W_A} hdh}$ with $0 \le h \le W_A$ and $0 \le W_A \le 1$

$$P(\tilde{A}) = \frac{1}{2}\frac{\int_0^1 h\left(\dfrac{a + b + (c - a)h}{2} + \dfrac{d + e - (e - c)h}{2}\right)dh}{\int_0^1 hdh} = \frac{a + 3b + 4c + 3d + e}{12}$$

III. Assumptions And Notations

The following assumptions are made:

1 . TheConsumptionrateD(t)attimetisassumedtobe

$$D(t) = \begin{cases} \alpha + \beta I(t) + ct, & 0 \le t \le t_d, t_d \le t \le t_1 \\ \alpha + ct, & t_1 \le t \le T \end{cases}$$

where$a$isa positive constant, b is the time-dependent consumption rate parameter, 0≤b≤1

2. The replenishment rate is infinite and lead time is zero.

3. Shortages are allowed and the backlogged rate is defined to be $\dfrac{1}{1 + \delta(T - t)}$ when inventory is negative. The backlogging parameter δ is a positive constant.

4. It isassumedthatduringcertainperiodoftimetheproducthasnodeterioration (i.e.,freshproducttime).Afterthisperiod,afraction,θ (0<θ<1),ofthe on-hand inventory deteriorates.

5. Product transactions are followed by instantaneous cash flow.

6. During the trade credit period, M, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the period, the retailer pays off all units bought, and starts to pay the capital opportunity cost for the items in stock.

The following notations are used:

$\tilde{K}$	-	Fuzzy Ordering cost of inventory, \$ per order.
$I(t)$	-	The inventory level at time t.
θ	-	Deterioration rate, a fraction of the onhand inventory,follows weibull two parameter distribution.
$\tilde{p}$	-	Fuzzy Purchase cost, \$ per unit
$\tilde{p}_1$	-	Fuzzy selling price per unit time
$\tilde{h}$	-	Fuzzy Holding cost excluding interest charges, \$ per unit/ year.
$\tilde{s}$	-	Fuzzy Shortage cost, \$ per unit/year
$\tilde{\pi}$	-	Fuzzy Opportunity cost due to lost sales, \$ per unit
T	-	The length of replenishment cycle.
M	-	Permissible delay in settling the accounts and $0 < M < 1$.
t_d	-	thelengthoftimeinwhich theproducthasnodeterioration(Freshproduct)
t_1		Time at which shortages starts, $0 \le t_1 \le T$.
$\tilde{I}_e$	-	Fuzzy Interest which can be earned, \$ /year

$\tilde{I}_r$	-	Fuzzy Interest charges which invested in inventory, \$ /year $I_r \geq I_e$
I_m	-	Maximum Inventory level.
I_b	-	Maximum amount of shortage demand to be backlogged.
$T\tilde{C}(t_1, T)$	-	The Fuzzy average total inventory cost per unit time.
$T\tilde{C}_1(t_1, T)$	-	The Fuzzy average total inventory cost per unit time for case(1) $0 < M \leq t_d$
$T\tilde{C}_2(t_1, T)$	-	The Fuzzy average total inventory cost per unit time for case(2) $t_d < M \leq t_1$
$T\tilde{C}_3(t_1, T)$	-	The Fuzzy average total inventory cost per unit time for case(3) $M > t_1$
$TC_{DG}(t_1, T)$	-	Defuzzified average total inventory cost per unit time.

IV. Model Formulation

The inventory system evolves as follows: I_m units of items arrive at the inventory system at the beginning of each cycle. During the time interval $[0, t_d]$, the inventory level is decreasing only owing to demand rate. The inventory level is dropping to zero due to demand and deteriorating during the time interval [t_d, t_1]. Then the shortage interval keeps to the end of the current order cycle. The whole process is repeated.

Based on the above description, the differential equation representing the inventory status is given by

$$\frac{dI(t)}{dt} = \begin{cases} -(\alpha + \beta I(t) + ct) & 0 \leq t \leq t_d, \\ -(\alpha + \beta I(t) + ct) - \theta I(t) & t_d \leq t \leq t_1, \\ \dfrac{-(\alpha + ct)}{1 + \delta(T - t)} & t_1 \leq t \leq T \end{cases} \tag{1}$$

With the boundary conditions $I(0)=I_m$, $I(t_1)=0$.
The solution of Eq.(1) is

$$I(t) = \begin{cases} I_1(t) & if \quad 0 \leq t \leq t_d, \\ I_2(t) & if \quad t_d \leq t \leq t_1, \\ I_3(t) & if \quad t_1 \leq t \leq T, \end{cases}$$

$$\frac{dI_1(t)}{dt} + \beta I_1(t) = -(\alpha + ct) \qquad 0 \leq t \leq t_d \tag{2}$$

$$\frac{dI_2(t)}{dt} + (\beta + \theta) I_2(t) = -(\alpha + ct) \qquad t_d \leq t \leq t_1 \tag{3}$$

$$\frac{dI_3(t)}{dt} = -\frac{-(\alpha + ct)}{1 + \delta(T - t)} \qquad t_1 \leq t \leq T \tag{4}$$

Solution of (2), (3), (4) is given by

$$I_1(t)=\left\{\frac{c}{\beta}(t_d-t)+\left(\frac{\alpha}{\beta}-\frac{c}{\beta^2}\right)\left(e^{-\beta t}-e^{-\beta t_d}\right)+\left[\frac{\alpha(\beta+\theta)-c}{(\beta+\theta)^2}\right]\left(e^{(\beta+\theta)(t_1-t_d)}-1\right)\right.$$
$$\left.+\frac{c}{\beta+\theta}\left(t_1e^{(\beta+\theta)(t_1-t_d)}-1\right)\right\},\qquad 0\le t\le t_d \tag{5}$$

$$I_2(t)=\left\{\left[\frac{\alpha(\beta+\theta)-c}{(\beta+\theta)^2}\right]\left(e^{(\beta+\theta)(t_1-t_d)}-1\right)+\frac{c}{\beta+\theta}\left(t_1e^{(\beta+\theta)(t_1-t_d)}-1\right)\right\},\qquad t_d\le t\le t_1 \tag{6}$$

$$I_3(t)=\left\{\frac{1}{\delta^2}\left[\alpha\delta+c(1+\delta T)\right]\left\{\log(1+\delta(T-t))-\log(1+\delta(T-t_1))\right\}+\frac{c}{\delta^2}\left[\delta(t-t_1)\right]\right\},\qquad t_d\le t\le t_1 \tag{7}$$

Order Quantity is given by

$$Q=I_m+I_b \tag{8}$$

The Fuzzy total inventory cost per cycle consist of the following elements.

(a) The Fuzzy cost of placing orders per cycle is $\tilde{K}$

(b) Fuzzy inventory holding cost HC per cycle is given by

$$HC=\tilde{h}\int_0^{t_d}I_1(t)dt+\tilde{h}\int_{t_d}^{t_1}I_2(t)dt$$

$$HC=\tilde{h}\left\{\frac{c}{\beta}\left(\frac{t_d^{\,2}}{2}\right)-\left(\frac{\alpha}{\beta}-\frac{c}{\beta^2}\right)\left(e^{-\beta t_d}\left(\frac{1}{\beta}+t_d\right)\right)-\frac{1}{\beta}+\left[\frac{\alpha(\beta+\theta)-c}{(\beta+\theta)^2}\right]\left[e^{(\beta+\theta)(t_1-t_d)}\left(t_d+\frac{1}{\beta+\theta}\right)-\frac{1}{\beta+\theta}-t_1\right]\right.$$
$$\left.+\frac{c}{\beta+\theta}\left[t_1e^{(\beta+\theta)(t_1-t_d)}\left(t_d+\frac{1}{\beta+\theta}\right)-\frac{t_1}{\beta+\theta}-t_1\right]\right\} \tag{9}$$

(c) The Fuzzy deterioration cost per cycle DC is given by

$$DC=\tilde{p}\,\theta(t)\int_{t_d}^{t_1}I_2(t)dt$$

$$=\tilde{p}\,\theta\left\{\left(\frac{\alpha(\beta+\theta)-c}{(\beta+\theta)^2}+\frac{ct_1}{\beta+\theta}\right)\left(\left(e^{(\beta+\theta)(t_1-t_d)}-1\right)+(t_d-t_1)\right)\right\} \tag{10}$$

(d) The Fuzzy shortage cost per cycle SC is given by

$$SC=\tilde{s}\int_{t_1}^{T}-I_3(t)dt$$

$$=-\frac{\tilde{s}}{\delta^2}\left[\alpha\delta+c(1+\delta T)\right]\left\{\delta(T-t_1)-\log(1+\delta(T-t_1))\right\}-\frac{\tilde{s}\,c}{\delta^2}\left[\frac{\delta T}{2}(T-2t_1)+\frac{\delta t_1^{\,2}}{2}\right] \tag{11}$$

(e) The Fuzzy opportunity cost per cycle due to lost sales OC is given by

$$OC=\tilde{\pi}\int_{t_1}^{T}\left(\alpha+ct-\frac{\alpha+ct}{1+\delta(T-t)}\right)dt$$

$$OC=\tilde{\pi}\left(\alpha(T-t_1)+\frac{c(T^2-t_1^2)}{2}+\frac{c(T-t_1)}{\delta}-\frac{\log(1+\delta T-t_1\delta)(\alpha+\frac{c}{\delta}+cT)}{\delta}\right) \quad (12)$$

CASE (1): $0<M\leq t_d$ (See figure 1)

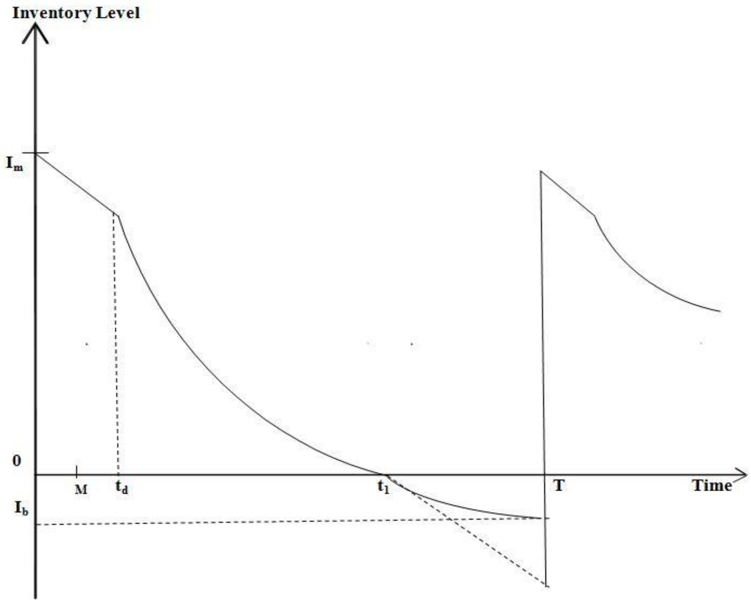


Figure: 1

When the end point of the credit period is shorter than or equal to the length of period with positive inventory stock of items ($M\leq t_1$), payment for goods is settled and retailer starts paying the interest for the goods still in stocks with annual rate $\tilde{I}_r$ Thus the fuzzy interest payable denoted by IP_1 and it is given by

$$IP_1=\tilde{p}\tilde{I}_r\int_{M}^{t_d}I_1(t)dt+\tilde{p}\tilde{I}_r\int_{t_d}^{t_1}I_2(t)dt$$

$$IP_1=\tilde{p}\tilde{I}_r\left(\frac{ct_d}{2\beta}+\left(\frac{\alpha}{\beta}-\frac{c}{\beta^2}\right)\left(\frac{-e^{-\beta t_d}}{\beta}-e^{-\beta t_d}t_d\right)+\frac{(\alpha(\beta+\theta)-c)(e^{(\beta+\theta)(t_1-t_d)}-1)(t_1-M)}{(\beta+\theta)^2}\right.$$
$$\left.+\frac{c(t_1e^{(\beta+\theta)(t_1-t_d)}-1)(t_1-M)}{(\beta+\theta)}-\frac{c\left(t_dM-M^2/2\right)}{\beta}-\left(\frac{\alpha}{\beta}-\frac{c}{\beta^2}\right)\left(-\frac{e^{-\beta M}}{\beta}-e^{-\beta t_d}M\right)\right) \quad (13)$$

We assume that during the time when the account is not settled, the retailer sells the goods and continues to accumulate sales revenue and earns the interest with a rate $\tilde{I}_e$ thus the Fuzzy interest earned per cycle is given by IE_1

$$IE_1=\tilde{p}_1\tilde{I}_e\int_0^M(\alpha+\beta I_1(t)+ct)tdt$$

$$IE_1 = \tilde{p}_1 \tilde{I}_e \left(\frac{M^2(\alpha + ct_d)}{2} - \frac{M^2\alpha}{2e^{\beta t_d}} + \frac{(\alpha\beta - c)(-\beta M e^{-\beta M} - e^{-\beta M})}{\beta^3} + \frac{cM^2}{2\beta e^{\beta t_d}} + \left(\frac{\beta^2 M^2 \alpha}{2(\beta+\theta)^2} + \frac{\alpha\beta\theta M^2}{2(\beta+\theta)^2} - \frac{c\beta M^2}{2(\beta+\theta)^2} \right)\left(\frac{e^{\beta t_1} e^{\theta t_1}}{e^{\beta t_d} e^{\theta t_d}} - 1 \right) + \frac{c\beta M^2}{2(\beta+\theta)}\left(\frac{t_1 e^{\beta t_1} e^{\theta t_1}}{e^{\beta t_d} e^{\theta t_d}} - 1 \right) - \left(\frac{-\alpha\beta + c}{\beta^3} \right) \right) \tag{14}$$

Thus the total Fuzzy annual cost which is a function of t_1 and T is given by

$$T\tilde{C}_1(t_1, T) = \frac{\tilde{K} + HC + DC + SC + OC + IP_1 - IE_1}{T}$$

$$= \frac{1}{T}\left\{ \begin{array}{l} \tilde{K} + \tilde{h}\left\{ \frac{c}{\beta}\left(\frac{t_d^2}{2}\right) - \left(\frac{\alpha}{\beta} - \frac{c}{\beta^2}\right)\left(e^{-\beta t_d}\left(\frac{1}{\beta} + t_d\right)\right) - \frac{1}{\beta} + \left[\frac{\alpha(\beta+\theta) - c}{(\beta+\theta)^2}\right]\left[e^{(\beta+\theta)(t_1 - t_d)}\left(t_d + \frac{1}{\beta+\theta}\right) - \frac{1}{\beta+\theta} - t_1 \right] \right. \\ \left. + \frac{c}{\beta+\theta}\left[t_1 e^{(\beta+\theta)(t_1-t_d)}\left(t_d + \frac{1}{\beta+\theta}\right) - \frac{t_1}{\beta+\theta} - t_1 \right]\right\} + \tilde{p}\,\theta\left\{\left(\frac{\alpha(\beta+\theta) - c}{(\beta+\theta)^2} + \frac{ct_1}{\beta+\theta}\right)\left(\left(e^{(\beta+\theta)(t_1-t_d)} - 1\right) + (t_d - t_1)\right)\right\} \\ - \frac{\tilde{s}}{\delta^2}[\alpha\delta + c(1+\delta T)]\{\delta(T - t_1) - \log(1 + \delta(T - t_1))\} - \frac{\tilde{s}\,c}{\delta^2}\left[\frac{\delta T}{2}(T - 2t_1) + \frac{\delta t_1^2}{2}\right] + \tilde{\pi}\left\{\alpha(T - t_1) + \frac{c(T^2 - t_1^2)}{2} \right. \\ \left. + \frac{c(T - t_1)}{\delta} - \frac{\log(1 + \delta T - t_1\delta)(\alpha + \frac{c}{\delta} + cT)}{\delta}\right\} + \tilde{p}\tilde{I}_r\left\{\frac{ct_d}{2\beta} + \left(\frac{\alpha}{\beta} - \frac{c}{\beta^2}\right)\left(\frac{-e^{-\beta t_d}}{\beta} - e^{-\beta t_d} t_d\right) \right. \\ \left. + \frac{(\alpha(\beta+\theta) - c)\left(e^{(\beta+\theta)(t_1-t_d)} - 1\right)(t_1 - M)}{(\beta+\theta)^2} + \frac{c\left(t_1 e^{(\beta+\theta)(t_1-t_d)} - 1\right)(t_1 - M)}{(\beta+\theta)} - \frac{c\left(t_d M - M^2/2\right)}{\beta} - \left(\frac{\alpha}{\beta} - \frac{c}{\beta^2}\right)\left(-\frac{e^{-\beta M}}{\beta} - e^{-\beta t_d} M\right)\right\} \\ - \tilde{p}_1\tilde{I}_e\left(\frac{M^2(\alpha + ct_d)}{2} - \frac{M^2\alpha}{2e^{\beta t_d}} + \frac{(\alpha\beta - c)(-\beta M e^{-\beta M} - e^{-\beta M})}{\beta^3} + \frac{cM^2}{2\beta e^{\beta t_d}} \right. \\ \left. + \left(\frac{\beta^2 M^2\alpha}{2(\beta+\theta)^2} + \frac{\alpha\beta\theta M^2}{2(\beta+\theta)^2} - \frac{c\beta M^2}{2(\beta+\theta)^2}\right)\left(\frac{e^{\beta t_1} e^{\theta t_1}}{e^{\beta t_d} e^{\theta t_d}} - 1\right) + \frac{c\beta M^2}{2(\beta+\theta)}\left(\frac{t_1 e^{\beta t_1} e^{\theta t_1}}{e^{\beta t_d} e^{\theta t_d}} - 1\right) - \left(\frac{-\alpha\beta + c}{\beta^3}\right)\right) \end{array} \right\} \tag{15}$$

Equation (15) is convex with respect to t_1, T i.e.

$$\frac{\partial T\tilde{C}_1(t_1, T)}{\partial t_1} = 0 \quad \text{………....(16)}$$

and $\frac{\partial T\tilde{C}_1(t_1, T)}{\partial T} = 0$ ………....(17)

provided they satisfy the sufficient conditions

$$\left[\frac{\partial T\tilde{C}_1(t_1, T)}{\partial t_1}\right]_{at\ (t_1^*, T^*)} > 0,$$

$$\left[\frac{\partial T\tilde{C}_1(t_1, T)}{\partial T}\right]_{at\ (t_1^*, T^*)} > 0$$

and $$\left[\left(\frac{\partial T\tilde{C}_1(t_1, T)}{\partial t_1}\right)\left(\frac{\partial T\tilde{C}_1(t_1, T)}{\partial T}\right) - \left[\frac{\partial^2 T\tilde{C}_1(t_1, T)}{\partial t_1 \partial T}\right]^2 \right]_{at\ (t_1^*, T^*)} > 0$$

4.1 Algorithm – 1 [Case(1)]

Step 1: Obtain the values of t_1 and T by fixing the value of M using the above equations.

Step 2: compare t_1 and M

1. If $M \le t_1$, t_1 is feasible. Then go to step (3).
2. If $M > t_1$, t_1 is not feasible. Set $t_1 = t_d$ in the equation (17) to calculate T and then go to step 3.

Step 3: Calculate the corresponding $T\tilde{C}_1\left(t_1^*, T^*\right)$

Case(2): $t_d < M \le t_1$ (See figure 2)

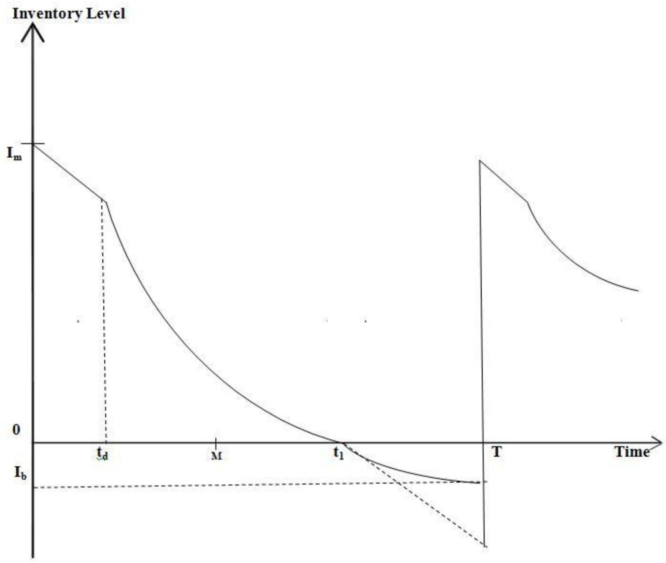


Figure: 2

Thus the Fuzzy interest payable denoted by IP_2 and it is given by

$$IP_2 = \tilde{p}\tilde{I}_r \int_M^{t_1} I_2(t)\,dt$$

$$= \tilde{p}\tilde{I}_r\left(\frac{c(t_1-1)(t_1-M)}{(\beta+\theta)}\right) \qquad (18)$$

The Fuzzy interest earned from the accumulated sales during this period

$$IE_2 = \tilde{p}_1\tilde{I}_e \int_0^M (\alpha + \beta I_2(t) + ct)\,t\,dt$$

$$= \tilde{p}_1\tilde{I}_e\left(\frac{cM^3}{3} + \frac{1}{2}\left(\alpha + \beta\left(\frac{(\alpha(\beta+\theta)-c)\left(e^{(\beta+\theta)(t_1-t_d)}-1\right)}{(\beta+\theta)^2} + \frac{c\left(t_1 e^{(\beta+\theta)(t_1-t_d)}-1\right)}{(\beta+\theta)}\right)\right)M^2\right) \qquad (19)$$

Thus the total Fuzzy annual cost which is a function of t_1 and T is given by

$$T\tilde{C}_2(t_1,T) = \frac{\tilde{K} + HC + DC + SC + OC + IP_2 - IE_2}{T}$$

$$= \frac{1}{T}\left\{\begin{array}{l} \tilde{K} + \tilde{h}\left\{ \frac{c}{\beta}\left(\frac{t_d^{\,2}}{2}\right) - \left(\frac{\alpha}{\beta} - \frac{c}{\beta^2}\right)\left(e^{-\beta t_d}\left(\frac{1}{\beta} + t_d\right)\right) - \frac{1}{\beta} + \left[\frac{\alpha(\beta+\theta) - c}{(\beta+\theta)^2}\right]\left[e^{(\beta+\theta)(t_1 - t_d)}\left(t_d + \frac{1}{\beta+\theta}\right) - \frac{1}{\beta+\theta} - t_1\right] \right. \\ \left. + \frac{c}{\beta+\theta}\left[t_1 e^{(\beta+\theta)(t_1 - t_d)}\left(t_d + \frac{1}{\beta+\theta}\right) - \frac{t_1}{\beta+\theta} - t_1\right]\right\} + \tilde{p}\,\theta\left\{\left(\frac{\alpha(\beta+\theta) - c}{(\beta+\theta)^2} + \frac{ct_1}{\beta+\theta}\right)\left(\left(e^{(\beta+\theta)(t_1 - t_d)} - 1\right) + (t_d - t_1)\right)\right\} \\ - \frac{\tilde{s}}{\delta^2}[\alpha\delta + c(1+\delta T)]\{\delta(T - t_1) - \log(1 + \delta(T - t_1))\} - \frac{\tilde{s}\,c}{\delta^2}\left[\frac{\delta T}{2}(T - 2t_1) + \frac{\delta t_1^{\,2}}{2}\right] + \tilde{\pi}\left\{\alpha(T - t_1) + \frac{c(T^2 - t_1^{\,2})}{2}\right. \\ \left. \qquad + \frac{c(T - t_1)}{\delta} - \frac{\log(1 + \delta T - t_1\delta)(\alpha + \frac{c}{\delta} + cT)}{\delta}\right\} + \tilde{p}\tilde{I}_r\left(\frac{c(t_1 - 1)(t_1 - M)}{(\beta+\theta)}\right) \\ \qquad - \tilde{p}_1\tilde{I}_e\left(\frac{cM^3}{3} + \frac{1}{2}\left(\alpha + \beta\left(\frac{(\alpha(\beta+\theta) - c)\left(e^{(\beta+\theta)(t_1 - t_d)} - 1\right)}{(\beta+\theta)^2} + \frac{c\left(t_1 e^{(\beta+\theta)(t_1 - t_d)} - 1\right)}{(\beta+\theta)}\right)\right)M^2\right) \end{array}\right\} \quad (20)$$

Equation (20) is convex with respect to t_1, T i.e.

$$\frac{\partial T\tilde{C}_2(t_1, T)}{\partial t_1} = 0 \qquad \text{………....(21)}$$

and $$\frac{\partial T\tilde{C}_2(t_1, T)}{\partial T} = 0 \qquad \text{………....(22)}$$

provided they satisfy the sufficient conditions

$$\left[\frac{\partial T\tilde{C}_2(t_1, T)}{\partial t_1}\right]_{at\ (t_1^*, T^*)} > 0,$$

$$\left[\frac{\partial T\tilde{C}_2(t_1, T)}{\partial T}\right]_{at\ (t_1^*, T^*)} > 0$$

and $$\left[\left(\frac{\partial T\tilde{C}_2(t_1, T)}{\partial t_1}\right)\left(\frac{\partial T\tilde{C}_2(t_1, T)}{\partial T}\right) - \left[\frac{\partial^2 T\tilde{C}_2(t_1, T)}{\partial t_1 \partial T}\right]^2\right]_{at\ (t_1^*, T^*)} > 0$$

4.2 Algorithm – 2 [case(2)]

Step 1: Obtain the values of t_1 and T by fixing the value of M using the above equations.

Step 2: compare t_1 and M

1. If $M \le t_1$, t_1 is feasible. Then go to step (3).
2. If $M > t_1$, t_1 is not feasible. Set $t_1 = M$ in the equation (22) to calculate T and then go to step 3.

Step 3: Calculate the corresponding $T\tilde{C}_2\left(t_1^*, T^*\right)$

Case (3) : $M > t_1$ (See figure 3)

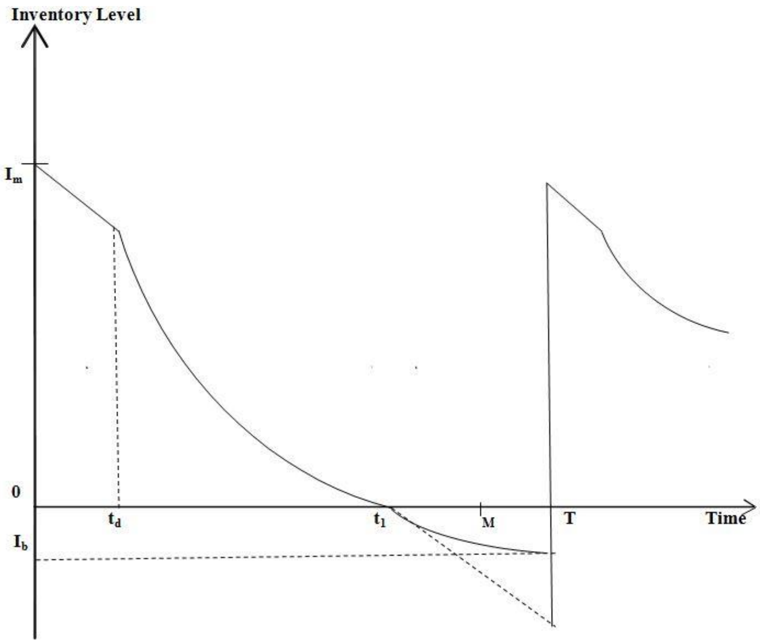


Figure: 3

In this case there is no Fuzzy interest payable but the Fuzzy interest earned from the accumulated sales during this period is given by

$$IE_3 = \tilde{p}_1 \tilde{I}_e \left\{ \left(\int_0^{t_1} (\alpha + \beta I(t) + ct)\,t\,dt \right) + (M - t_1)\left(\int_0^{t_1} (\alpha + \beta I(t) + ct)\,dt \right) \right\}$$

$$= \tilde{p}_1 \tilde{I}_e \left\{ \frac{1}{2(\beta+\theta)^2 \beta^2} \left(\theta \left(2\beta^4 c t_1^2 - 4Mc\beta^2 - 2c\beta^4 t_d t_1^2 - 2e^{-(\beta+\theta)(-t_1+t_d)} t_1^2 \alpha \beta^4 \right) \right) \right\} \qquad (23)$$

Thus the total Fuzzy annual cost which is a function of t_1 and T is given by

$$T\tilde{C}_3(t_1, T) = \frac{\tilde{K} + HC + DC + SC + OC - IE_3}{T}$$

$$= \frac{1}{T}\left\{ \begin{array}{l} \tilde{K} + \tilde{h}\left\{ \frac{c}{\beta}\left(\frac{t_d^2}{2}\right) - \left(\frac{\alpha}{\beta} - \frac{c}{\beta^2}\right)\left(e^{-\beta t_d}\left(\frac{1}{\beta} + t_d\right)\right) - \frac{1}{\beta} + \left[\frac{\alpha(\beta+\theta) - c}{(\beta+\theta)^2}\right]\left[e^{(\beta+\theta)(t_1 - t_d)}\left(t_d + \frac{1}{\beta+\theta}\right) - \frac{1}{\beta+\theta} - t_1\right] \right. \\ \left. + \frac{c}{\beta+\theta}\left[t_1 e^{(\beta+\theta)(t_1 - t_d)}\left(t_d + \frac{1}{\beta+\theta}\right) - \frac{t_1}{\beta+\theta} - t_1\right] \right\} + \tilde{p}\,\theta\left\{\left(\frac{\alpha(\beta+\theta) - c}{(\beta+\theta)^2} + \frac{ct_1}{\beta+\theta}\right)\left(\left(e^{(\beta+\theta)(t_1 - t_d)} - 1\right) + (t_d - t_1)\right)\right\} \\ - \frac{\tilde{s}}{\delta^2}[\alpha\delta + c(1 + \delta T)]\{\delta(T - t_1) - \log(1 + \delta(T - t_1))\} - \frac{\tilde{s}\,c}{\delta^2}\left[\frac{\delta T}{2}(T - 2t_1) + \frac{\delta t_1^2}{2}\right] \\ + \tilde{\pi}\left(\alpha(T - t_1) + \frac{c(T^2 - t_1^2)}{2} + \frac{c(T - t_1)}{\delta} - \frac{\log(1 + \delta T - t_1\delta)(\alpha + \frac{c}{\delta} + cT)}{\delta} \right) \\ - \tilde{p}_1 \tilde{I}_e \left\{ \frac{1}{2(\beta+\theta)^2\beta^2}\left(\theta\left(2\beta^4 c t_1^2 - 4Mc\beta^2 - 2c\beta^4 t_d t_1^2 - 2e^{-(\beta+\theta)(-t_1+t_d)} t_1^2 \alpha\beta^4\right)\right) \right\} \end{array} \right\}$$

(24)

Equation (24) is convex with respect to t_1, T i.e.

$$\frac{\partial T\tilde{C}_3(t_1, T)}{\partial t_1} = 0 \qquad \ldots\ldots\ldots\text{....(25)}$$

and $\frac{\partial T\tilde{C}_3(t_1, T)}{\partial T} = 0$ ………....(26)

provided they satisfy the sufficient conditions

$$\left[\frac{\partial T\tilde{C}_3(t_1, T)}{\partial t_1}\right]_{at\ (t_1^*, T^*)} > 0,$$

$$\left[\frac{\partial T\tilde{C}3\ (t_1, T)}{\partial T}\right]_{at\ (t_1^*, T^*)} > 0$$

and $$\left[\left(\frac{\partial T\tilde{C}_3(t_1, T)}{\partial t_1}\right)\left(\frac{\partial T\tilde{C}_3(t_1, T)}{\partial T}\right) - \left[\frac{\partial^2 T\tilde{C}_3(t_1, T)}{\partial t_1 \partial T}\right]^2\right]_{at\ (t_1^*, T^*)} > 0$$

4.2 Algorithm – 3 [case(3)]

Step 1: Obtain the values of t_1 and T by fixing the value of M using the above equations.

Step 2: compare t_1 and M

1. If $t_1<M$, t_1 is feasible. Then go to step (3).
2. If $t_1 \geq M$, t_1 is not feasible. Set t_1 = M in the equation (26) to calculate T and then go to step 3.

Step 3: Calculate the corresponding $T\tilde{C}_3(t_1^*, T^*)$

Our aim is to find the optimal values of $t_1$and T which minimize $T\tilde{C}(t_1^*, T^*)$

$$T\tilde{C}(t_1^*, T^*) = Min\ \{T\tilde{C}_1(t_1^*, T^*), T\tilde{C}_2(t_1^*, T^*), T\tilde{C}_3(t_1^*, T^*)\}$$

V. Numerical Examples

To illustrate the preceding theory the following examples are presented.

Example:

Expressing the fuzzy inventory costs (pentagonal fuzzy numbers) in relevant units.

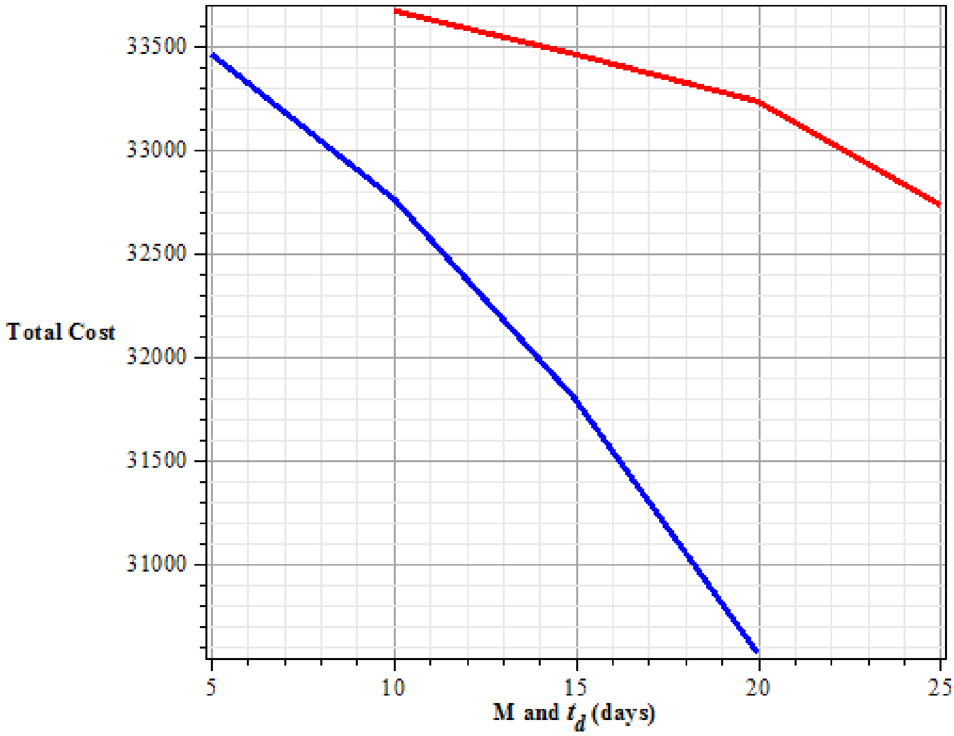
Let $\tilde{K}$ =(100,150,200,250,300), $\tilde{p}$ =(150,200,250,300,350), $\tilde{p}_1$ =(260,270,280,300,320), $\tilde{h}$ =(85,90,95,100,105), t_d =0.04, $\tilde{s}$ =(210,220,230,240,250), θ=0.08, α=1000, β=0.6, $\tilde{\pi}$ =(180,185,190,195,200), $\tilde{I}_r$ =(0.11,0.12,0.13,0.14,0.15), $\tilde{I}_e$ =(0.12,0.13,0.14,0.15,0.16), c=0.4, δ =.56, M=5/365(years).

Applying the algorithm given in section 4, we find that , t_1^*=0.1771, T^*=0.2884, $TC_{DG}(t_1, T)$ =33,460 and Q^*=69 units.

Table Effect of Total cost function with respect to the following parameters.

Parameter		T	t_1	TC	Q
M (in days)	5	0.2884	0.1771	33,460	69
	10	0.2915	0.1735	32,763	59
	15	0.2933	0.1660	31,797	42
	20	0.2937	0.1546	30,569	20

t_d (in days)	10	0.2992	0.1899	33,671	83
	15	0.2884	0.1771	33,460	69
	20	0.2775	0.1640	33,234	53
	25	0.2555	0.1372	32,735	22
δ	0.54	0.2642	0.0962	28,191	72
	0.56	0.2884	0.1771	33,460	69
	0.58	0.3123	0.2473	38,035	67
	0.60	0.3355	0.3085	42,023	58
θ	0.072	0.3136	0.1930	32,497	76
	0.080	0.2884	0.1771	33,460	69
	0.088	0.2631	0.1607	34,394	61
	0.096	0.2375	0.1438	35,290	52



Effect of M and t_d on Total Cost function:

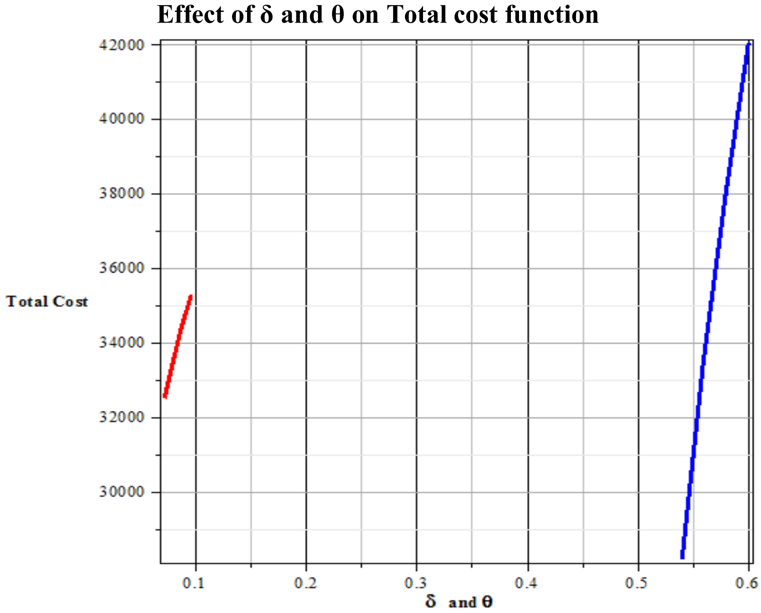


Figure: 5

5.1 :Managerial Implications

Based on the numerical examples considered above we now study the effects of change in M , δ and t_d on the optimal values of t_1,T and $T\tilde{V}C$ (t_1,T).

The results are summarized above. Based on, the observations can be made as follows:

1. With an increase in the credit period M(other parameters are fixed), it can be found that each of cycle length order quantity and total inventory cost decreases. It implies that the longer the credit period is the shorter the replenishment cycle, lower the order quantity and total cost. From the economical point of view, if the supplier provides permissible delay in payments, the retailer will order lower quantity in order to take the benefits of credit period frequently.

2. When the fresh product time t_d increases and other parameters remain unchanged, the optimal total annual cost decreases. That is, the longer the fresh product time is, the lower total cost would be. It implies that the model with non-instantaneous deteriorating item always has smaller total annual inventory cost than instantaneous items. If the retailer can extend effectively the length of time the product has no deterioration for few days or months the total cost, order quantity reduced obviously.

3. Increasing the backlogging parameter δ or equivalently decreasing the backlogging rate reduces order quantity and increases the total average inventory cost. It implies that the retailer should restrict the backlogging parameter with the aim of reducing the average total inventory cost.

4. When the deterioration rate θ is increasing, the optimal cost is increasing and the order quantity is decreasing.

Hence if the retailer can effectively reduce the deteriorating rate of an item by improving equipment of store house, the total annual inventory cost will be lowered.

VI. Conclusion

The main purpose of this study is to frame a suitable model that will help the retailer to determine the optimal replenishment policy for non-instantaneous deteriorating items, when the supplier offers a permissible delay in payments under fuzzy environment. This model will suits to situations where shortages were allowed (Partially Backlogged). From sensitivity analysis carried out some managerial insights are obtained. The retailer can reduce total annual inventory cost, when supplier provides a permissible delay in payments, improving storage conditions for non-instantaneous deteriorating and increasing backlogging rate (or equivalently decreasing the backlogging parameter).

Thus, this model incorporates some realistic features that are likely to be associated with some kinds of inventory. The model is very useful in their retail business. It can be used for electronic components, fashionable clothes, domestic goods and other products which are more likely with the characteristics above.

References

[1] K.V.Geetha and R.Uthayakumar, Economic Design of inventory policy non-instantaneous deteriorating items permissible delay in payments, Journal of Computational and Applied Mathematic 233(2010)2492-2505.

[2] R.Uthayakumar and K.V.Geetha , A Replenishment policy for Non-instantaneous deteriorating inventory system with partial backlogging, Tamsi Oxford Journal of Mathematical Sciences 25(3) (2009)313-332.

[3] R. P. Covert and G. C. Philip, An EOQmodel for items with weibulldistribution deterioration, AIIE Transactions, **5**(1973), 323-326.

[4] P. M. Ghare and G. H. Schrader, A modelforexponentiallydecayinginventory system, InternationalJournal of Production Research, **21**(1963), 49-46.

[5] S. K. Goyal and B. C. Giri, The production-inventory problem of a product with time varying demand, production and deterioration rates, European Journal of Operational Research, **147**(2003), 549-557.

[6] R. H. Hollier and K. L. Mak, Inventory replenishment policies for deteriorating items in a declining market, International Journal of Production Economics,**21** (1983), 813-826.

[7] K. S. Park, Inventory models with partial backorders,International Journal of Systems Science, **13**(1982), 1313-1317.

[8] G. C. Philip, Ageneralized EOQmodel for items with weibull distribution,AIIE Transactions, **6**(1974), 159-162.

[9] S. Sana, S. K. Goyal, and K. S. Chaudhuri, A production-inventory model for a deteriorating itemwith trended demand and shortages, EuropeanJournalof Operational Research, **157**(2004), 357-371.

[10] Y. K. Shah, An Order-level lot size inventory model for deteriorating items, AIIE Transactions, **9**(1977), 108-112.

[11] H. M. Wee, Adeterministic lot-size inventory model for deteriorating items with shortages and a decliningmarket, Computers and Operations Research,**22**(1995), 345-356.

[12] K. S. Wu, L.Y. Ouyang, and C. T. Yang, An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging,International Journal of Production Economics, **101**(2006),369-384.

[13] S. K. Goyal and B. C. Giri, Recent trendsin modeling of deteriorating inventory, European Journal of Operational Research, **134**(2001), 1-16.