

Reduced Fuzzy Soft Matrices in Medical Diagnosis

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Abstract: Today is a world of uncertainty with its associated problems, which can be well handled by Soft set theory which is a new emerging mathematical tool. In this paper, Sanchez's approach for medical diagnosis using the representation of Reduced Fuzzy Soft Matrix is introduced. It has less computational work compared with available methods. This is a new approach in the medical diagnosis field by employing the interval valued Fuzzy Soft set.

Keywords: Soft matrix, Fuzzy soft matrix, Interval valued fuzzy soft matrix Reduced fuzzy soft matrix.

I. Introduction

In real life situation, most of the problems in economics, social science, environment etc, have various uncertainties. There are theories viz, theory of probability, evidence, fuzzy set[18], intuitionistic fuzzy set[1,2], vague set, interval mathematics, rough set for dealing with uncertainties. These theories have their own difficulties as pointed out by Molodtsov[11]. In 1999 Molodtsov[11] initiated a novel concept of soft set theory which is completely new approach for modeling vagueness and uncertainties. Soft set theory has a rich potential for application in solving practical problems in economics, social science, medical science etc. Later on Maji et al [5] have studied the theory of fuzzy soft set. Majumdar et al[8] have further generalised the concept of fuzzy soft sets. Maji et al[7] extended fuzzy soft sets to intuitionistic fuzzy soft sets. Yang et al[15] presented the concept of the interval-valued fuzzy soft sets by combining the interval valued fuzzy sets and soft set.

Matrices play an important role in the broad area of science and engineering. However the classical matrix theory sometimes fails to solve the problems involving uncertainties. Fuzzy matrices play a vital role in scientific Development. In [17], Yong et al initiated a matrix representation of a fuzzy soft set and applied it in certain decision making problems. In[9] Borah et al extended fuzzy soft matrix theory and its application. In [3] Chetia et al proposed intuitionistic fuzzy soft matrix theory. Accordingly in[12,13], Rajarajeswari et al proposed new definitions for intuitionistic fuzzy soft matrices and its types. Also extended some operations on it.

The field of medicine is the most fruitful and interesting area of applications of fuzzy theory. There are variety of models involving fuzzy matrices to deal with different complicated aspects of medical diagnosis. Sanchez formulated the diagnostic models involving fuzzy matrices representing the medical knowledge between symptoms and diseases. Chetia [4] have studied Sanchez's approach of medical diagnosis through interval-valued fuzzy soft sets.

It is not always feasible to make winding up due to the fact that the Interval Valued Fuzzy Soft Matrix given an interval valued data to describe the degree of membership. In order to improve this, put forward reduction set that is a compromised one. A reduction is interpreted as a minimum subset that provides the same descriptive ability as the entire set.

II. Preliminaries

Basic Notion of Interval valued Fuzzy Soft Matrices and Reduced Fuzzy Soft Matrices are studied.

2.1. Soft set [11] Suppose that U is an initial universe set and E is a set of parameters, let $P(U)$ denotes the power set of U . A pair (F, E) is called a soft set over U where F is a mapping given by

$F: E \rightarrow P(U)$. Clearly, a soft set is a mapping from parameters to $P(U)$, and it is not a set, but a parameterized family of subsets of the Universe.

2.2. Fuzzy soft set [6]

Let U be an initial Universe set and E be the set of parameters. Let $A \subseteq E$. A pair (F, A) is called fuzzy soft set over U where F is a mapping given by $F: A \rightarrow I^U$, where I^U denotes the collection of all fuzzy subsets of U .

2.3. Interval valued Fuzzy soft set [16]

Let U be an initial Universe set and E be the set of parameters. Let $A \subseteq E$. A pair (F, A) is called fuzzy soft set over U where F is a mapping given by $F: A \rightarrow I^U$, where I^U denotes the collection of all interval valued fuzzy subsets of U .

2.4. Fuzzy Soft Matrix [17]

Let $U=\{c_1,c_2,c_3,\dots,c_m\}$ be the Universal set and E be the set of parameters given by $E=\{e_1,e_2,e_3,\dots,e_n\}$. Let $A \subseteq E$ and (F,A) be a fuzzy soft set in the fuzzy soft class (U,E) . Then fuzzy soft set (F,A) in a matrix form as $A_{m \times n}=[a_{ij}]_{m \times n}$ or

$$A = [a_{ij}] \quad i=1, 2, \dots, m, j=1, 2, 3, \dots, n$$

Where $a_{ij} = \begin{cases} \mu_j(c_i) & \text{if } e_j \in A \\ 0 & \text{if } e_j \notin A \end{cases}$ $\mu_j(c_i)$ represents the membership of c_i in the fuzzy set $F(e_j)$.

2.5 Interval valued fuzzy soft matrix (IVFSM)[14]

Let $U=\{c_1,c_2,c_3,\dots,c_m\}$ be an Universal set and E be the set of parameters given by $E=\{e_1,e_2,e_3,\dots,e_n\}$. Let $A \subseteq E$ and (F,A) be an interval valued fuzzy soft set over U , where F is a mapping given by $F: A \rightarrow I^U$, where I^U denotes the collection of all interval valued fuzzy subsets of U . Then the interval valued fuzzy soft set can be expressed in matrix form as

$$\tilde{A}_{m \times n}=[\tilde{a}_{ij}]_{m \times n} \text{ or } \tilde{A}=[a_{ij}] \quad i=1,2,\dots,m, j=1,2,3,\dots,n$$

where $a_{ij} = \begin{cases} [\mu_{jL}(c_i), \mu_{jU}(c_i)] & \text{if } e_j \in A \\ [0, 0] & \text{if } e_j \notin A \end{cases}$

$[\mu_{jL}(c_i), \mu_{jU}(c_i)]$ represents the membership of c_i in the interval valued fuzzy set $F(e_j)$.

Note: If $\mu_{jU}(c_i) = \mu_{jL}(c_i)$ then the **IVFSM** reduces to an **FSM**.

2.6 Reduced Fuzzy Soft set [15]

Let $U=\{c_1, c_2, c_3, \dots, c_m\}$ be an Universal set and E be the set of parameters given by $E=\{e_1, e_2, e_3, \dots, e_n\}$. Let $A \subseteq E$ and (F,A) be an Interval Valued Fuzzy Soft Set over U , where F is a mapping given by $F: A \rightarrow I^U$, where I^U denotes the collection of all Interval Valued Fuzzy subsets of U . Then the Interval Valued Fuzzy Soft set is

$$F(e_j) = \left\{ \left(c_i, [\mu_{jL}(c_i), \mu_{jU}(c_i)] \right), \forall e_j \in A \text{ and } \forall c_i \in U \right\}, i=1, 2, \dots, m, j=1, 2, 3, \dots, n$$

$[\mu_{jL}(c_i), \mu_{jU}(c_i)]$ represents the membership of c_i in the Interval Valued Fuzzy set $F(e_j)$.

Let $w_1, w_2 \in [0,1], w_1 + w_2 = 1$.

The vector $W = (w_1, w_2)$ is called weighted vector. The Fuzzy Soft set (F_w, A) over U such that

$$F_w(e_j) = \left\{ \left(c_i, w_1 \mu_{jL}(c_i) + w_2 \mu_{jU}(c_i) \right), \forall c_i \in U, \forall e_j \in A \right\}, i=1, 2, \dots, m, j=1, 2, 3, \dots, n$$

is called **Reduced Fuzzy Soft set** of an Interval Valued Fuzzy Soft set with respect to the weighted vector. By adjusting the value of w_1 and w_2 an interval valued Fuzzy Soft set can be converted into Reduced Fuzzy Soft set. Then the Reduced Fuzzy Soft Set (F_w, A) over U in a matrix form as

$$\tilde{A}_R = [a_{ij}]_{m \times n} \text{ where}$$

$$a_{ij} = \begin{cases} w_1 \mu_{jL}(c_i) + w_2 \mu_{jU}(c_i) & \text{if } e_j \in A \\ 0 & \text{if } e_j \notin A \end{cases} \quad i=1, 2, \dots, m, j=1, 2, \dots, n$$

so it can identify any Reduced Fuzzy Soft set (F_w, A) in the Fuzzy Soft class (U,E) by its Reduced Fuzzy Soft Matrix \tilde{A}_R .

Let $\tilde{A}_R = [a_{ij}]_{m \times n} \in \mathbf{RFSM}$. Then \tilde{A}_R is called

- i) **Pessimistic Reduced Fuzzy Soft Matrix** if $w_1 = 1, w_2 = 0$.
- ii) **Optimistic Reduced Fuzzy Soft Matrix** if $w_1 = 0, w_2 = 1$.
- iii) **Neutral Reduced Fuzzy Soft Matrix** if $w_1 = 0.5, w_2 = 0.5$.

III. Methodology

In this section, Sanchez’s approach for medical diagnosis using the representation of Reduced Fuzzy Soft Matrix is introduced. This proposed new method has less computational work compared with available methods. It is very much better than the approach of medical diagnosis through the Fuzzy Soft set and its extensions.

Assume that there is a set of m patients $P = \{p_1, p_2, p_3, \dots, p_m\}$ with a set of n symptoms $S = \{s_1, s_2, s_3, s_4, \dots, s_n\}$ related to a set of k diseases $D = \{d_1, d_2, d_3, \dots, d_k\}$. Apply Interval Valued Fuzzy Soft matrix to diagnose which patient is suffering from what disease. Construct a Interval Valued Fuzzy Soft set (F, P) over S where F is a mapping $F: P \rightarrow IV^S$, IV^S is the collection of all Interval Valued Fuzzy subsets of S . This Interval Valued Fuzzy Soft set gives a matrix \tilde{R}_{sym} called patient symptom matrix, then construct another Interval Valued Fuzzy Soft set (G, S) over D where G is a mapping $G: S \rightarrow IV^D$, IV^D is the collection of all Interval Valued Fuzzy subsets of D . This Interval Valued Fuzzy Soft set gives a matrix \tilde{R}_{occ} called symptom–disease matrix, where each element denote the weight of the symptoms for a certain disease. Compute the complements $(F, P)^c$ and $(G, S)^c$ and their matrices \tilde{R}_{sym}^c and \tilde{R}_{occ}^c called non symptom matrix and non occurrence matrix respectively. Compute $\tilde{R}_{R_1} = \tilde{R}_{R_{sym}} * \tilde{R}_{R_{occ}}$ which is the maximum membership of occurrence of Symptoms of the diseases. Compute $\tilde{R}_{R_2} = \tilde{R}_{R_{sym}}^c * \tilde{R}_{R_{occ}}$ which is the maximum membership of non Symptoms of the diseases. Compute $\tilde{R}_{R_3} = \tilde{R}_{R_{sym}} * \tilde{R}_{R_{occ}}^c$ which is the maximum membership of non occurrence of the diseases. Find the Addition matrix $\tilde{R}_{R_4} = \tilde{R}_{R_2} + \tilde{R}_{R_3}$.

Finally find (p_i, d_j) at which $\max_{i,j} \tilde{R}_{R_1}$ with $\min_{i,j} \tilde{R}_{R_4} \forall i$ then conclude that the patient p_i is suffering from disease d_j . In case it occurs for more than one (p_i, d_j) , then reassess the symptoms to break the tie.

IV. Reduced Fuzzy Soft Matrix -Algorithm

The Reduced Fuzzy Soft Matrix Algorithm is provided as

- Step1:** Input the Interval Valued Fuzzy Soft set (G,S) over the set P of symptoms and obtain the Interval Valued Fuzzy Soft Matrix \tilde{R}_{sym} corresponding to the set (G,S) which gives membership interval to the patient for the symptoms .
- Step2:** Write the Interval Valued Fuzzy Soft complement set $(G,S)^c$ and obtain the Interval Valued Fuzzy Soft complement matrix \tilde{R}_{sym}^c corresponding to $(G,S)^c$ which gives membership interval to the patient for non-symptoms.
- Step3:** Input the Interval Valued Fuzzy Soft set (F,D) over the set S of disease and obtain the Interval Valued Fuzzy Soft Matrix \tilde{R}_{occ} corresponding to the set (F,D) which gives membership interval to the symptoms for the disease .
- Step4:** Write the Interval Valued Fuzzy Soft complement set $(F,D)^c$ and obtain the Interval Valued Fuzzy Soft complement matrix \tilde{R}_{occ}^c corresponding to $(F,D)^c$ membership interval to the non-symptoms for the disease.
- Step5:** Input a weighted vector $W = (w_1, w_2)$ and convert the Interval Valued Fuzzy Soft Matrix into any one of the Reduced Fuzzy Soft Matrices.
- Step6:** Compute the following product matrices for Reduced Fuzzy Soft Matrices
 i) $\tilde{R}_{R_1} = \tilde{R}_{R_{sym}} * \tilde{R}_{R_{occ}}$ ii) $\tilde{R}_{R_2} = \tilde{R}_{R_{sym}}^c * \tilde{R}_{R_{occ}}$ iii) $\tilde{R}_{R_3} = \tilde{R}_{R_{sym}} * \tilde{R}_{R_{occ}}^c$
- Step7:** Compute the Addition matrix $\tilde{R}_{R_4} = \tilde{R}_{R_2} + \tilde{R}_{R_3}$.
- Step8:** Find (p_i, d_j) at which $\max_{i,j} \tilde{R}_{R_1}$ with $\min_{i,j} \tilde{R}_{R_4} \forall i$.

Then conclude that the patient p_i is suffering from disease d_j . Incase it occurs for more than one (p_i, d_j) , then reassess the symptoms to break the tie.

V. Illustrative Example

There are diseases having some common symptoms. Our goal is to develop a methodology and algorithm using Reduced Fuzzy Soft Matrix to help general practitioners in diagnosing and predicting patient's condition from certain rules based on experience.

- Sanchez's approach for medical diagnosis using fuzzy set is applied to diagnose which patient is suffering from what disease.
- Suppose there are three patients in a hospital with symptoms temperature, headache, cough and stomach problem. Let the possible diseases related to the above symptoms be viral fever and malaria. Now take $P = \{p_1, p_2, p_3\}$ as the universal set where p_1, p_2 and p_3 represents patients. Let $S = \{s_1, s_2, s_3, s_4\}$ as the set of symptoms where s_1, s_2, s_3, s_4, s_5 represents symptoms temperature, headache, cough, stomach problem respectively.

$$\text{Let the patient Symptom matrix is } \tilde{R}_{sym} = \begin{matrix} & \begin{matrix} s_1 & s_2 & s_3 & s_4 \end{matrix} \\ \begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} & \begin{pmatrix} [0.6, 0.9] & [0.3, 0.5] & [0.8, 1] & [0.6, 0.9] \\ [0.3, 0.5] & [0.3, 0.7] & [0.2, 0.4] & [0.3, 0.5] \\ [0.6, 0.8] & [0.2, 0.6] & [0.5, 0.7] & [0.2, 0.5] \end{pmatrix} \end{matrix}$$

$$\text{and Symptom disease matrix is } \tilde{R}_{occ} = \begin{matrix} & \begin{matrix} d_1 & d_2 \end{matrix} \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{matrix} & \begin{pmatrix} [0.7, 1] & [0.6, 0.9] \\ [0.1, 0.4] & [0.4, 0.6] \\ [0.5, 0.6] & [0.3, 0.6] \\ [0.2, 0.4] & [0.8, 1] \end{pmatrix} \end{matrix}$$

$$\tilde{R}_{N_{sym}}^c = \begin{matrix} & \begin{matrix} s_1 & s_2 & s_3 & s_4 \end{matrix} \\ \begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} & \begin{pmatrix} 0.25 & 0.6 & 0.1 & 0.25 \\ 0.6 & 0.5 & 0.7 & 0.6 \\ 0.3 & 0.6 & 0.4 & 0.65 \end{pmatrix} \end{matrix} \quad \tilde{R}_{N_{occ}}^c = \begin{matrix} & \begin{matrix} d_1 & d_2 \end{matrix} \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{matrix} & \begin{pmatrix} 0.15 & 0.25 \\ 0.75 & 0.5 \\ 0.45 & 0.55 \\ 0.7 & 0.55 \end{pmatrix} \end{matrix}$$

Convert the Interval Valued Fuzzy Soft Matrix into Neutral Reduced Fuzzy Soft Matrix by taking the weighted vector as $w_1 = 0.5$ and $w_2 = 0.5$

Compute the following Product matrices

- i) $\tilde{R}_{N_1} = \tilde{R}_{N_{sym}} * \tilde{R}_{N_{occ}}$ ii) $\tilde{R}_{N_2} = \tilde{R}_{N_{sym}}^c * \tilde{R}_{N_{occ}}$ (non symptom) iii) $\tilde{R}_{N_3} = \tilde{R}_{N_{sym}} * \tilde{R}_{N_{occ}}^c$ (non occurrence)

$$\tilde{R}_{N_1} = \tilde{R}_{N_{sym}} * \tilde{R}_{N_{occ}} = \begin{matrix} & \begin{matrix} d_1 & d_2 \end{matrix} \\ \begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} & \begin{pmatrix} 0.75 & 0.75 \\ 0.4 & 0.5 \\ 0.7 & 0.7 \end{pmatrix} \end{matrix} \quad \tilde{R}_{N_4} = \tilde{R}_{N_2} + \tilde{R}_{N_3} = \begin{matrix} & \begin{matrix} d_1 & d_2 \end{matrix} \\ \begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} & \begin{pmatrix} 0.7 & 0.55 \\ 0.6 & 0.6 \\ 0.45 & 0.55 \end{pmatrix} \end{matrix}$$

From the matrices $\tilde{R}_{N_1}, \tilde{R}_{N_4}$ anyone have diagnostic conclusion. To make a confirmed diagnosis of a disease d_j for patient p_i if the matrix \tilde{R}_{N_1} have maximum membership 1. But this is not the case for any of three patients. To make an excluded diagnosis for a disease d_j for patient p_i if the matrix \tilde{R}_{N_4} have maximum membership 1. But this is also not possible for any of three patients. In the above Illustrative example, an acceptable diagnostic hypothesis by using at which $(p_i, d_j) \max_{i,j} \tilde{R}_{N_1}$ with $\max_{i,j} \tilde{R}_{N_4}$ for all i occurs. In \tilde{R}_{N_1} first patient has same membership 0.75 for both the disease, but in \tilde{R}_{N_4} , minimum (0.55) is for second disease. In \tilde{R}_{N_4} the second patient, has membership 0.6 for both the disease, but in \tilde{R}_{N_1} maximum (0.5) is for second disease. In \tilde{R}_{N_1} the third patient has same membership 0.7 for both the disease, but in \tilde{R}_{N_4} minimum (0.45) is for first

disease. Hence it has concluded that the patient p_1 and p_2 is suffering from the disease d_2 (malaria) whereas p_3 is from d_1 .

VI. Conclusion

From the above work it has been concluded that the Interval Valued Fuzzy Soft Matrix can be reduced to Fuzzy Soft Matrix, which holds a membership value instead of an interval value. This output is very supportive and encouraging in making a fair conclusion as it contains only one membership value rather than interval. The Reduced Fuzzy Soft Matrix algorithm is very simple, less computation, easy understandable. This feature highlights to be applied to many domains like pattern recognition, face identification, speech recognition, predictive analytics and many more decision making process.

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