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Steady Nanofluid Layer Induced By Gyrotactic Microorganisms Containing Wall Temperature Variations

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Abstract: *The problem of free convective steady boundary layer flows over a solid horizontal flat plate nested in a porous medium filled with nanofluid containing gyrotactic microorganisms is considered. Exponent of the temperature, the nanoparticle volume fraction and the density of motile microorganisms are introduced to make quantities dimensionless. The impacts of considered exponent and bioconvection parameters on the dimensionless temperature, velocity, nanoparticle concentration and density of motile microorganisms along with Nusselt, Sherwood and motile microorganism numbers are tabulated and shown graphically. For regular fluid and also for isothermal case, the results are compared with the existing data while excellent compatibility is found.*

I. Introduction

Research in free convection boundary layer flows has become a popular area of research in different industrial processes and engineering in recent years. It has gained interest for its numerous applications in fibrous insulation, food processing and storage, geophysical systems, thermal insulation, electro-chemistry, underground disposal of nuclear or non-nuclear waste, metallurgy, cooling system of electronic devices etc. according to Aziz at al.[1].The books by Nield and Bejan[2],Pop and Ingham[3], Ingham and pop[4],Vadasz[5],Vafai[6,7] contain outstanding reviews of the topic.

In this paper the behavior of boundary layer flow contained gyro tactic microorganisms is our main concern. This region, where there is a velocity profile in the flow due to the shear stress at the wall, we call the boundary layer and the boundary layer flow is a viscous fluid which is in the neighborhood of a body in contact with the fluid and in motion relative to the fluid. There are two types of boundary layer fluids which are Newtonian &non-Newtonian.Schowalter [8] discussed boundary layer theory in 1960 following Metzner [9] who presented applications of non-newtonian flow behavior in chemical engineering in 1955.Boundary layer flow behavior for non-newtonian fluid is also discussed in [10-11].

The study of airborne microorganisms has a history stretching back hundreds of years, with the earliest mention in 1676 by Antony van Leeuwenhoek (Gregory, 1971). Louis Pasteur was the first to intentionally observe the microbial content of the air, an endeavor that lead to major advances in medicine and disease control, as the spread of many diseases was traced to the aerial dispersal of bacteria (Pasteur, 1860a,b).The study of microorganism is called microbiology. Now-a-days a major development has been occurred in the field of microbiology. Scientists develop new models to predict the fluid motions associated with aquatic microorganisms. Number of mechanisms are developed that compel microorganisms to swim in certain directions depending on the ambient condition.Neild and Bejan [2] defined bioconvection as “Pattern formation in suspensions of microorganisms such as bacteria and algae due to up swimming of the microorganisms”.Gyrotactic microorganisms such as Cnivalis swim upward in still water due to the fact that their center of mass is located behind the center of buoyancy of them.A detailed discussions of bioconvection in suspensions of gyrotactic/oxytactic microorganisms is presented in [12-14]

Our aim is to analyze the work of Aziz et al.[1] for the case of exponents of temperature, nanoparticle volume fraction and the density of motile microorganisms. Also for the different values of exponent we showed the behavior of these three parameters. If the value of exponent is equal to zero, the we have the similar results as found in the paper of Aziz et al.[1].

Nomenclature

b Chemo taxis constant
$\bar{C}$ Nanoparticle volume fraction
$\bar{C}_w$ Wall nanoparticle volume fraction
$\bar{C}_\infty$ Ambient nanoparticle volume fraction
D_B Brownian diffusion coefficient
D_n Diffusivity of microorganisms

D_T Thermophoretic diffusion coefficient
$f(\eta)$ Dimensionless stream function
g Acceleration due to gravity
k Thermal conductivity
K Permeability of the porous mediCharacteristic length
Lb bioconvection Lewis number
Le Lewis number
$\bar{n}_n$ Volume fraction of motile microorganisms
Nb Brownian motion parameter fraction
N_{n_x} Local density number of motile microorganisms
Nt Thermophoresis parameter
N_{u_x} Local Nusselt number
$\bar{p}$ Pressure
Pe bioconvection peclet number
Pr Prandtl number
$\bar{q}_m$ wall mass flux
$\bar{q}_n$ wall motile microorganisms flux
$\bar{q}_w$ Wall heat flux
Ra Rayleigh number for the porous medium
Ra_x local Rayleigh number for the porous medium
Rb bioconvection Rayleigh number
Re_x local Reynolds number material
Sh_x local Sherwood number

$\bar{T}$ Nanofluid temperature
$\bar{T}_w$ Wall temperature
$\bar{T}_\infty$ Ambient temperature
$\bar{u}$,$\bar{v}$ velocity components along $\bar{x}$ &$\bar{y}$ axes
W_c Maximum cell swimming speed
$\bar{x}$,$\bar{y}$ Cartesian coordinates ($\bar{x}$-axis is aligned along the horizontal surface and $\bar{y}$ - axis is normal to it)

Greek Symbols

α_m Effective thermal diffusivity of the porous medium
$\varphi(\eta)$ Rescale nanoparticle volume
η Similarity variable
γ Average volume of a microorganism
$\theta(\eta)$ Dimensionless Temperature
ν Kinematic viscosity of fluid
ρ_f Fluid density
ρ_p Nanoparticle mass density
$(\rho C)_f$ heat capacity of the fluid
$(\rho C)_p$ heat capacity of nanoparticle
τ ratio of effective heat capacity of the nanoparticle material and heat capacity of the fluid.
$\sigma(\eta)$ rescaled density of motile microorganisms
λ Exponent of temperature, volume and density
Ψ stream function

II. Basic Equations

According to the work Aziz [1],we have the following governing equations:

Continuity Equation: $\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} = 0$ (1)

Momentum (Darcy Equation) :

$$\frac{\partial\bar{u}}{\partial\bar{y}} - \frac{\partial\bar{v}}{\partial\bar{x}} = \mp\frac{(1-\bar{C}_\infty)g\rho_{f0}K\beta}{\mu}\frac{\partial\bar{T}}{\partial\bar{x}} \pm \frac{(\rho_P-\rho_{fo})gK}{\mu}\frac{\partial\bar{C}}{\partial\bar{x}} \pm \frac{g\gamma\Delta\rho K}{\mu}\frac{\partial\bar{n}}{\partial\bar{x}} \quad (2)$$

Thermal Energy Equation:

$$\bar{u}\frac{\partial\bar{T}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{T}}{\partial\bar{y}} = \left(\frac{\partial^2\bar{T}}{\partial\bar{x}^2} + \frac{\partial^2\bar{T}}{\partial\bar{y}^2}\right) + \tau\{D_B(\frac{\partial\bar{T}}{\partial\bar{x}}\frac{\partial\bar{C}}{\partial\bar{x}} + \frac{\partial\bar{T}}{\partial\bar{y}}\frac{\partial\bar{C}}{\partial\bar{y}}) + (\frac{D_T}{\bar{T}_\infty})[(\frac{\partial\bar{T}}{\partial\bar{x}})^2 + (\frac{\partial\bar{T}}{\partial\bar{y}})^2]\} \quad (3)$$

Oxygen Conservation Equation:

$$\bar{u}\frac{\partial\bar{C}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{C}}{\partial\bar{y}} = D_B\left(\frac{\partial^2\bar{C}}{\partial\bar{x}^2} + \frac{\partial^2\bar{C}}{\partial\bar{y}^2}\right) + (\frac{D_T}{\bar{T}_\infty})(\frac{\partial^2\bar{T}}{\partial\bar{x}^2} + \frac{\partial^2\bar{T}}{\partial\bar{y}^2}) \quad (4)$$

Conservation Equation for microorganisms:

$$\frac{\partial}{\partial\bar{x}}[\bar{n}\bar{u} + \bar{n}\tilde{\bar{u}} - D_n(\frac{\partial\bar{n}}{\partial\bar{x}} + \frac{\partial\bar{n}}{\partial\bar{y}})] + \frac{\partial}{\partial\bar{y}}[\bar{n}\bar{v} + \bar{n}\tilde{\bar{v}} - D_n(\frac{\partial\bar{n}}{\partial\bar{x}} + \frac{\partial\bar{n}}{\partial\bar{y}})] = 0 \quad (5)$$

Subject to the boundary conditions

$\bar{v} = 0,\ \bar{T} = \bar{T}_w\ ,\ \bar{C} = \bar{C}_w\ ,\ \bar{n} = \bar{n}_w$ at $\bar{y} = 0$

$\bar{T} = \bar{T}_\infty\ ,\ \bar{C} = \bar{C}_\infty\ ,\ \bar{n} = \bar{n}_\infty$ as $\bar{y} = \infty$ (6)

Where the signs $\pm$ refer to the flow above a heated or below a cooled flat plate, respectively. We introduce now the following dimensionless quantities

$$X = \frac{\bar{x}}{l}\ , y = Ra^{\frac{1}{3}}\frac{\bar{y}}{l}\ , u = Ra^{\frac{-2}{3}}(\frac{l}{\alpha_m})\bar{u}\ , v = Ra^{\frac{-1}{3}}(\frac{l}{\alpha_m})\bar{v} \quad (7)$$

$$\theta = \frac{\bar{T}-\bar{T}_\infty}{Ax^\lambda}\ ,\ \varphi = \frac{\bar{C}-\bar{C}_\infty}{Bx^\lambda}\ ,\ \sigma = \frac{\bar{n}-\bar{n}_\infty}{Cx^\lambda}$$

Where A, B, C are constants whose values depend on the properties of fluid.

x is the distance ,λ is the exponent of temperature, volume, concentration and density and Ra is the Rayleigh number for a porous medium which is defined by

$$Ra = \frac{(1-\bar{C}_\infty)gK\rho_{fo}\beta Ax^\lambda l}{\mu\alpha_m} \quad (8)$$

Substituting (6) into equations (1)-(5) and assuming the boundary layer approximation ($Ra \to \infty$),we obtain the following boundary layer equations for the problem considered.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (9)$$

$$\frac{\partial u}{\partial y} = \mp\frac{\partial\theta}{\partial x} \mp \frac{\theta\lambda}{x} \pm Nr\frac{\partial\varphi}{\partial x} \pm \frac{\varphi\lambda}{x} \pm Rb\frac{\partial\sigma}{\partial x} \pm \frac{\sigma\lambda}{x} \quad (10)$$

$$u\frac{\partial\theta}{\partial x} + u\frac{\theta\lambda}{x} + v\frac{\partial\theta}{\partial y} = \frac{\partial^2\theta}{\partial y^2} + Nb\frac{\partial\theta}{\partial y}\frac{\partial\varphi}{\partial y} + Nt\,(\frac{\partial\theta}{\partial y})^2 \quad (11)$$

$$Le[u\frac{\partial\varphi}{\partial x} + u\frac{\varphi\lambda}{x} + v\frac{\partial\varphi}{\partial y}] = \frac{\partial^2\varphi}{\partial y^2} + \frac{Nt}{Nb}\frac{\partial^2\theta}{\partial y^2} \quad (12)$$

$$u\frac{\partial\sigma}{\partial x} + u\frac{\sigma\lambda}{x} + v\frac{\partial\sigma}{\partial x} + \frac{\rho_e}{Lb}\frac{\partial}{\partial x}[\sigma\frac{\partial\varphi}{\partial y}] = \frac{1}{Lb}\frac{\partial^2\sigma}{\partial y^2} \quad (13)$$

The boundary conditions (13) becomes

$V = 0,\ \ \theta = 1,\ \ \varphi = 1, \sigma = 1,$ at y= 0 (14)

$u \to 0,,\ \ \theta \to 0,\ \ \varphi \to 0, \sigma \to 0,$ as y$\to\infty$

re Lb is the bioconvection Lewis number Le is the traditional Lewis number,Pe is the bioconvection Péclet number He Nr is the buoyancy ratio parameter,Rb is the bioconvection Rayleigh number,Nb is the Brownian motion parameter and Nt is the thermophoresis parameter.These parameters are defined as follows.

$$Lb = \frac{\alpha_m}{D_n}\ , Le = \frac{\alpha_m}{D_B}\ , Pe = \frac{bW_c}{D_n}\ , Nr = \frac{(\rho_p - \rho_{f\infty})Bx^\lambda}{\rho_{f\infty}(1-\overline{C_\infty})\beta Ax^\lambda},\quad Rb = \frac{\gamma Cx^\lambda\Delta\rho}{\rho_{f\infty}\beta Ax^\lambda(1-\overline{C_\infty})}$$

$$Nb = \frac{\tau D_B Bx^\lambda}{\alpha_m}\ , Nt = \frac{\tau D_T Ax^\lambda}{\alpha_m\overline{T_\infty}} \quad (15)$$

Equation (9)-(13) with boundary conditions (14) admit a similarity solution of the form

$$\psi = x^{\frac{1}{3}}f(\eta), \theta = \theta(\eta), \varphi = \varphi(\eta), \sigma = \sigma(\eta), \eta = x^{\frac{-2}{3}}y \quad (16)$$

Where ψ is the stream function , which is defined in the usual way as $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$

Thus the velocity components u and v can be expressed as $u = x^{\frac{-1}{3}}f'(\eta),\ v = -\frac{x^{\frac{-2}{3}}}{3}\left(f - 2\eta f'(\eta)\right)$ (17)

Where prime represents differentiation with respect to η .

Substituting (16) into (10)-(13), we obtain the following ordinary differential equations

$$f'' - \frac{2}{3}\eta(\theta' - Nr\varphi' - Rb\sigma') + \lambda\theta - Rb\sigma\lambda = 0 \quad (18)$$
$$\theta'' + f'\theta\lambda + \frac{1}{3}f\theta' + Nb\theta'\varphi' + Nt\theta'^2 = 0 \quad (19)$$
$$\varphi'' + \frac{Nt}{Nb}\theta'' + \frac{Le}{3}f\varphi' + Le\varphi\lambda f' = 0 \quad (20)$$
$$\sigma'' + \frac{Lb}{3}f\sigma' - Pe(\sigma'\varphi' + \varphi''\sigma) + Lbf'\lambda\sigma = 0 \quad (21)$$

Subject to the boundary conditions

$$f(0) = 0, \theta(0) = \varphi(0) = \sigma(0) = 1$$
$$f'(\infty) = 0, \theta(\infty) = 0, \varphi(\infty) = 0, \sigma(\infty) = 0 \quad (22)$$

It is interesting to demonstrate that for $\lambda = 0$, we get isothermal case which is investigated by Aziz et al. [1] In this study, the local Nusselt number Nu_x,the Sherwood number Sh_x and the local density number of the motile microorganisms Nn_x are the quantities of our main concern and are defined as

$$Nu_x = \frac{\bar{x}\overline{q_w}}{kAx^{\lambda}}, Sh_x = \frac{\bar{x}\overline{q_m}}{D_B Bx^{\lambda}}, Nn_x = \frac{\bar{x}\overline{q_n}}{D_n Cx^{\lambda}} \quad (23)$$

Where $\overline{q_w}$, $\overline{q_m}$ and $\overline{q_n}$ are the wall heat, the wall mass and wall motile microorganisms fluxes, respectively and are defined as

$$\overline{q_w} = -k\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0}, \overline{q_m} = -D_B\left(\frac{\partial C}{\partial \bar{y}}\right)_{\bar{y}=0}, \overline{q_n} = -D_n\left(\frac{\partial \bar{n}}{\partial \bar{y}}\right)_{\bar{y}=0} \quad (24)$$

Using variables (7),(16),(23) and (24), we obtain

$$Ra_x^{\frac{-1}{2}}Nu_x = -\theta'(0), Ra_x^{\frac{-1}{2}}Sh_x = -\varphi'(0), Ra_x^{\frac{-1}{2}}Nn_x = -\sigma'(0) \quad (25)$$

Where, $Ra_x = \frac{(1-\overline{C_\infty})gK\rho_{f\infty}\beta Ax^{\lambda}\bar{x}}{\mu\alpha_m}$ is the local Rayleigh number.

III. Numerical Solution

We transformed the governing equations partial differential equations (10)-(14) into ordinary differential equations (15)-(22) respectively using similarity solutions. Then we solved it numerically using Maple 14.0. By **dsolve** command, the type of BVP or IVP problem is easily identified and appropriate algorithm is applied. The validity and accuracy of Maple's algorithm have been repeatedly justified in several recent research papers. We will see further confirmation when the present results for the special cases are compared with the published result investigated by Aziz at el. [1].We investigated our numerical solution for different values exponents of the temperature, the nanoparticle volume fraction and the density of motile microorganisms.

The asymptotic boundary conditions given by equation (22) were replaced by using a value of 10 for the similarity variable η_{max} as follows:

$$\eta_{max} = 10, \ f'(10) = 0, \theta(10) = 0, \varphi(10) = 0, \sigma(10) = 0 \quad (26)$$

IV. Results And Discussion

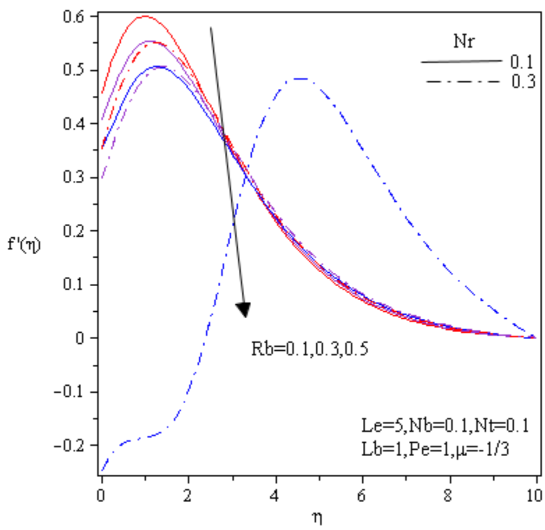


Fig:1

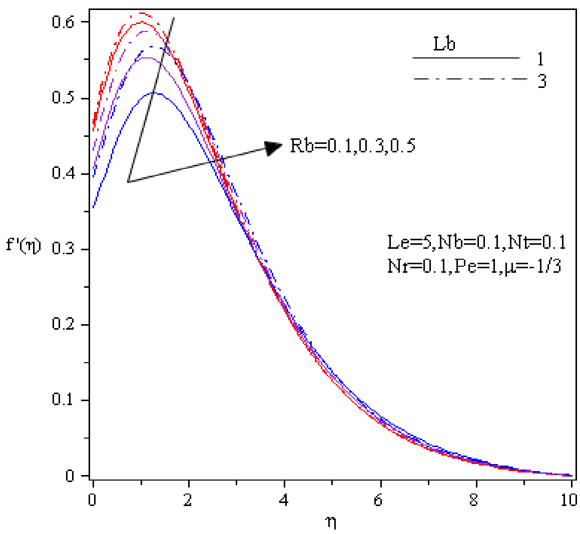
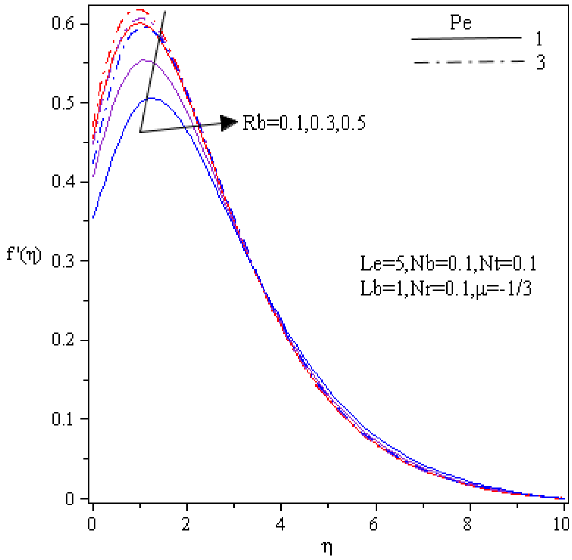


Fig-2



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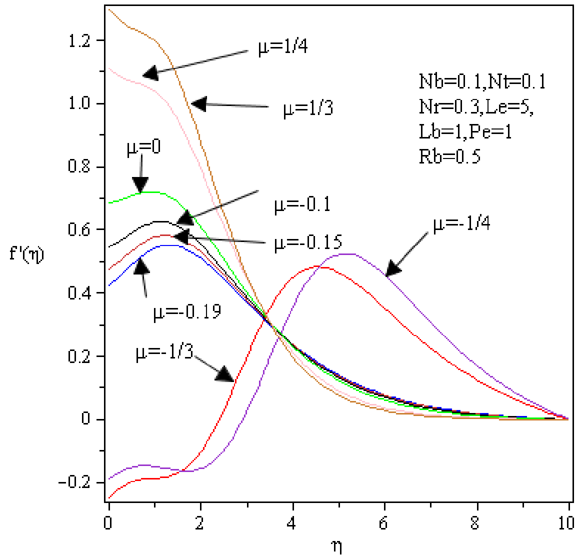

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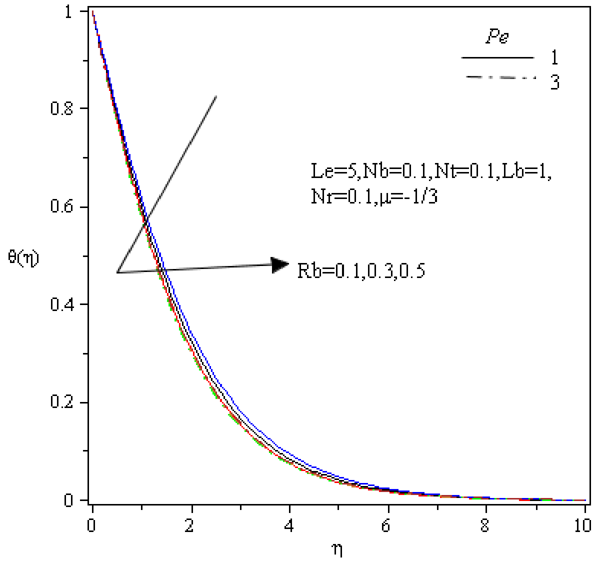

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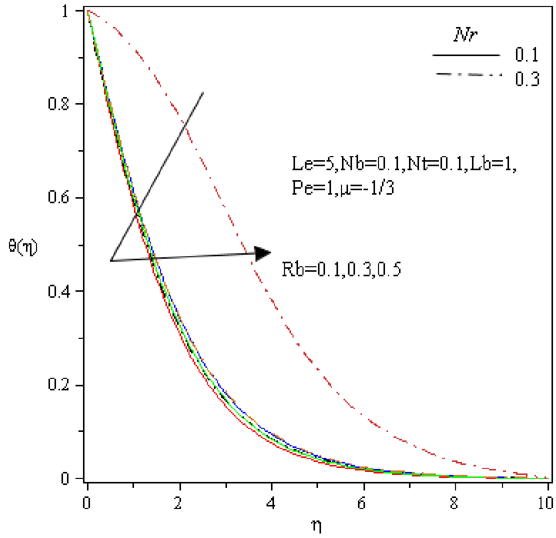

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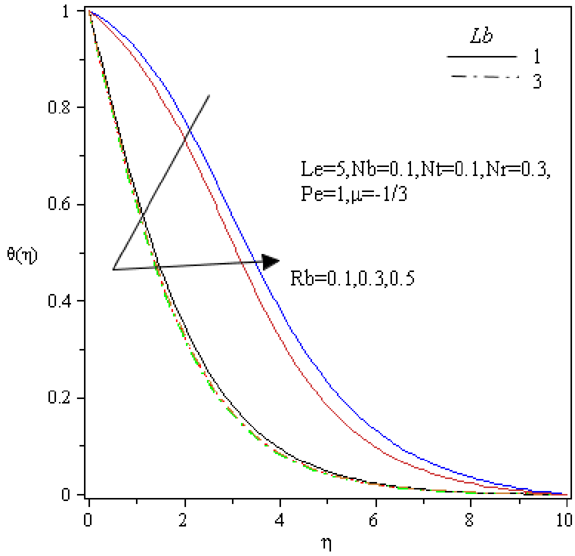

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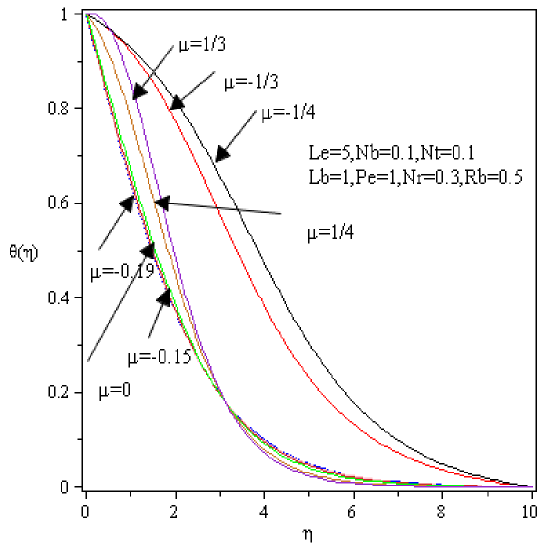

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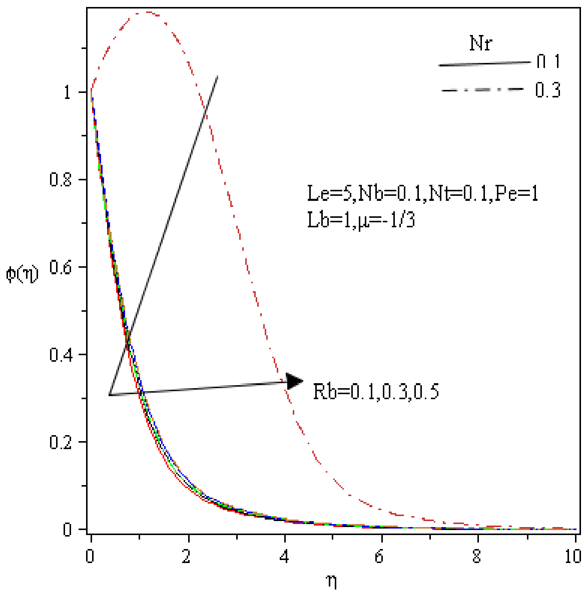
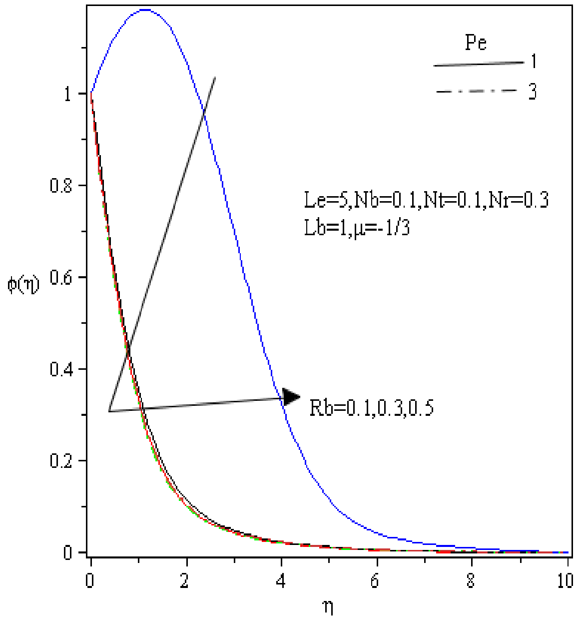


Fig:9



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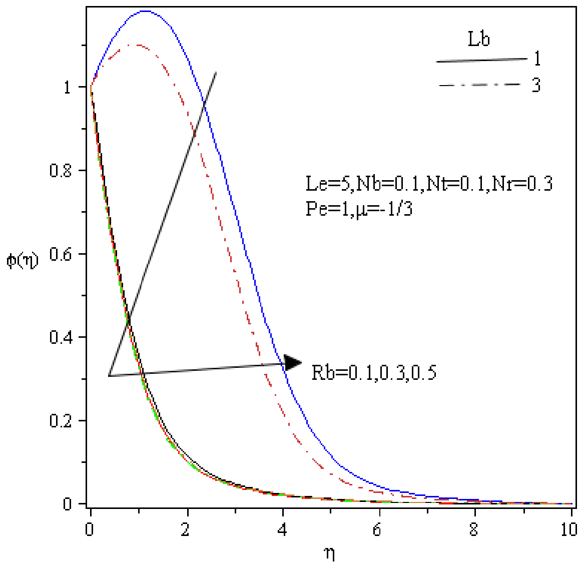

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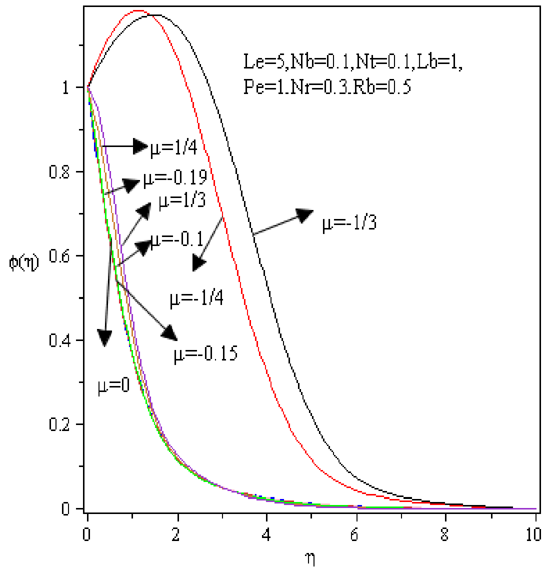


Fig:12

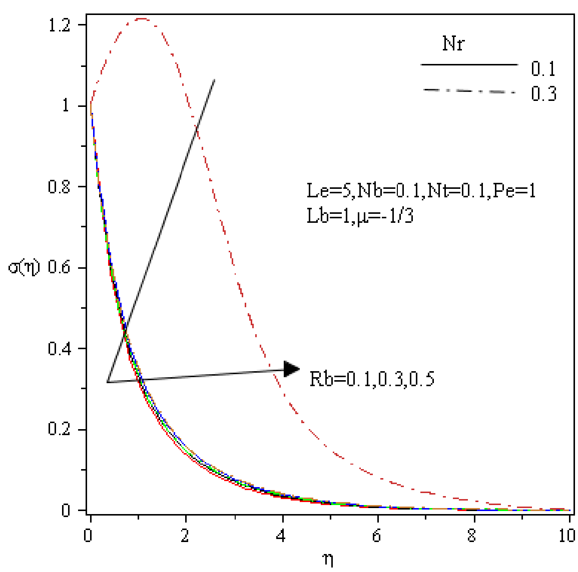
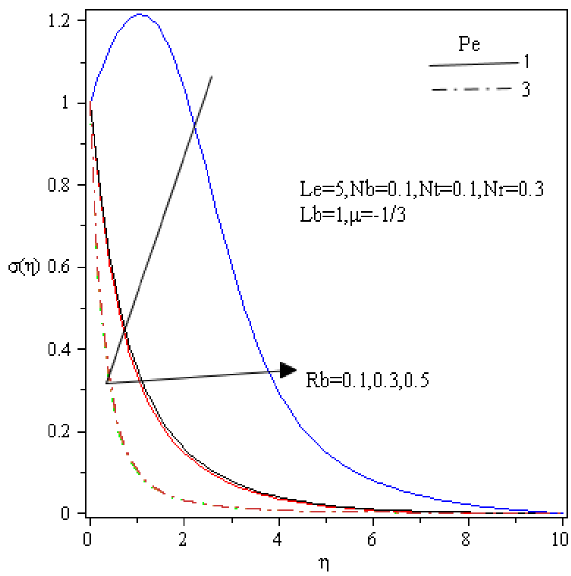


Fig:13



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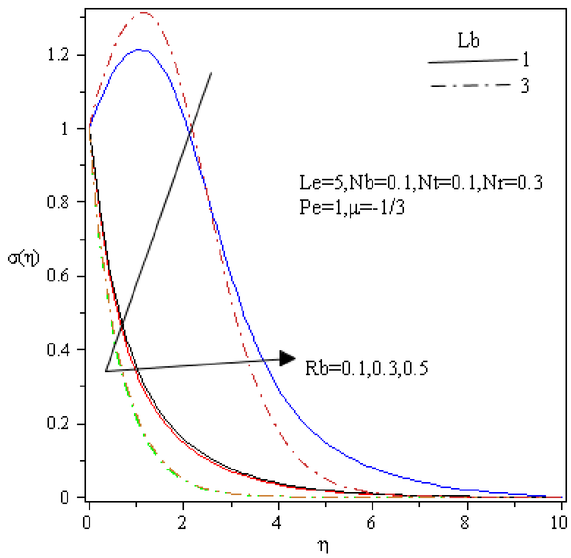

Fig:15

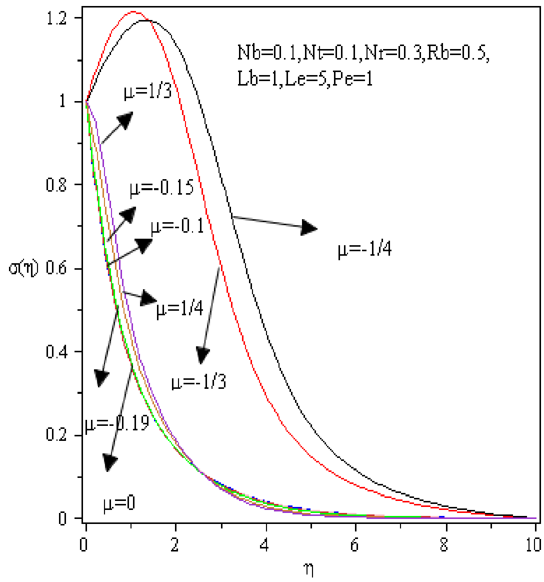

Fig:16

$$Nb = 0.1, Nt = 0.1, Nr = 0.3, Le = 0.5, Lb = 1, Pe = 1, Rb = 0.5$$

λ	$f'(0)$	$\theta(0)$	$-\theta'(0)$	$\varphi(0)$	$-\varphi'(0)$	$\sigma(0)$	$-\sigma'(0)$
$-\frac{1}{4}$	-0.18926	0.99999	0.05097	0.99999	-0.23548	0.99999	-0.26701
$-\frac{1}{3}$	-0.24849	0.99999	0.04489	0.99999	-0.32627	0.99999	-0.37745
-0.19	0.42393	0.99999	0.38848	0.99999	0.85612	0.99999	1.12273
-0.15	0.47774	1.00000	0.38141	1.00000	0.85025	1.00000	1.11365
-0.1	0.54596	1.00000	0.36854	1.00000	0.83359	1.00000	1.09021
0	0.68769	0.99999	0.32809	0.99999	0.76756	0.99999	1.00079
$\frac{1}{4}$	1.11086	0.99999	0.10064	0.99999	0.34074	0.99999	0.43437
$\frac{1}{3}$	1.29833	0.99999	-0.05141	0.99999	0.05151	0.99999	0.05154

Table-1: Effects of exponents on dimensionless variables

	$Ra_x^{-1/2}Nu_x$				$Ra_x^{-1/2}Sh_x$			
	Aziz et al.[1]	Present Results			Aziz et al.[1]	Present Results		
		$\lambda=-\frac{1}{3}$	$\lambda=0$	$\lambda=\frac{1}{3}$		$\lambda=-\frac{1}{3}$	$\lambda=0$	$\lambda=\frac{1}{3}$
Nr	$Nb=0.3, Nt=0.1$							
0.1	0.32586	0.43023	0.32586	-0.09197	1.48225	1.71261	1.48225	0.03067
0.2	0.32393	0.42474	0.32393	-0.09149	1.46686	1.67644	1.46686	0.03051
0.3	0.32195	0.41899	0.32195	-0.09100	1.45106	1.63846	1.45106	0.03034
0.4	0.31992	0.41296	0.31992	-0.09050	1.43482	1.59839	1.43482	0.03018

Table-2: Comparison of results of dimensionless local Nusselt and local Sherwood number for regular fluid when $Rb=0, Le=10$ and different values of exponents λ.

Fig:1-3 are drawn to show effect of bioconvection parameters on dimensionless velocity for a value of exponent λ.In fig:[2],[3] velocity decreases since Peclet numbers and Lewis numbers have the great influence at the surface with an increase in the bioconvection Rayleigh number. As a result velocity becomes zero for a certain value of Rb.Similarly in fig:1 for the buoyancy parameter velocity decreases at the surface but for a certain value of Rb,it increases. In fig:4 the behavior of exponent λ for different values is shown with the present of bioconvection parameters on dimensionless velocity. When λ=-1/3 and $\lambda=-1/4$,we see that there are two separation points in velocity profile.Velocity profile increases when η is close to 2 and decreases when η is close to 5.For $\lambda=-0.19,-0.15,-0.1$ and 0,we observe very subtle difference in the velocity profile. For these values velocity profile started to decrease when η is close to 1 which is the only separation point.For $\lambda=\frac{1}{3}$ and $\lambda=\frac{1}{4}$,velocity profile decreases with the increase of η.

Effects of bioconvection parameters on dimensionless temperature are shown in fig 5-7 where a specific value of λ is taken. Temperature profile indicates weak dependence on Rayleigh and Peclet numbers together but the effect of Nr and Lb is more detectable for higher values of Rb. Dimensionless temperature profile for different values of exponent λ is shown in fig 8. Here for all values of λ,we see that temperature profile is decreasing with the increase of η where there is no separation point.

Similarly the nanoparticle concentration profiles are shown in fig 9-11 where boundary layer thickness for the dimensionless density of motile microorganisms is relatively insensitive to change. In fig 12, we see profile of dimensionless nanoparticle concentration for different values of exponent λ. When λ=-1/3 and $\lambda=-1/4$,we see that concentration profile increases at first but when η is closed to 2,it started decreasing while for other values of λ,the profile decreases with the increase of η.

In fig 13, profile of dimensionless density of motile microorganisms has weak dependence on buoyancy and bioconvection parameter. Boundary layer thickness decreases for certain value of Pe which is shown in fig:14. In fig:15, Lb has comparatively strong influence on buoyancy and bioconvection parameter. The behavior of different values of exponent λ for profile of dimensionless density of motile microorganisms is illustrated in fig:16 which behaves the same way of nanoparticle concentration profile.

From Table 1, we can observe that for $\eta=0$ when λ increases, dimensionless velocity profile also increases, but profiles of dimensionless temperature, nanoparticle concentration and density of motile microorganisms remain always close to 1.Moreover, dimensionless local Nusselt number decreases for the increase of λ whereas dimensionless local Sherwood number and local density number decreses.

Table 2 compares the present result for three different values of exponent λ with the results of Aziz et al.[1] when bioconvection is not present.So for the isothermal case when $\lambda=0$,our result coincides with the result of Aziz et al.[1] which approves that the present numerical approach is correct.

V. Conclusion

In this paper we introduced the exponent of temperature, volume, nanoparticle concentration and density of motile microorganisms .It has remarkable impact on the velocity profile whereas on other profiles the different value has no appreciable effect.

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