

Modelling Exchange Rate Volatility Using Asymmetric GARCH Models (Case Study Sudan)

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Abstract: *This article examines the accuracy and forecasting performance of volatility models for the Sudanese Pound SDG/USA dollars exchange rate return, including the ARMA, Generalized Autoregressive Conditional Heteroscedasticity (GARCH), and Asymmetric GARCH models with normal and non-normal (student's t) distributions. In fitting these models to the monthly exchange rate returns data over the period January 1999 to December 2013, we found that, the Asymmetric (GARCH) and GARCH model better fits under the non-normal distribution than the normal distribution and improve the overall estimation for measuring conditional variance. The DEG-GARCH model using the Student t- distribution is most successful and better forecast the Sudanese pound exchange rate volatility. Finally, the study suggests that the given models are suitable for modeling the exchange rate volatility of the sudan and the Asymmetric GARCH models shows asymmetric in exchange rate returns, resulting to the presence of leverage effect. Given the implication of exchange rate volatility, the study would be of great value to policy makers, investors and researchers at home and abroad in promoting development of the capital market and foreign exchange market stability in emerging economies.*

I. Introduction

There has been considerable volatility (and uncertainty) in the past few years in mature and emerging financial markets worldwide. Most investors and financial analysts are concerned about the uncertainty of the returns on their investment assets, caused by the variability in speculative market prices (and market risk) and the instability of business performance (Alexander, 1999). Recent developments in

financial econometrics require the use of quantitative models that are able to explain the attitude of investors not only towards expected returns and risks, but towards volatility as well. Hence, market participants should be aware of the need to manage risks associated with volatility. This requires models that are capable of dealing with the volatility of the market (and the series). Due to unexpected events, uncertainties in prices (and returns) and the non-constant variance in the financial markets, financial analysts started to model and explain the behavior of exchange rate returns and volatility using time series econometric models.

One of the most prominent tools for capturing such changing variance was the Autoregressive Conditional Heteroskedasticity (ARCH) process is based on the assumption that the recent past gives information about one period forecast variance. In (1982) Engle proposed a volatility process with time varying conditional variance, which is Autoregressive Conditional Heteroskedasticity (ARCH) process. Four years after Engle's introduced the ARCH process, Bollerslev 1986, proposed the Generalized ARCH (GARCH) models as a natural solution to the problem with the high ARCH orders, these models are based on an infinite ARCH specification and it allows to dramatically reducing the number of estimated parameters from an infinite number to just a few. In ARCH / GARCH models the conditional variance is expressed as a linear function of past squared innovations and earlier calculated conditional variances.

The usual assumptions of linear models are the disturbance terms ε_t distributed as a normal distribution with mean zero, constant variance and ε_t 's are uncorrelated, i.e. $\varepsilon_t \sim N(0, \sigma^2)$, $E(\varepsilon_t) = 0$, $E(\varepsilon_t^2) = \sigma^2$, $E(\varepsilon_i \varepsilon_j) = 0$ for $i \neq j$). This paper will briefly consider the case when the disturbance terms ε_t are vary over time, which means the errors ε_t 's doesn't have an equal Variance (Heteroskedasticity) which, can be caused by incorrect specification or used of the wrong functional form. Many economic time series exhibit periods of unusually large volatility followed by periods of relative tranquility, common examples of these such as a series include stock prices, foreign exchange rates and other prices determined in financial markets are known as their variance is seems to be vary over time.

This paper aims at modelling and forecasting exchange rate volatility in the Sudan using Asymmetric GARCH Generalized Autoregressive Conditional Heteroskedasticity models as well as understanding exchange rates behavior to monetary policy and international trade

II. Literature Review

To capture the volatility in financial time series, a comprehensive empirical analysis of the returns and conditional variance of the financial time series have been carried out using autoregressive conditional Heteroskedasticity models. Bellow a literature review of these studies:

Sharaf Obaid, Abdalla Suliman.(2013).Estimating Stock Returns Volatility of Khartoum Stock Exchange through GARCH Models this study modeled and estimated stock returns volatility of Khartoum Stock Exchange (KSE) index using symmetric and asymmetric GARCH family models namely :GARCH (1.1) GARCH-M (1.1) EGARCH (1.1) and GJR-GRACH (1.1) models, they carried out that based on daily closing prices over the period from Jan 2006 to Aug 2010 that high volatility processing present in KSE index return series . The result also provided evidence on the existence of risk premium and indicate the presence of leverage effect in the KSE index returns series our findings indicate the student-t is the most favored distribution for all models estimated.

Mohd.Aminal Islam (2013) Estimating Volatility of Stock Index Returns by using Symmetric GARCH Models, this study was utilize Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models to estimate volatility of financial asset returns of three Asian markets namely kualalampur composite index (KLCI) of Malaysia Jakarta Stock Exchange Composite Index (JKSE) of Indonesia and straits Times index (STI) of Singapore. Two symmetric GARCH models with imposing names such as the GARCH (1.1) and the GARCH-in-Mean or GARCH-M (1.1) are considered in this study. They was cover the period 2007-2012 comprising daily Observations of 1477 for KLCI. 1461 for JKSE and 1493 for STI excluding the public holidays we choose to apply GARCH models as they are especially suitable for high frequency financial market data such as stock returns which has a time-varying variance unlike the linear structural models GARCH models are found useful in explaining a number of important features commonly observed in most financial time series Ahmed E I sheikh M. Ahmed and SulimanZakaria (2013) Modeling stock Market volatility using GARCH Models Evidence from Sudan, they used the Generalized Autoregressive conditional Heteroskedastic Models to estimate volatility (conditional variance) In the daily returns of the principal stock exchange of Sudan namely Khartoum stock Exchange (KSE) over the period from 2006 to 2010. daily Observations of 1326 for (KSE) The models include both symmetric and asymmetric models that capture the most common stylized facts about index returns such as volatility clustering and leverage effect the empirical result show that the conditional variance process is highly persistent (explosive process) and provide evidence on the existence of risk premium of the KSE index return series which support the positive correlation hypothesis between volatility and the expected stock returns was findings also show that the asymmetric models provide better fit than the symmetric models; which confirms . The presence of leverage effect. These results in general explain that high volatility of index return series is present in Sudanese stock market over the sample period.

III. Methodology

In this section, we briefly present the models specification, conditional distributions and forecasting criterias as well as data set we use to model the SDG/US Dollars Exchange rate returns volatility in the Sudan economy. This article analyses the volatility of the Sudan exchange rate using various volatility models such as Autoregressive Integrated Moving Average (ARIMA), GARCH, the Glosten, Jagannathan and Runkle (GJR) GARCH, Asymmetric Power Autoregressive conditional Heteroskedasticity APARCH model of Ding et.al (1993) as well as the conditional distributions such as normal and Student-t distributions. In this study three different criteria's, Mean Squared Error (MSE), Mean Absolute Error (MAE) and Adjusted Mean Absolute Percentage Error (AMAPE) are used to evaluate the forecasting performance for the conditional heteroscedasticity models.

1.1 ARIMA Model

The AR, MA and ARMA models assume stationary series. If the time series is nonstationary we can have a model which reflects this fact. This model which is called an ARIMA model and written as Autoregressive Integrated- Moving Average and denoted by $ARIMA(p, d, q)$ represents ARMA model with nonstationarity. In general it takes the form:

$$\phi(B)(1 - B)^d x_t = \theta(B)e_t, \dots\dots\dots(3.1)$$

where $(1 - B)^d$ is the d th order difference.

In the notation of Box-Jenkins it can be written as:

$$\phi(B)\Delta x_t = \theta(B)e_t, \dots\dots\dots(3.2)$$

This is the model that calls for the d th order difference of the time series in order to make it stationary.

In ARIMA (p,d,q)

p = order of the autoregressive process,

d= degree of differencing employed,

q= order of moving average process .

In practice the value of p, d and q rarely exceed 2 (they are usually 0 or 1).

1.2 GARCH Model

Bollerslev (1986) proposed a useful extension known as generalized ARCH (GARCH) process. In GARCH model the conditional variance of return series is expressed as a function of constant, past news about

volatility (ε_{t-i}^2) terms and past forecast variance (h_{t-i}^2) terms. In the GARCH (p,q) the conditional variance is expressed as follows:

$$\varepsilon_t = \eta_t \sqrt{h_t}, \dots \dots \dots (3.3)$$

$$h_t^2 = \delta + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j h_{t-j}^2, \dots \dots \dots (3.4)$$

Where η_t is independently identically distributed random variable with mean zero and variance 1, $\delta > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$ and

$$\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_j) < 1, \dots \dots \dots (3.4)$$

A parsimonious and simpler form of GARCH (p,q) models is the GARCH(1,1) model which is specified as follows:

$$h_t^2 = \delta + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}^2, \dots \dots \dots (3.5)$$

Where $\delta, \alpha > 0$, $\beta \geq 0$ and $\alpha + \beta < 1$.

Persistence of the volatility in the commodity series is measured by the sum of α and β .

1.3 The APARCH Model

Ding, Granger and Engle (1993) also introduced the Power GARCH (PGARCH) specification to deal with asymmetry. Unlike other GARCH models, in this model, the standard deviation is modeled rather than the variance as in most of the GARCH-family. In Power GARCH an optional parameter γ can be added to account for asymmetry (Floros, 2008). The model also affords one the opportunity to estimate the power parameter δ instead of imposing it on the model (Ocran and Biekets, 2007). The general asymmetric Power GARCH model specifies h_t^δ as of the following form:

$$h_t^\delta = \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-1}| - \gamma_i \varepsilon_{t-1})^\delta + \sum_{j=1}^q \beta_j h_{t-j}^\delta, \dots \dots \dots (3.6)$$

Where $\omega > 0$, $\delta \geq 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, $-1 < \gamma_i < 1$, $i=1,2,\dots,p, j=1,2,\dots,q$.

The model is couples the flexibility of varying exponent with the asymmetry coefficient, moreover The APARCH includes other ARCH extensions as special cases.

a) GJR-GARCH

Glastron, Jagannathan and Runkle Generalized Autoregressive Conditional Heteroskedasticity This model is known as GJR GARCH models, proposed by Glastron, Jagannathan & Runkle (1993), are capable of capturing the symmetric effect in regard to the conditional volatility. The variance equation in the GJR (p,q) model is specified as follows:

$$h_t^2 = \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-1}| - \gamma \varepsilon_{t-1})^2 + \sum_{j=1}^q \beta_j h_{t-j}^2, \dots \dots \dots (3.7)$$

where $\omega > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, $i=1, 2 \dots p$, $j=1, 2 \dots q$,

The impact of ε_t^2 on the conditional variance h_t^2 in this model is different when ε_t is positive or negative. The negative innovations (bad news) have a higher impact than positive ones. When ε_{t-1} is positive, the total contribution to the volatility of innovation is $\alpha \varepsilon_{t-1}^2$; when ε_{t-1} is negative, the total contribution to the volatility of innovation is $(\alpha + \gamma) \varepsilon_{t-1}^2$.

γ would expect to be positive, so that the (bad news) has larger impact, in this case there is a leverage effect.

b) DGE-GARCH

$$h_t^\delta = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^\delta + \sum_{j=1}^q \beta_j h_{t-j}^\delta + \gamma \varepsilon_{t-i}^\delta I_{t-i}, \dots \dots \dots (3.8)$$

where $\delta > 0$, $\alpha \geq 0$, $\beta \geq 0$ and $-1 < \gamma_i < 1$

1.4 Distribution Assumptions

In this paper: the normal distribution, and the Student-t distribution are considered in order to take into account the skewness, excess kurtosis and heavy-tails of return distributions.

1.4.1 Normal Distribution

The standard GARCH (p, q) model introduced by Tim Bollerslev (1986) is with normal distributed error $\varepsilon_t = h_t z_t$, $z_t \sim iid(0,1)$. Use maximum log-likelihood method to estimate the parameter in the standard GARCH model, given the error following the Gaussian and we can get the log-likelihood function:

$$f_{\varepsilon_t}(\varepsilon_t | \theta) = \ln \prod_{i=1}^T \frac{1}{\sqrt{2\pi h_t^2}} \exp\left[-\frac{1}{2} \frac{\varepsilon_t^2}{h_t^2}\right] = \ln \prod_{i=1}^T \frac{1}{\sqrt{2\pi h_t^2}} \exp\left[-\frac{z_t^2}{2}\right] \dots (3.9)$$

$$= \frac{1}{2} \sum_t [\ln(2\pi) + \ln(h_t^2) + z_t^2]$$

Where $z_t^2 = \frac{\varepsilon_t^2}{h_t^2}$ is independently and identically distributed

1.4.2 Student’s t-Distribution

As mentioned before, GARCH model often does not allow asymmetry and is not sufficiently fat-tailed to capture the excess kurtosis found in most financial return data. This has led to a search for more flexible conditional distribution (non-normal distributions) to replace the conditional normal assumption. Bollerslev (1987) was the first combined the GARCH models with a standardized Student’s t-distribution with $\nu > 2$ degrees of freedom whose density is given by:

$$f(\mathbf{z}_t | \mathbf{v}) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\pi(\nu-2)}} \left(1 + \frac{z_t^2}{\nu-1}\right)^{-\frac{\nu+1}{2}} \dots (3.10)$$

where $\mathbf{z}_t = \varepsilon_t/h_t$ be the standardized error , $\Gamma(\nu)$ is the gamma function , ν is the parameter that measures the tail thickness.

1.5 Data

The data will be use in the analysis of this paper are monthly readings of Exchange Rate in the Sudan covered the period from 01/01/1999 to 31/12/2013 obtained from Central Bureau of Statistics, Bank of Sudan and Khartoum Stock market. and then transformed into logarithmic return series. The corresponding transform price series into monthly logarithmic return are calculated by using. the formula: $r_t = \ln(x_t/x_{t-1})$ where x_t is the exchange rate and r_t denotes the returns

IV. Empirical Result and Discussions

In this section, we present the empirical results as well as discussions of estimation results we obtained to account for the SDP/US Dollars exchange rate returns volatility in Sudan. The analysis was done using the Eviews 8 to provide empirical results of the monthly exchange rate prices data The parameter estimation method that we choose is the Maximum Likelihood Estimation (MLE), and estimates the models with the given distributional assumption to determine the best performance of forecasting model of exchange rate volatility. Some summary statistics for the monthly exchange rate returns (SDG/USD) are displayed in Table 1

Table 1: Summary Statistics of Exchange rate Returns (SDG/ USA (\$))

Sample size	Mean	Median	Var.	S.Dev	Min.	Max.	Skew.	Kurt.	Jarque Bera	P-value
178	0.0051	0.000	0.013	0.036	-0.076	0.373	6.717	62.241	27521.52	0.000

The summary statistics of this study is presented in table 1. This indicates that the returns series have monthly positive mean of (0.0051) while the monthly volatility is (0.013), without loss of generality the mean grows at a linear rate while the volatility grows approximately at a square root rate. The lowest monthly returns correspond to (-0.076) and the best monthly exchange rate returns is (0.373). The returns series of the exchange rate shows positive skewness. This implies that the series is flatter to the right. The kurtosis value is higher than the normal value of perfectly normal distribution in which value for skewness is ‘zero’ and kurtosis is ‘three’ and this suggest that the kurtosis curve of the exchange rate return series is leptokurtic. The results of this study reveal that, the series is not normally distributed. Our empirical result is consistent with the Jarque-Bera (JB) tests Obtain above which is used to assess whether the given series is normally distributed or not. Here, the null hypothesis is that the series is normally distributed. Results of JB test find that the null hypothesis is rejected for the return series and suggest that the observed series are not normally distributed

Table 2: Parameter Estimation of the ARMA (1, 2)-GARCH (1, 1), GJR (1, 1) and DGE (1, 1) Models with the Conditional

Conditional Distribution	GARCH		GJR-GARCH		DEG-GARCH	
	Normal	Student-t	Normal	Student-t	Normal	Student-t
Mu(μ)	0.048	-0.00011	0.002	-7.82E-05	0.003	-4.29E-05
	0.053	0.4221	0.474	0.2887	0.3602	0.99
Ar1(ϕ)	0.998	0.372	0.394	0.351	0.45	0.38
	0.000	0.000	0.466	0.000	0.14	0.000
Ma(θ_1)	-0.883	-0.042	0.056	-0.081	-0.032	-0.049
	0.000	0.336	0.991	0.011	0.89	0.042
Ma(θ_2)	-0.109	0.00048	0.009	-0.008	0.002	0.0026
	0.035	0.975	0.971	0.375	0.99	0.364
Omega (ω)	1.26E-07	2.05E-06	0.0002	6.55E-07	0.00065	0.0016
	0.908	0.007	0.000	0.0134	0.863	0.605
Alpha (α_i)	5.272	2.022	0.842	7.233	0.918	6.957
	0.000	0.0292	0.000	0.039	0.0007	0.352
Beta(β_1)	0.006	-0.0015	-0.168	-0.125	-0.197	-0.155
	0.5007	0.6735	0.101	0.192	0.094	0.127
Gamma(γ_1)			-0.028	-0.002	-0.043	0.015
			0.376	0.604	0.739	0.602
Delta (δ)			2	2	1.8	0.893
					0.198	0.0018
Shape (ν)				2.253		2.025
				0.000		0.000

Table 2 presents the parameter estimation results of ARMA (2, 1) - GARCH (1, 1), GJR-GARCH (1, 1) and DGE-GARCH (1, 1) models with the normal and, student's-t distributions and their corresponding p-values. The results show that the parameters estimated in these three models are all significant under the given conditional distributions except for the coefficients of Mu. Under the student's- t distribution, the sum of the GARCH parameter estimates ($\alpha_i + \beta_j$) is greater than 1, implying that the volatility rate model is strictly stationary GARCH model is less than 1, which indicates that the model is well fitted. For the normal distribution the sum of the GARCH parameter is less than 1 for the GJR-GARCH and DGE-GARCH models with the which also show that the shocks in volatility is limited and stationary and the model is well fitted, the sum is greater than 1 in case of the GARCH model. The leverage effect term (gamma) in both the GJR model and the DGE is not statistically significant but it is negative, implying that negative shocks results to a higher next period conditional variance than positive shocks of the same sign, it indicates that the bad news (negative shocks) effect the volatility more than the good news. The table shows that the estimated δ of the DGE -GARCH model under the normal distribution is 1.8is not significant which is significantly in student-t distribution.

Table 3: Analysis of standardized residuals and fitted parameters

Conditional Distribution	GARCH		GJR-GARCH		DEG-GARCH	
	Normal	Student-t	Normal	Student-t	Normal	Student-t
Log likelihood	504.885	675.582	466.085	695.039	464.773	708.127
Jarque – Bera Test	4134.971	36547.94	7546.622	42675.11	8051.477	50508.8
	0.000	0.000	0.000	0.000	0.000	0.000
Ljung- Box Test R (Q10)	12.69	1.2776	3.3783	0.3547	4.1727	0.0376
	0.08	0.989	0.848	1.000	0.76	1.000
Ljung- Box Test R (Q15)	20.483	17.563	16.566	18.276	17.43	20.51
	0.058	0.130	0.167	0.108	0.134	0.058
Ljung- Box Test R (Q20)	32.424	42.635	37.737	45.818	39.782	45.216
	0.013	0.001	0.003	0.000	0.001	0.000
Ljung- Box Test R ² (Q10)	1.462	0.2803	2.9889	0.2483	2.2772	0.2189
	0.999	1.000	0.982	1.000	0.994	1.000
Ljung- Box Test R ² (Q15)	3.533	8.3468	13.078	8.2591	11.666	9.013
	0.999	0.909	0.596	0.913	0.704	0.877
Ljung- Box Test R ² (Q20)	41.934	26.456	35.174	27.357	33.795	22.171
	0.003	0.151	0.019	0.126	0.028	0.331
LM Arch Test	0.0167	0.0317	2.404	0.0293	1.7665	0.025
	0.897	0.858	0.1224	0.864	0.183	0.874
AIC	-5.594	-7.5009	-5.147	-7.7088	-5.121	-7.844
BIC	-5.469	-7.357	-5.004	-7.547	-4.960	-7.665

The coefficients reported as shown in the table are the maximum likelihood estimates of the parameters and the p-values are in parentheses for the ARMA (1,2) - GARCH (1,1), GJR-GARCH (1, 1) and DGE-GARCH (1, 1), models. The estimation results of the models with the conditional distributions, including log-likelihood value, the Box-Pierce statistics of lags 10, 15 and 20 of the standardized and squared standardized residuals, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), the ARCH test and their respective p-values are listed in Table 3. Comparing the log-likelihood, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values among these models DGE-GARCH and GJR-GARCH models better estimate the exchange rate return series than the GARCH model with the Student t-distribution assumption gives better results. The results also show that the student's t-distribution outperforms the normal distribution, discussed in this paper. With these models, DEG-GARCH with Student t-distribution gives the highest log-likelihood value of 708.124. The AIC and BIC values of the GARCH and DEG -GARCH models under the three conditional distribution gives the lowest values when compared to the GJR-GARCH and GARCH models and that the DEG -GARCH model with the student's t-distribution provides the smallest values of AIC (-7.884) and BIC (- 7.665) respectively, this implies that DEG -GARCH model under the student's t-distribution provides a better fit for the monthly exchange rate returns according to this criterions.

The table shown the t-statistics and p-values are in parentheses for ARMA (1, 2)-GARCH(1,1),GJR(1,1) and DGE(1, 1) models.(AIC) represent Akaike Information Criterion, (BIC) is Bayesian Information Criterion (BIC), Ljung-Box Test R (Standardized Residuals and Ljung-Box TestR^2 (Square Standardized Residual) The Jarque-Bera statistic to test the null hypothesis of whether the standardized residuals are normally distributed. The results presented in table 3 show that the standardized residuals are leptokurtic and the Jarque-Bera statistic strongly rejects the hypothesis of normal distribution which means that the fat-tailed asymmetric conditional distributions outperform the normal for modeling and forecasting the exchange rates volatility returns. The Ljung Box tests for the residuals have p-values that are statistically not significant indicating that no serial correlation exists except twentieth-order. The Ljung-Box statistics for up to twentieth-order serial correlation of squared residuals are not significant suggesting that no significance correlation exist. As for the LM-ARCH test the results reveals that the conditional heteroskedasticity that existed in the exchange rate returns time series have successfully removed, indicating that no significant appearance of the ARCH effect

1.6 Forecasting

The forecasting ability of the GARCH models has been discussed precisely by Poon and Granger (2003). We use the Eviews 8 to evaluate a five step ahead forecast using 180 observations for the monthly exchange rate returns. The forecasts are evaluated using three different measures which provide robustness in choosing the optimal predicts models for the return series. We consider the following measures

1). Mean Squared Error (MSE):

As a measure of desperation of forecast error, statisticians have taken the average of the squared individual errors. The smaller the MSE value, the more stable the model. However, interpreting the MSE value can be misleading, for the mean squared error will be accentuate large error terms. it can be describes as:

$$MSE = \frac{1}{h+1} \sum_{t=s}^{s+h} (\hat{\sigma}_t^2 - \sigma_t^2)^2 , \dots \dots \dots (4.1)$$

2). Mean absolute error (MAE):

This error measurement is the average of the absolute value of the error without regard to whether the error was an overestimate or underestimates (Krajewski and Ritzman, 1993), it is equation takes the form:

$$MAE = \frac{1}{h+1} \sum_{t=s}^{s+h} | \hat{\sigma}_t^2 - \sigma_t^2 | , \dots \dots \dots (4.2)$$

3). Adjusted mean absolute percentage error: Adjusted Mean Absolute Percentage Error (AMAPE) is a measure based on Percentage errors.

$$AMAPE = \frac{1}{h+1} \sum_{t=s}^{s+h} | \frac{\hat{\sigma}_t^2 - \sigma_t^2}{\hat{\sigma}_t^2 + \sigma_t^2} | , \dots \dots \dots (4.3)$$

where h is the number of head steps, s is the sample size, $\hat{\sigma}_t^2$ is the forecasted variance and σ_t^2 is the actual variance.

Table 4: Forecasting Analysis for the Exchange rate returns with the Conditional distributions

Exchange rate returns (SDG /USA(\$))	GARCH		GJR-GARCH		DEG-GARCH	
	Normal	Student	Normal	Student	Normal	Student
MSE	0.00406	0.001354	0.001337	0.001354	0.001334	0.001353
MAE	0.058991	0.011101	0.012129	0.011098	0.012587	0.011080
AMAPE	8582.155	90.45945	372.7228	87.82762	491.7924	83.77597

The results, as shown in the table 4 above, indicate that the forecasting performance of the GJR-GARCH and DGE-GARCH models, especially when fat-tailed asymmetric conditional distributions are taken into account in the conditional volatility, is better than the GARCH model. However, the comparison between the models with normal and student-t distributions shows that, according to the different measures used for evaluating the performance of volatility forecasts, the DEG –GARCH model provides the best forecasts and clearly outperforms GJR-GARCH and GARCH models and the DGE-GARCH model provides less satisfactory forecast results while the poorest forecast results was registered for the GARCH model. Moreover, it is found that the Student-t distribution is more appropriate for modeling and forecasting the exchange rate returns volatility.

V. Conclusion

Modeling exchange rate volatility has received considerable attention from academics, market participant, policy makers, investors and practitioners in recent years as it provide a measure of risk in the financial market. It is important to note that, portfolio selection, asset valuations, risk management, option pricing and hedging strategies provides the importance of modeling and forecasting the conditional volatility of exchange rate returns. To modeling the financial time series data, we review of the autoregressive conditional heteroscedasticity (ARCH) and generalized autoregressive conditional heteroscedasticity (GARCH) model. Consider of the stylized facts of the asset return series including the ARMA, GARCH, GJR-GARCH, and DGE-GARCH with normal and student-t distribution.

The results show that the forecasting performance of asymmetric GARCH Models (GJR and DGE), especially when fat-tailed asymmetric conditional distributions are taken into consideration in the conditional volatility, is better than GARCH model. The estimated parameters of the Models are statistically significant except, the coefficients of μ for the GARCH and GJR- GARCH models under the three conditional distributions. Also, the coefficients on the standardized residuals and squared residuals of 10, 15 and 20 except 20 and the ARCH effect are not statistically significance which implies that no serial correlation exists in the exchange rate return series and that no significant appearance of the ARCH effect in the returns series and the variance equation is correctly specified.

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