Soret and Dufour Effects on Steady MHD Flow in Presence of Heat Source through a Porous Medium over a Non-Isothermal Stretching Sheet

Bishwa Ram Sharma¹, Animesh Aich²

¹Department of Mathematics, Dibrugarh University, Dibrugarh-786004, Assam, India
²Department of Mathematics, Dibrugarh University, Dibrugarh-786004, Assam, India

Abstract: Heat and mass transfer characteristics of MHD forced convection flow over a non-isothermal stretching sheet embedded in a porous medium subjected to a heat source is investigated numerically by taking into account the Soret and Dufour effects. The velocity, temperature and concentration profiles are drawn for various values of permeability parameter, heat source parameter, magnetic field parameter, Soret and Dufour numbers, thermal stratification parameter. Numerical results of rate of flow, heat and mass transfer for different parameters are presented in tabular form and the results are depicted graphically.

Keywords: Chemical reaction, heat and mass transfer, heat source, magnetic field, Soret and Dufour effects, thermal stratification.

I. Introduction

Convective flow through porous media has attracted considerable attention in last several decades due to its many important engineering, environmental and geophysical applications. The combined heat and mass transfer problems with chemical reactions are investigated by many researchers in recent years. Seddeek and Almushigeh [1] investigated the effects of radiation and variable viscosity on MHD free convective flow and mass transfer over a stretching sheet with chemical reaction. Kandasamy et al. [2] presented group analysis for Soret and Dufour effects on free convective heat and mass transfer with thermophoresis and chemical reaction over a porous stretching surface in the presence of heat source/sink. Pal and Talukdar [3] presented the combined effects of Joule heating and chemical reaction on unsteady magnetohydrodynamic mixed convection with viscous dissipation over a vertical plate in the presence of porous media and thermal radiation. Joneidi et al. [4] presented analytical treatment of MHD free convection flow over a stretching sheet with chemical reaction. Anjalidevi and Kandasamy [5] studied effects of chemical reaction, heat and mass transfer on laminar flow along a semi-infinite horizontal plate. Seddeek [6] took into account the homogenous chemical reaction of first-order in boundary layer hydromagnetic flow with heat and mass transfer over a heat surface. Various other effects have been included in his analysis such as thermophoresis, variable viscosity and heat generation absorption. Abel et.al. [7] studied the two-dimensional boundary layer problem in mixed convection of an incompressible viscoelastic fluid immersed in porous medium over a stretching sheet. Vyas and Srivastava [8] investigated the radiation effect on MHD flow over a nonisothermal stretching sheet in a porous medium. Vajravelu and Hadjinicolaou [9] studied the convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream. Ibrahim and Reddy [10] analysed the similarity solution of heat and mass transfer for natural convection over a moving vertical plate with internal heat generation and a convective boundary condition in the presence of thermal radiation, viscous dissipation and chemical reaction. Shateyi and Motsa [11] studied the thermal radiation effects on heat and mass transfer over an unsteady stretching surface. Cortell [12] studied the effects of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet. Effects of chemical reaction and thermal stratification on MHD free convective heat and mass transfer over a vertical stretching surface embedded in a porous media considering Soret and Dufour numbers provided by Mansour et.al.[13]. Mansour and Chamkha [14] studied the effects of Soret and Dufour numbers on free convection over isothermal and adiabatic stretching surfaces embedded in porous media. Sharma and Aich [15] studied the Influence of Chemical Reaction, Magnetic field and Radiation on Heat and Mass Transfer by Free Convection Flow near the Lower Stagnation Point of an Isothermal Horizontal Circular Cylinder in a Porous Medium considering Soret and Dufour Effects, Sharma and Borgohain [16] analyzed the Influence of chemical reaction, Soret and Dufour effects on heat and mass transfer of a binary fluid mixture in porous medium over a rotating disk. Soret and Dufour effects on MHD slip flow with thermal radiation over a porous rotating infinite disk given by Anjali Devi and Uma Devi [17]. Postelnicu [18] has analyzed the effect of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour Effects neglecting the effect of magnetic field. Sharma [19] has analyzed the Soret and Dufour effects on separation of a binary fluid mixture in MHD natural convection in
porous media neglecting the effect of chemical reaction term. Effects of Chemical Reaction on Dissipative Radiative MHD Flow through a Porous Medium over a Nonisothermal Stretching Sheet was studied by Ibrahim [20] neglecting the soret and dufour effects.

The objective of this paper is to study the effects of heat source, Soret and Dufour effects on MHD boundary layer flow over a nonisothermal stretching sheet embedded in porous medium.

II. Mathematical Formulation

We consider steady, laminar, two dimensional MHD forced convective flow together with heat and mass transfer of a viscous incompressible electrically conducting fluid along a stretching sheet placed at the bottom of fluid saturated porous medium. The effects of thermal-diffusion, diffusion-thermal, heat source, a first order homogenous chemical reaction, radiation, viscous dissipation and uniform magnetic field on flow, heat and mass transfer are taken into account. The x-axis is along the sheet and y-axis is perpendicular to it as shown in the figure 1. Two equal and opposite forces are applied along the sheet so that the wall is stretched, keeping the position of the origin unaltered. The stretching velocity varies linearly with the distance from the origin. A uniform magnetic field of strength \( B_0 \) is applied normal to the sheet. The ambient temperature and concentration far away from the surface of the sheet which is assumed to be uniform is \( T_\infty \) and \( C_\infty \) where \( T_\infty > T_w \) and \( C_w > C_\infty \).

Fig. 1: Physical model and coordinate system

Further it is assumed that both the fluid and the porous medium are in local thermal equilibrium. The fluid is considered to be gray absorbing-emitting radiations but non scattering medium and the Rosseland approximation is used to describe the radiative heat flux in y-direction. Under the Boussinesq approximation, the governing equations are given by

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{1}{\rho} \left( \frac{\partial p}{\partial y} + \rho g \beta (T - T_\infty) + \rho g \beta (C - C_\infty) \right) - \frac{\varepsilon B_0 u}{\kappa} - \frac{v u}{k} \quad (1)
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{k^*}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\rho_0 (T - T_\infty)}{\rho c_p} + \frac{2 \kappa_T}{\rho c_p} \frac{\partial^2 C}{\partial y^2} \quad (2)
\]

\[
\frac{\partial T}{\partial x} + \frac{\partial v}{\partial y} = - \frac{\alpha^*}{\rho c_p} \frac{\partial T}{\partial y} + \frac{\rho k_T}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - K_1 (C - C_\infty) \quad (3)
\]

where \( u \) and \( v \) are fluid velocity components in the x and y directions respectively, \( g \) is acceleration due to gravity, \( C \) and \( T \) denotes the concentration and temperature of the fluid, \( \rho \) is the density of the fluid, \( \beta \) is the coefficient of thermal expansion, \( \beta^* \) is coefficient of concentration expansion, \( k^* \) is the permeability of porous medium, \( \sigma \) is the fluid electrical conductivity, \( k \) and \( C_p \) are the thermal conductivity and specific heat of the fluid, \( C_i \) is the concentration susceptibility, \( q_T \) is the radiative heat flux, \( D \) is mass diffusivity, \( k_T \) is thermal diffusion ratio, \( T_m \) is mean fluid temperature, \( K_1 \) is dimensional of chemical reaction.

The radiative heat flux \( q_T \) under Rosseland approximation is given by the expression

\[
q_T = - \frac{4 \sigma^* \varepsilon^*}{\lambda} \frac{\partial T^4}{\partial y} \quad (5)
\]

where \( \sigma^* \) is Stefan-Boltzmann constant and \( \varepsilon^* \) is the mean absorption coefficient.

It is assumed that temperature differences within the flow are sufficiently small such that \( T^4 \) can be expanded in a Taylor’s series about \( T_\infty \) and after rejecting higher order terms, we have

\[
T^4 = 4T^3_\infty - 3T^2_\infty \quad (6)
\]
The initial and boundary conditions are
\[
\begin{align*}
u = & cx, v = 0, T = T_w, C = C_w & \quad \text{at } y = 0, \\
u \rightarrow & 0, T \rightarrow T_{\infty}, C \rightarrow C_{\infty} & \quad \text{as } y \rightarrow \infty
\end{align*}
\] (7)

The similarity transformation is given by
\[
\psi = \sqrt{cv}x f(\eta), \quad \eta = \frac{z^c}{\sqrt{v}}, \quad \theta(\eta) = \frac{T - T_w}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C_w - C_{\infty}}{C_w - C_{w_0}}
\]
\[
T_w - T_\infty = dx^c, \quad C_w - C_{\infty} = dx^d
\]
\] (8)

where \(c\) is a constant called the thermal stratification parameter, \(d\) is a constant called the wall concentration exponent, \(c, d\) and \(d_1\) are all constant.

The stream function is introduced by the equations
\[
u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}
\] (9)

If we introduce the relations (8) into the equations (2), (3) and (4), then we have
\[
f'' + 
\]
\[
\theta'' + \frac{2Prd_1}{d_2 + 4d} (f \theta' - \alpha \theta + Ec f'^2 + Q \theta + D_\theta \theta'') = 0
\]
\[
\phi'' + Sc(f \phi' - \delta \phi + S_c \phi') = 0
\] (10)

where \(M\) is the magnetic field parameter, \(Gr = \frac{g \theta^2 (T_w - T_\infty)}{xc^2}\) is the Grashoff number, \(Gc = \frac{g \phi^2 (C_w - C_{\infty})}{xc^2}\) is the modified Grashoff number, \(K = \frac{v}{k c}\) is the permeability parameter, \(Pr = \frac{\nu \kappa}{\sigma}\) is the Prandtl number, \(Q = \frac{Q_c}{Q r_\phi}\) is the heat generation or absorption parameter, \(R = \frac{k'k}{4 \sigma T_\infty^2}\) is the radiation parameter, \(Ec = \frac{C_r (T_w - T_\infty)}{C_r (T_w - T_\infty)}\) is the Eckert number, \(D_\phi = \frac{C_r (C_w - C_{\infty})}{C_r (C_w - C_{\infty})}\) is the Dufour number, \(Sc = \frac{nu}{d}\) is the Schmidt number, \(S_c = \frac{nu}{d}\) is the Soret number and \(\gamma = \frac{\alpha}{c}\) is the chemical reaction parameter.

The initial and boundary conditions (7) becomes
\[
f' = 1, \quad \theta = 0, \quad \phi = 0 & \quad \text{at } \eta = 0, \quad \tilde{f}' = 0, \quad \theta = 0, \quad \phi = 0 & \quad \text{as } \eta \rightarrow \infty
\] (13)

III. Results and Discussions

The ordinary differential equations (10), (11) and (12) have been solved numerically by using bvp4c solver of MATLAB. From the process of numerical computation, the local skin friction, the local Nusselt number and the local Sherwood number which are proportional to \(f''\), \(\theta''\) and \(\phi''\) respectively are worked out and their numerical values are presented in tabular form. Numerical calculations for \(f''\), \(\theta''\) and \(\phi''\) have been carried out by taking various values of parameters \(K, M, Q, D_\phi, S_c\) and \(\alpha\) by considering
\[
Gr = 2.0, Gc = 2.0, Pr = 0.7, R = 1.0, \delta = 0.1, Ec = 0.5, Sc = 0.6, \gamma = 0.5.
\]

Several Cases considered are:

Case I: \(M = 1.0, Q = 0.5, D_\phi = 0.5, S_c = 0.5, \alpha = 1.0\) with \(K = (0.5,1.0,1.5)\).

Case II: \(K = 0.5, Q = 0.5, D_\phi = 0.5, S_c = 0.5, \alpha = 1.0\) with \(M = (0.5,1.0,1.5)\).

Case III: \(M = 1.0, K = 0.5, D_\phi = 0.5, S_c = 0.5, \alpha = 1.0\) with \(Q = (0.5,0.8,1.0)\).

Case IV: \(M = 1.0, K = 0.5, Q = 0.5, S_c = 0.5, \alpha = 1.0\) with \(D_\phi = (0.5,1.0,1.5)\).

Case V: \(M = 1.0, K = 0.5, Q = 0.5, D_\phi = 0.5, \alpha = 1.0\) with \(S_c = (0.5,1.0,1.5)\).

Case VI: \(M = 1.0, K = 0.5, Q = 0.5, D_\phi = 0.5, S_c = 0.5, \alpha = (1.0,2.0,3.0)\).

The effects of radiation and chemical reaction have already been discussed by [20]. So to save the space, we have avoided the repetition of graph.

Case 1: Fig. 2(a)-(c) exhibit velocity, temperature and concentration profile for various values of permeability parameter \(K\). It is observed that the velocity, temperature and concentration decrease exponentially from their maximum values at the surface to their minimum values at the end of the boundary layer. Fig. 2(a) exhibits that with the increase in the values of \(K\), velocity of the fluid decreases in the boundary layer region \(0 < \eta < 4\) and reverse effect is observed from \(\eta = 4\) onwards. Fig. 2(b) exhibits that with the increase in the value of \(K\), temperature of the fluid increases in the boundary layer region \(0 < \eta < 10\). Fig. 2(c) exhibits that with the
increase in the value of $K$, concentration of the fluid slightly increases in the boundary layer region $0 < \eta < 3$ but no effect is observed from $\eta = 3$ onwards.

Case 2: Fig. 3(a)-(c) exhibit velocity, temperature and concentration profile for various values of magnetic field parameter $M$. It is observed that the velocity, temperature and concentration decrease exponentially from their maximum values at the surface to their minimum values at the end of the boundary layer. Fig. 3(a) exhibits that with the increase in the values of $M$, velocity of the fluid decreases in the boundary layer region $0 < \eta < 4$ and reverse effect is observed from $\eta = 4$ onwards. Fig. 3(b) exhibits that with the increase in the value of $M$, temperature of the fluid increases in the boundary layer region $0 < \eta < 10$. Fig. 3(c) exhibits that with the increase in the value of $M$, concentration of the fluid slightly increases in the boundary layer region $0 < \eta < 5$ but no effect is observed from $\eta = 5$ onwards.

Case 3: Fig. 4(a)-(c) exhibit velocity, temperature and concentration profile for various values of heat source parameter $Q$. It is observed that the velocity, temperature and concentration decrease exponentially from their maximum values at the surface to their minimum values at the end of the boundary layer. Fig. 4(a) exhibits that with the increase in the values of $Q$, velocity of the fluid increases in the boundary layer region $0 < \eta < 10$. Fig. 4(b) exhibits that with the increase in the value of $Q$, temperature of the fluid increases in the boundary layer region $0 < \eta < 10$. Fig. 4(c) exhibits that with the increase in the value of $Q$, concentration of the fluid decreases in the boundary layer region $0 < \eta < 5$ but no effect is observed from $\eta = 5$ onwards.
region $0 < \eta < 10$. Fig. 4(c) exhibits that with the increase in the value of $Q$, concentration of the fluid slightly decreases in the boundary layer region $0 < \eta < 4$ but no effect is observed from $\eta = 4$ onwards.

**Case 4:** Fig. 5(a)-(c) exhibit velocity, temperature and concentration profile for various values of Dufour number $D_f$. It is observed that the velocity, temperature and concentration decrease exponentially from their maximum values at the surface to their minimum values at the end of the boundary layer. Fig. 5(a) exhibits that with the increase in the values of $D_f$, velocity of the fluid increases in the boundary layer region $0 < \eta < 5$ but no effect is observed from $\eta = 5$ onwards. Fig. 5(b) exhibits that with the increase in the value of $D_f$, temperature of the fluid increases in the boundary layer region $0 < \eta < 6$ but no effect is observed from $\eta = 6$ onwards. Fig. 5(c) exhibits that with the increase in the value of $D_f$, concentration of the fluid slightly decreases in the boundary layer region $0 < \eta < 3$ but no effect is observed from $\eta = 3$ onwards.

**Case 5:** Fig. 6(a)-(c) exhibit velocity, temperature and concentration profile for various values of Soret number $S_r$. It is observed that the velocity, temperature and concentration decrease exponentially from their maximum values at the surface to their minimum values at the end of the boundary layer. Fig. 6(a) exhibits that with the
increase in the values of $S_r$, velocity of the fluid increases in the boundary layer region $0 < \eta < 6$ but no effect is observed from $\eta = 6$ onwards. Fig. 6(b) exhibits that with the increase in the value of $S_r$, temperature of the fluid decreases in the boundary layer region $0 < \eta < 8$ but no effect is observed from $\eta = 8$ onwards. Fig. 6(c) exhibits that with the increase in the value of $S_r$, concentration of the fluid increases in the boundary layer region $0 < \eta < 8$ but no effect is observed from $\eta = 8$ onwards.

![Graph](image1.png)

(a) (b) (c)

Fig 6: Effect of Soret number $S_r$ on (a) the velocity profile and (b) the temperature profile and (c) the concentration profile

**Case 6:** Fig. 7(a)-(c) exhibit velocity, temperature and concentration profile for various values of thermal stratification parameter $\alpha$. It is observed that the velocity, temperature and concentration decrease exponentially from their maximum values at the surface to their minimum values at the end of the boundary layer. Fig. 7(a) exhibits that with the increase in the values of $\alpha$, velocity of the fluid decreases in the boundary layer region $0 < \eta < 7$ but no effect is observed from $\eta = 7$ onwards. Fig. 7(b) exhibits that with the increase in the value of $\alpha$, temperature of the fluid decreases in the boundary layer region $0 < \eta < 8$ but no effect is observed from $\eta = 8$ onwards. Fig. 7(c) exhibits that with the increase in the value of $\alpha$, concentration of the fluid slightly increases in the boundary layer region $0 < \eta < 3$ but no effect is observed from $\eta = 3$ onwards.

![Graph](image2.png)

(a) (b) (c)

Fig 7: Effect of thermal stratification parameter $\alpha$ on (a) the velocity profile and (b) the temperature profile and (c) the concentration profile
TABLE 1: The values of rate of flow, heat and mass transfer in terms of local skin friction \( f'(0) \), local Nusselt number \(-\theta'(0)\) and local Sherwood number \(-S'(0)\) for selected values of \(K, M, Q, D_f, S_r\) and \(\alpha\).

<table>
<thead>
<tr>
<th>(K)</th>
<th>(M)</th>
<th>(Q)</th>
<th>(D_f)</th>
<th>(S_r)</th>
<th>(\alpha)</th>
<th>(f'(0))</th>
<th>(-\theta'(0))</th>
<th>(-S'(0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>0.1586</td>
<td>0.2976</td>
<td>1.0592</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>-0.0593</td>
<td>0.2617</td>
<td>1.0442</td>
</tr>
<tr>
<td>1.5</td>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>-0.2589</td>
<td>0.2251</td>
<td>1.0315</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>0.3986</td>
<td>0.3320</td>
<td>1.0770</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>0.1586</td>
<td>0.2976</td>
<td>1.0592</td>
</tr>
<tr>
<td>0.5</td>
<td>1.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>-0.0593</td>
<td>0.2617</td>
<td>1.0442</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>0.1586</td>
<td>0.2976</td>
<td>1.0592</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>0.8</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>0.2175</td>
<td>0.1541</td>
<td>1.0941</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>0.2679</td>
<td>0.0361</td>
<td>1.1214</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>0.1586</td>
<td>0.2976</td>
<td>1.0592</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>0.1901</td>
<td>0.1792</td>
<td>1.0892</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>0.5</td>
<td>1.5</td>
<td>1.0</td>
<td>1.0</td>
<td>0.2222</td>
<td>0.0529</td>
<td>1.1210</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>0.1586</td>
<td>0.2976</td>
<td>1.0592</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>0.1649</td>
<td>0.3055</td>
<td>1.0393</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>0.5</td>
<td>1.5</td>
<td>1.0</td>
<td>1.0</td>
<td>0.1719</td>
<td>0.3137</td>
<td>1.0175</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
<td>2.0</td>
<td>0.0889</td>
<td>0.5453</td>
<td>0.9953</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
<td>3.0</td>
<td>1.0</td>
<td>0.0407</td>
<td>0.7375</td>
<td>0.9441</td>
</tr>
</tbody>
</table>

IV. Conclusions

In this work, heat source, thermal-diffusion and diffusion-thermal effects on MHD heat and mass transfer over a non-isothermal stretching sheet embedded in a porous medium has been investigated. From our investigation as obvious from table 1, we can conclude that the rate of flow decreases with increase in permeability parameter, magnetic field parameter and thermal stratification parameter but increases with increase in heat source parameter, Dufour number and Soret number. It can also be concluded from table 1 that the rate of heat transfer decreases in magnitude with increase in permeability parameter, magnetic field parameter, heat source parameter, Dufour number but increases in magnitude with increase in Soret number and thermal stratification parameter. We can also conclude from table 1 that the rate of mass transfer decreases in magnitude with increase in permeability parameter, magnetic field parameter, Soret number and thermal stratification parameter but increases in magnitude with increase in heat source parameter and Dufour number.

REFERENCES


DOI: 10.9790/5728-12115360 www.iosrjournals.org
Soret and Dufour effects on steady MHD flow in presence of heat source through a porous medium ...


