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A Note on Total and Global Accurate Domination In Fuzzy Graphs

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Abstract: *An attempt is made in this paper to define fuzzy accurate domination number, fuzzy accurate total domination number, fuzzy global accurate domination number and some exact value for $\gamma_{fatd}(G)$. Also relationship between $\gamma_{fgad}(G)$ and other known domination parameters are explored.*
Keywords: *Fuzzy accurate domination, Fuzzy accurate total domination, Fuzzy global accurate domination.*

I. Introduction

The study of dominating sets in graphs was begun by Orge and Berge, V.R.Kulli wrote on theory of domination in graphs. The concept of fuzzy sets and fuzzy relations was introduced by L.A.Zadeh in 1965 [1] and further studied in [2]. It was Resenfeld[5] who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975.The domination number is introduced by cockayne and Hedetniemi Resenfeld introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. A.Somasundaram and S.Somasundaram[24] and introduced the domination number for several classes of fuzzy graphs and obtained bounds for the same. NagoorGani and Chandrasekaran discussed domination in fuzzy graph using strong arc.This concept of Accurate domination number was introduced by V.R.Kulli and M.B.Kattiman. Among the various applications to the theory of domination in fuzzy graphs, here we consider the fault tolerant property in communication network, that is, even if any communication link to a station is failed, still it can communicate the message to that station. We also discuss the fuzzy accurate domination number and fuzzy accurate total domination number and fuzzy global accurate domination number of the fuzzy graph.

Preliminaries

A fuzzy subset of a non empty set V is a mapping $\sigma : V\to[0,1]$.A fuzzy relation on V is a fuzzy subset of $V\times V$. A *fuzzy graph* $G = (\sigma , \mu)$ is a pair of function $\sigma : V\to[0,1]$ and $\mu : V \times V \to[0,1]$, where $\mu (u ,v) \leq \sigma(u)\Lambda\, \sigma (v)$ for all $u ,v \in V$. The *order p* and *size q* of the fuzzy graph $G = (\sigma , \mu)$ are define by $p = \sum_{v\in V} \sigma(v)$ and $q = \sum_{u,v\in E} \mu(u,v)$. The *complement* of a fuzzy graph $G = (\sigma , \mu)$ is a fuzzy graph $G = (\sigma , \mu)$ where $\sigma = \sigma$ and $\overline{\mu} (u ,v) = \sigma(u)\, \Lambda\, \sigma (v) - \mu (u ,v)$ for all u ,v in V. The fuzzy cardinality of a fuzzy subset D of V is $|D|_f = \sum_{v\in D} \sigma(v)$. Let $G = (\sigma , \mu)$ be a fuzzy graph on V. Let $x , y \in V$. We say that x dominates y in G if $\mu (uv) = \sigma(u)\Lambda\, \sigma (v)$. The *strength* of connectedness between two nodes u ,v in a fuzzy graph G is $\mu^\infty(u ,v) = \sup\{\mu^k(u ,v): k = 1,2,3\ldots\ldots\}$ where $\mu^k(u ,v) = \sup \{\mu(u,u_1)\Lambda\, \mu(u_1,u_2)\Lambda \ldots\ldots\Lambda\, \mu(u_{k-1},v)\}$, An arc (u ,v) is said to be a *strong arc* if $\mu(u ,v) \geq \mu^\infty(u ,v)$ and the node V is said to be a strong neighbor of u. If $\mu(u ,v) = 0$ for every $v\in V$, then u is called *isolated node*. A node cover of a graph G is a nodes that covers all the sizes and an size cover of G is a set of sizes that covers all the nodes. The node (size) covering number $(\alpha_0(G),\alpha_1(G))$ of G is minimum cardinality of a node (size) cover. A set S of nodes of G is independent if no two nodes in S are adjacent. The independence number $\beta_0(G)$ of G is the maximum cardinality of an independent set. A β_0 set is a maximum independent set. A set F of nodes of G is independent if no two sizes in F are adjacent. The size independence number $\beta_1(G)$ of G is the maximum cardinality among the independent sets of sizes. Let G be a fuzzy graph and u be a node in G then there exists node v such that (u , v) is a strong arc then *u dominates v*. A subset S of V is called a *dominating set* in G if for every $v \notin S$, there exists $u \in S$ such that u dominates v. The minimum fuzzy cardinality of a dominating set in G is called the *domination number* of G and is denoted by $\gamma (G)$. Let $G = (\sigma , \mu)$ be a fuzzy graph. A subset D of V is said to be *fuzzy dominating set* of G if for every $v \in V-D$, there exists $u \in D$ such that (u ,v) is a Strong arc. A fuzzy dominating set D of a fuzzy graph G is called *minimal dominating set* of G, if for every node $v\in D$, D - {v} is not a dominating set. The *domination number* $\gamma(G)$ is the minimum cardinalities taken over all minimal dominating sets nodes of G. A dominating set D of a graph G is an *accurate dominating set*, if V-D has no dominating set of cardinality $|D|$. The accurate domination number $\gamma_a (G)$ of G is the minimum cardinality of an accurate dominating set. A *total dominating set* T of G is a dominating set such that the induced subgraph < T > has no isolates. The *total domination* *number* $\gamma_t(G)$ of G is the minimum cardinality of a total dominating set. A total dominating set D of G is an *accurate total dominating set* if V-D has no total dominating set of cardinality $|D|$. The *accurate total domination number* $\gamma_{at}(G)$ of G is the minimum cardinality of an accurate total dominating set. A dominating set D of a graph G is a *global dominating set* if D is also a dominating set of $\bar{G}$. The *global domination number* $\gamma_g(G)$ of G is the minimum cardinality of global dominating set. An accurate dominating set D of a graph G is a *global accurate dominating set*, if D is also an accurate dominating set of $\bar{G}$. The *global accurate domination number* $\gamma_{ga}(G)$ of G is the minimum cardinality of global accurate dominating set. Let G: (σ, μ) be a fuzzy graph. The *degree* of a node u is $d_G(u) = \sum_{u \neq v} \mu(uv)$. Since $\mu(uv) > 0$ for $uv \in E$ and $\mu(uv) = 0$ for $uv \notin E$, This is equivalent to $d_G(u) = \sum_{uv \in E} \mu(uv)$. The *minimum degree* of G is $\delta(G) = \wedge \{d(v) / v \in V\}$. The *maximum degree* of G is $\Delta(G) = \vee \{d(v) / v \in V\}$.

II. Main Results

Accurate Total Domination in Fuzzy Graphs:

Definition: 2.1

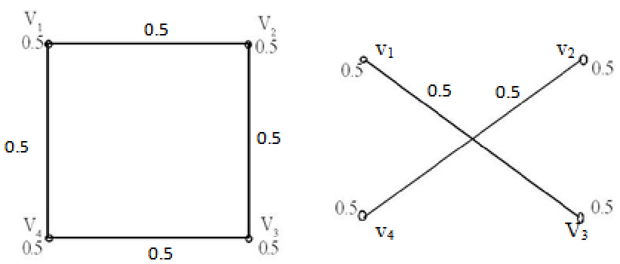
A dominating set D of a fuzzy graph G is an *fuzzy accurate dominating set*, if V-D has no dominating set of cardinality $|D|$. The *fuzzy accurate domination number* $\gamma_{fa}(G)$ of G is the minimum cardinality of an accurate dominating set.

Definition: 2.2

A fuzzy total dominating set D of a graph G = (V,E) is an *fuzzy accurate total dominating set*, if V-D has no fuzzy total dominating set, of cardinality $|D|$. The *fuzzy accurate total domination number* $\gamma_{fat}(G)$ of G is the minimum cardinality of an fuzzy accurate total dominating set.

Example: 2.3

The graph G in Figure 1 has $\{v_1, v_2\}$ as a γ_f -set and $\{v_1, v_2, v_3\}$as a γ_{fa} –set and $\{v_1, v_2, v_3\}$ γ_{fat}-set. Thus G has $\gamma_f(G) = 1$, $\gamma_{ft}(G) = 1$, $\gamma_{fa}(G) = 1.5$, $\gamma_{fat}(G) = 1.5$



G V4 fig.1 $\bar{G}$ V3

0.5

Proposition: 2.4

For any fuzzy graph G without isolated nodes, $\gamma_{fa}(G) \leq \gamma_{fat}(G)$.

Proof: Every fuzzy accurate total dominating set is an fuzzy accurate dominating set.

Thus $\gamma_{fa}(G) \leq \gamma_{fat}(G)$.

Proposition: 2.5

For any fuzzy graph G without isolated nodes, $\gamma_{ft}(G) \leq \gamma_{fat}(G)$.

Proof: Every fuzzy accurate total dominating set is an fuzzy total dominating set.

Thus $\gamma_{ft}(G) \leq \gamma_{fat}(G)$.

Theorem: 2.6

For any fuzzy graph G without isolated nodes, $\gamma_{fat}(G) \leq \lfloor \frac{p}{\Delta+1} \rfloor + 1$.

Theorem: 2.7

For any fuzzy graph G, $\gamma_{fa}(G) \leq p - \gamma_f(G) + 1$.

Proof: Let D be a minimum fuzzy dominating set of G. Then for any node $v \in D$, $(V-D) \cup \{v\}$ is an fuzzy accurate total dominating set of G.

Thus $\gamma_{fa}(G) \leq |(V-D) \cup \{v\}| = p - \gamma_f(G) + 1$.

Theorem: 2.8

For any fuzzy graph G, $\frac{p}{\Delta+1} \leq \gamma_{fa}(G) \leq \frac{p\Delta}{\Delta+1} + 1$.

Proof: It is know that $\frac{p}{\Delta+1} \leq \gamma_f(G)$ and since $\gamma_f(G) \leq \gamma_{fa}(G)$, by previous theorem,

$$\gamma_{fa}(G) \le p - \gamma_f(G) + 1$$
$$\le p - \frac{p}{\Delta+1} + 1$$
$$\le \frac{p\Delta}{\Delta+1} + 1$$

Thus $\frac{p}{\Delta+1} \le \gamma_{fa}(G) \le \frac{p\Delta}{\Delta+1} + 1$.

Proposition: 2.9

For any fuzzy graph G, then $\gamma_{fa}(G) \le \alpha_0(G) + 1$.

Proof: Let S be a node cover. We consider the following two cases.

Case1. Suppose $|S| < \frac{p}{2}$. Clearly S is an fuzzy accurate dominating set of G.

Thus $\gamma_{fa}(G) \le \alpha_0(G) + 1$.

Case2. Suppose $|S| = \frac{p}{2}$. Then for any node $v \in V\text{-}S$, $S \cup \{v\}$ is an fuzzy accurate dominating set of G.

Thus $\gamma_{fa}(G) \le \alpha_0(G) + 1$.

Proposition: 2.10

For any fuzzy graph G without isolated nodes, $\gamma_{fat}(G) \le \gamma_{ft}(G) + \Delta(G) - 1$.

Proof: Let D be a γ_{ft}-set of G. We consider the following two cases.

Case1: If V-D is not a fuzzy total dominating set of G,
Then D is an fuzzy accurate total dominating set.
This implies that $\gamma_{fat}(G) = \gamma_{ft}(G)$, Thus $\gamma_{fat}(G) \le \gamma_{ft}(G) + \Delta(G) - 1$.

Case2: If V-D is a fuzzy total dominating set and S is the set of all nodes of V-D adjacent to a node $v \in D$, then $D \cup S$ is an fuzzy accurate total dominating set of G.
Thus $\gamma_{fat}(G) \le |D \cup S| = \gamma_{ft}(G) + \deg v - 1$. (Since $\deg v \le \Delta(G)$),
Thus $\gamma_{fat}(G) \le \gamma_{ft}(G) + \Delta(G) - 1$.

Theorem: 2.11

For any fuzzy graph of G without isolated nodes, $\gamma_{ft}(G) \le \gamma_{fat}(G) \le 2\gamma_f(G)$.

Theorem: 2.12

If G is a fuzzy graph having p order, q size and maximum degree Δ, then $p - q \le \gamma_{fat}(G) \le p + \Delta(G)$.

Theorem: 2.13

If G is a fuzzy graph without isolated nodes, then $\gamma_{fat}(G) \le \frac{2p}{\Delta(G)+1}$

Proof: Let G be a fuzzy graph have no isolated nodes and let D be a fuzzy accurate total dominating set of G. Further, let $t = \Sigma\rho(e)$, where e is an size in G which having one node in D and the other in V–D.
Since $\Delta(G) \ge \deg(v)$ for all $v \in D$, and each node in D has at least one neighbor in D,
we have $t \ge (\Delta(G) - 1)\,|D| = (\Delta(G) - 1)\,\gamma_{fat}(G)$. Also, since each node in V-D is adjacent to at least two nodes in D, We have $t \le 2\,|V-D| = 2\,(p - \gamma_{fat}(G))$.
Hence $2\,(p - \gamma_{fat}(G)) \le (\Delta(G) + 1)\,\gamma_{fat}(G)$.

III. Global Accurate Domination in Fuzzy Graphs:

Definition: 3.1

An fuzzy accurate dominating set D of a graph G is a *fuzzy global accurate dominating set*, if D is also an fuzzy accurate dominating set of $\bar{G}$. The *fuzzy global accurate domination number* $\gamma_{fga}(G)$ of G is the minimum cardinality of fuzzy global accurate dominating set.

Theorem: 3.2

An fuzzy accurate dominating set D of a fuzzy graph G is a fuzzy global accurate dominating set if and only if the following conditions holds:

(i) for each node $v \in V-D$, there exists a node $u \in D$ such that u is not adjacent to v.
(ii) there exists a node $w \in D$ such that w is adjacent to all nodes in V–D.

Theorem: 3.3

Let G be a fuzzy graph such that neither G and $\bar{G}$ have an isolated node. Then

(i) $\gamma_{fga}(G) = \gamma_{fga}(\bar{G})$

(ii) $\frac{(\gamma fa(G) + \gamma fa(\bar{G}))}{2} \le \gamma_{fa}(G) \le \gamma_{fa}(G) + \gamma_{fa}(\bar{G})$

Theorem: 3.4

Let G be a fuzzy graph such that neither G nor $\bar{G}$ have an isolated node. Then $\gamma_{fa}(G) \le \gamma_{fga}(G)$.

Proof: Every fuzzy global accurate dominating set is an fuzzy accurate dominating set.

Thus $\gamma_{fa}(G) \leq \gamma_{fga}(G)$.

Theorem: 3.5 Let G be a fuzzy graph such that neither G nor $\bar{G}$ have an isolated node. Then $\gamma_{fg}(G) \leq \gamma_{fga}(G)$.

Proof: Every fuzzy global accurate dominating set is an fuzzy global dominating set

Thus $\gamma_{fa}(G) \leq \gamma_{fga}(G)$.

Theorem: 3.6

Let D be a fuzzy accurate dominating set of G. If there exist two nodes u $\in$ V-D and v $\in$D such that u is adjacent only to the nodes of D and v is adjacent only to the nodes of V-D. Then $\gamma_{fga}(G) \leq \gamma_{fa}(G) + 1$.

Proof: Let D be a γ_{fa}–set of G. If these exists a node u inV-D such that u is adjacent only to the nodes of D, then D $\cup$\{u\} is a fuzzy global accurate dominating set of G.

Thus $\gamma_{fga}(G) \leq |D \cup \{u\}|$

$\leq |D| + 1$

(or) $\gamma_{fga}(G) \leq \gamma_{fa}(G) + 1$.

Theorem: 3.7

Let G be a fuzzy graph without isolated nodes. Then $\gamma_{fga}(G) \leq \alpha_0(G) + 1$.

Proof: Let S be a maximum independent set of nodes in G.

Then for any node v$\in$S, \{V–S\}$\cup$\{v\} is a fuzzy global accurate dominating set of G,

Thus $\gamma_{fga}(G) \leq |(V-S) \cup \{v\}|$

$\leq |V-S| + 1$

$\leq p - \beta_0(G) + 1$

(or) $\gamma_{fga}(G) \leq \alpha_0(G) + 1$.

Proposition: 3.8

For any fuzzy graph G without isolated nodes, $(2q - p(p-3))/2 \leq \gamma_{ga}(G) \leq p - \beta_0(G) + 1$.

Proof: Let D be a minimum fuzzy global accurate dominating set.

Then every node in V–D is not adjacent to at least one node in D. This implies $q \leq \frac{p(p-1)}{2} - (p - \gamma_{ga}$) and the lower bound follows.

To establish upper bound.

Let S be an independent set with β_0 nodes. Since G has no isolated nodes, V-D is a dominating set of G.

Clearly, for any node v $\in$ S is a fuzzy global dominating set G and the upper bound follows.

Proposition: 3.9

For any fuzzy graph of G without isolated nodes, then $\gamma(G) + \gamma_{fga}(G) \leq p + 1$.

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