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Strongly Soft g^{**}-Closed Sets In Soft $\check{C}ech$ Closure Space

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Abstract: *In this paper, we introduce strongly soft g^{**}-closed sets and strongly soft g^{**}-open sets in soft Čech closure spaces, which are defined over an initial universe with a fixed set of parameters and studied some of their basic properties. Also, we show that every strongly soft ∂-closed set is strongly soft g^{**}-closed.*
Keywords: *Soft set, Strongly soft g^{**}-closed set, Strongly soft g^{**}-open set.*

I. Introduction

Fuzzy sets [1], theory of rough sets [2], theory of vague sets [3], theory of intuitionistic fuzzy sets [4], and theory of interval mathematics [5,6] are the tools, which are dealing with uncertainties. But all these theories have their own difficulties, namely inadequacy of parameterization. In 1999, D. Molodtsov [6] introduced the notion of soft set to deals with inadequacy of parameterization. Later, he applied this theory to several directions [7,8].

Levine [9] introduced generalized closed sets in topological space in order to extend some important properties of closed sets to a large family of sets. For instance, it was shown that compactness, normality and completeness in a uniform space are inherited by g-closed subsets.

E. $\check{C}ech$ [10] introduced the concept of closure spaces. In $\check{C}ech's$ approach the operator satisfies idempotent condition among Kuratowski axioms. This condition need not hold for every set A of X. When this condition is also true, the operator becomes topological closure operator. Thus the concept of closure space is the generalization of a topological space. In 2010, Chawalit Boonpok [11] introduced generalized closed sets in $\check{C}ech$ closure spaces.

R. Gowri and G. Jegadeesan [12,13,14,15,16] introduced and studied the concept of lower separation axioms, higher separation axioms, soft generalized closed sets, soft ∂-closed sets, strongly soft g-closed sets and strongly soft ∂-closed sets in soft $\check{C}ech$ closure spaces.

In this paper, we introduce the strongly soft g^{**}-closed sets in soft $\check{C}ech$ closure spaces. Also, we investigate some of their basic properties.

II. Preliminaries

In this section, we recall the basic definitions of soft Čech closure spaces.

Definition 2.1 [12]. Let X be an initial universe set, A be a set of parameters. Then the function $k: P(X_{F_A}) \to P(X_{F_A})$ defined from a soft power set $P(X_{F_A})$ to itself over X is called Čech Closure operator if it satisfies the following axioms:

(C1) $k(\emptyset_A) = \emptyset_A$.
(C2) $U_A \subseteq k(U_A)$
(C3) $k(U_A \cup V_A) = k(U_A) \cup k(V_A)$

Then (X, k, A) or (F_A, k) is called a soft Čech closure space.

Definition 2.2 [12]. A soft subset U_A of a soft Čech closure space (F_A, k) is said to be soft k-closed (soft closed) if $k(U_A) = U_A$.

Definition 2.3 [12]. A soft subset U_A of a soft Čech closure space (F_A, k) is said to be soft k-open (soft open) if $k({U_A}^C) = {U_A}^C$.

Definition 2.4 [12]. A soft set $Int(U_A)$ with respect to the closure operator k is defined as $Int(U_A) = F_A - k(F_A - U_A) = [k({U_A}^C)]^C$. Here $U_A^C = F_A - U_A$.

Definition 2.5 [12]. A soft subset U_A in a soft Čech closure space (F_A , k) is called Soft neighbourhood of e_F if $e_F \in Int(U_A)$.

Definition 2.6 [12]. If (F_A, k) be a soft $\check{C}ech$ closure space, then the associate soft topology on F_A is $\tau = \{U_A{}^C : k(U_A) = U_A\}$.

Definition 2.7 [12]. Let (F_A, k) be a soft $\check{C}ech$ closure space. A soft $\check{C}ech$ closure space (G_A, k^*) is called a soft subspace of (F_A, k) if $G_A \subseteq \mathrm{F_A}$ and $k^*(U_A) = k(U_A) \cap G_A$, for each soft subset $U_A \subseteq \mathrm{G_A}$.

Definition 2.8 [16]. Let U_A be a soft subset of a soft $\check{C}ech$ closure space (F_A, k) is said to be

1. Soft semi-open set if $U_A \subseteq k[int(U_A)]$ and a soft semi-closed set if $int(k[U_A]) \subseteq U_A$.
2. Soft regular-open set if $int(k[U_A]) = U_A$ and a soft regular-closed set if $U_A = k[int(U_A)]$.
3. Soft pre-open set if $U_A \subseteq int(k[U_A])$ and a soft pre-closed set if $k[int(U_A)] \subseteq U_A$.
4. Soft α-open set if $U_A \subseteq int(k[int(U_A)])$ and soft α-closed set if $k[int(k[U_A])] \subseteq U_A$.
5. Soft semi pre-open (soft β-open) set if $U_A \subseteq k[int(k[U_A])]$ and soft semi pre-closed set if $int(k[int(U_A)]) \subseteq U_A$.

The smallest soft $\check{C}ech$ semi-closed set containing U_A is called soft $\check{C}ech$ semi-closure of U_A with respect to k and it is denoted by $k_s(U_A)$.
The largest soft $\check{C}ech$ semi-open set contained in U_A is called soft $\check{C}ech$ semi-interior of U_A with respect to k and it is denoted by $int_s(U_A)$.
The smallest soft $\check{C}ech$ pre-closed set containing U_A is called soft $\check{C}ech$ pre-closure of U_A with respect to k and it is denoted by $k_p(U_A)$.
The largest soft $\check{C}ech$ pre-open set contained in U_A is called soft $\check{C}ech$ pre-interior of U_A with respect to k and it is denoted by $int_p(U_A)$.
The smallest soft $\check{C}ech$ α-closed set containing U_A is called soft $\check{C}ech$ α-closure of U_A with respect to k and it is denoted by $k_\alpha(U_A)$.
The largest soft $\check{C}ech$ α-open set contained in U_A is called soft $\check{C}ech$ α-interior of U_A with respect to k and it is denoted by $int_\alpha(U_A)$.
The smallest soft $\check{C}ech$ semi pre-closed set containing U_A is called soft $\check{C}ech$ semi pre-closure of U_A with respect to k and it is denoted by $k_{sp}(U_A)$.
The largest soft $\check{C}ech$ semi pre-open set contained in U_A is called soft $\check{C}ech$ semi pre-interior of U_A with respect to k and it is denoted by $int_{sp}(U_A)$.

Definition 2.9 [16]. A soft subset U_A of a soft $\check{C}ech$ closure space (F_A, k) is said to be soft semi-generalized closed set (briefly soft sg-closed) if $k_s(U_A) \subseteq G_A$, whenever $U_A \subseteq G_A$, G_A is soft semi open in F_A.

Definition 2.10 [16]. A soft subset U_A of a soft $\check{C}ech$ closure space (F_A, k) is said to be soft generalized semi-closed set (briefly soft gs-closed) if $k_s(U_A) \subseteq G_A$, whenever $U_A \subseteq G_A$, G_A is soft open in F_A.

Definition 2.11 [16]. A soft subset U_A of a soft $\check{C}ech$ closure space (F_A, k) is said to be soft generalized semi pre-closed set (briefly soft gsp-closed) if $k_{sp}(U_A) \subseteq G_A$, whenever $U_A \subseteq G_A$, G_A is soft open in F_A.

Definition 2.12 [16]. A soft subset U_A of a soft $\check{C}ech$ closure space (F_A, k) is said to be soft generalized pre-closed set (briefly soft gp-closed) if $k_p(U_A) \subseteq G_A$, whenever $U_A \subseteq G_A$, G_A is soft open in F_A.

Definition 2.13 [16]. Let (F_A, k) be a soft $\check{C}ech$ closure space. A soft subset $U_A \subseteq F_A$ is called soft generalized pre-regular closed (briefly soft gpr-closed) set if $k_p(U_A) \subseteq G_A$, whenever $U_A \subseteq G_A$ and G_A is soft regular open subset of (F_A, k).

Definition 2.14 [14]. A soft subset U_A of a soft $\check{C}ech$ closure space (F_A, k) is said to be soft generalized closed (briefly soft g-closed) set if $k[U_A] \subseteq G_A$, whenever $U_A \subseteq G_A$ and G_A is soft open subset of (F_A, k).

Definition 2.15 [15]. A soft subset U_A of a soft $\check{C}ech$ closure space (F_A, k) is said to be soft ∂-closed set if $k[U_A] \subseteq G_A$, whenever $U_A \subseteq G_A$ and G_A is soft g-open subset of (F_A, k).

Definition 2.16 [16]. Let (F_A, k) be a soft $\check{C}ech$ closure space. A soft subset $U_A \subseteq F_A$ is called a strongly soft generalized closed (briefly strongly soft g-closed) set if $k[int(U_A)] \subseteq G_A$, whenever $U_A \subseteq G_A$ and G_A is soft open subset of (F_A, k).

Definition 2.17 [16]. Let (F_A, k) be a soft $\check{C}ech$ closure space. A soft subset $U_A \subseteq F_A$ is called a strongly soft ∂-closed set if $k[int(U_A)] \subseteq G_A$, whenever $U_A \subseteq G_A$ and G_A is soft g-open subset of (F_A, k).

Definition 2.18 [16]. Let (F_A, k) be a soft $\check{C}ech$ closure space. A soft subset $U_A \subseteq F_A$ is called soft regular generalized closed (briefly soft rg-closed) set if $k[U_A] \subseteq G_A$, whenever $U_A \subseteq G_A$ and G_A is soft regular open subset of (F_A, k).

III. Strongly Soft g^{**}-Closed Sets

Definition 3.1. Let (F_A, k) be a soft Čech closure space. A soft subset $U_A \subseteq F_A$ is called a strongly soft g^{**}-closed set if $k[int(U_A)] \subseteq G_A$, whenever $U_A \subseteq G_A$ and G_A is soft ∂-open subset of F_A .

Example 3.2. Let the initial universe set $X = \{u_1, u_2\}$ and $E = \{x_1, x_2, x_3\}$ be the parameters.
Let $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$.Then $P(X_{F_A})$ are
$F_{1A} = \{(x_1, \{u_1\})\}, F_{2A} = \{(x_1, \{u_2\})\}, F_{3A} = \{(x_1, \{u_1, u_2\})\}, F_{4A} = \{(x_2, \{u_1\})\}, F_{5A} = \{(x_2, \{u_2\})\}$,
$F_{6A} = \{(x_2, \{u_1, u_2\})\}, F_{7A} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}, F_{8A} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$,
$F_{9A} = \{(x_1, \{u_2\}), (x_2, \{u_1\})\}, F_{10A} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}, F_{11A} = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}$,
$F_{12A} = \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\}, F_{13A} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\}$,
$F_{14A} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}, F_{15A} = F_A, F_{16A} = \emptyset_A$.

An operator $k: P(X_{F_A}) \to P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
$k(F_{1A}) = k(F_{5A}) = F_{8A}, k(F_{2A}) = F_{3A}, k(F_{3A}) = k(F_{9A}) = k(F_{13A}) = F_{13A}, k(F_{4A}) = F_{4A}, k(F_{7A}) = F_{7A}$,
$k(F_{6A}) = k(F_{8A}) = k(F_{11A}) = F_{11A}, k(F_{10A}) = F_{14A}, k(F_{12A}) = k(F_{14A}) = k(F_A) = F_A, k(\emptyset_A) = \emptyset_A$.
Here, strongly soft g^{**}- closed sets are, $\emptyset_A, F_{1A}, F_{3A}, F_{4A}, F_{6A}, F_{7A}, F_{8A}, F_{9A}, F_{10A}, F_{11A}, F_{12A}, F_{13A}, F_A$.

Theorem 3.3. In a soft Čech closure space (F_A, k), every soft closed set is strongly soft g^{**}-closed.

Proof. The proof is obvious from the definition (2.2) of soft closed set.

Result 3.4. The converse of the above theorem (3.3) is not true as shown in the following example.

Example 3.5. In example 3.2, here $F_{3A} = \{(x_1, \{u_1, u_2\})\}$ is strongly soft g^{**}-closed but not soft closed.

Theorem 3.6. In a soft Čech closure space (F_A, k), every strongly soft ∂-closed set is strongly soft g^{**}-closed but not converse.

Proof. The proof follows from the definition (2.17) of strongly soft ∂-closed set.

Example 3.7. Let us consider the soft subsets of F_A that are given in example 3.2.
An operator $k: P(X_{F_A}) \to P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
$k(F_{1A}) = k(F_{7A}) = k(F_{8A}) = k(F_{11A}) = F_{11A}, k(F_{4A}) = k(F_{5A}) = k(F_{6A}) = F_{6A}, k(F_{2A}) = F_{10A}$,
$k(F_{9A}) = k(F_{10A}) = k(F_{12A}) = F_{12A}, k(F_{3A}) = k(F_{13A}) = k(F_{14A}) = k(F_A) = F_A, k(\emptyset_A) = \emptyset_A$.
Here $F_{7A} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}$ is strongly soft g^{**}-closed set but not strongly soft ∂-closed.

Theorem 3.8. In a soft Čech closure space (F_A, k), every soft ∂-closed set is strongly soft g^{**}-closed but not converse.

Proof. The proof is obvious.

Example 3.9. Let us consider the soft subsets of F_A that are given in example 3.2. An operator $k: P(X_{F_A}) \to P(X_{F_A})$is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
$k(F_{1A}) = F_{1A}, k(F_{2A}) = k(F_{9A}) = F_{12A}, k(F_{4A}) = F_{4A}, k(F_{5A}) = k(F_{8A}) = F_{14A}, k(F_{7A}) = F_{7A}$,
$k(F_{3A}) = k(F_{6A}) = k(F_{10A}) = k(F_{11A}) = k(F_{12A}) = k(F_{13A}) = k(F_{14A}) = k(F_A) = F_A, k(\emptyset_A) = \emptyset_A$.
Here $F_{3A} = \{(x_1, \{u_1, u_2\})\}$ is strongly soft g^{**}-closed set but not soft ∂-closed.

Theorem 3.10. In a soft Čech closure space (F_A, k), every soft g-closed subset of F_A is strongly soft g^{**}-closed.

Proof. The proof is obvious.

Result 3.11. The following example shows that the above theorem (3.10) is not true.

Example 3.12. In example 3.2, here $F_{3A} = \{(x_1, \{u_1, u_2\})\}$ is strongly soft g^{**}-closed set but not soft g-closed.

Result 3.13. Let U_A and V_A are two non-empty strongly soft g^{**}-closed subsets of a soft Čech closure space (F_A, k), then the following example shows that, $U_A \cap V_A$ and $U_A \cup V_A$ need not be strongly soft g^{**}-closed.

Example 3.14. In example 3.7, take, $U_A = F_{7A}$ and $V_A = F_{8A}$.

Then, $U_A \cap V_A = F_{7A} \cap F_{8A} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\} \cap \{(x_1, \{u_1\})\}, \{(x_2, \{u_2\})\}$.

$= \{(x_1, \{u_1\})\}$

$= F_{1A}$, which is not a strongly soft g^{**}-closed subset in F_A .

Example 3.15. In example 3.9, take , $U_A = F_{2A}$ and $V_A = F_{5A}$.

Then, $U_A \cup V_A = F_{2A} \cup F_{5A} = \{(x_1, \{u_2\})\} \cup \{(x_2, \{u_2\})\}$.

$= \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$

$= F_{10A}$, which is not a strongly soft g^{**}-closed subset in F_A .

Theorem 3.16. Let (F_A, k) be a soft Čech closure space and let $U_A \subseteq F_A$. If U_A is strongly soft g^{**}-closed, then $k[int(U_A)] - U_A$ has no non-empty soft closed subset.

Proof. Suppose that, U_A is strongly soft g^{**}-closed. Let V_A be a soft closed subset of $k[int(U_A)] - U_A$. Then, $V_A \subseteq k[int(U_A)] \cap (F_A - U_A)$ and so $U_A \subseteq (F_A - V_A)$. Consequently, $V_A \subseteq F_A - k[int(U_A)]$. Since, $V_A \subseteq k[int(U_A)]$. Then, $V_A \subseteq k[int(U_A)] \cap (F_A - k[int(U_A)]) = \emptyset_A$. Thus, $V_A = \emptyset_A$. Therefore, $k[int(U_A)] - U_A$ contains no non-empty soft closed set.

Result 3.17. The converse of the above theorem (3.16) is not true as shown in the following example.

Example 3.18. In example 3.7, take $U_A = F_{2A}$.

$k[int(U_A)] - U_A = \{(x_1, \{u_2\}), (x_2, \{u_2\})\} - \{(x_1, \{u_2\})\}$

$=\{(x_2, \{u_2\})\}$, which does not contain non-empty soft closed subset of F_A .

But, $F_{2A} = \{(x_1, \{u_2\})\}$ is not strongly soft g^{**}-closed.

Theorem 3.19. Let $U_A \subseteq H_A \subseteq F_A$ and if U_A is strongly soft g^{**}-closed in F_A, then U_A is strongly soft g^{**}-closed relative to H_A.

Proof. Let $U_A \subseteq H_A \subseteq F_A$ and suppose that U_A is strongly soft g^{**}-closed in F_A . Let $U_A \subseteq H_A \cap G_A$, where G_A is soft ∂-open in F_A. Since, U_A is strongly soft g^{**}-closed in F_A , $U_A \subseteq G_A$ implies $k[int(U_A)] \subseteq G_A$. That is $H_A \cap k[int(U_A)] \subseteq H_A \cap G_A$, where $H_A \cap k[int(U_A)]$ is closure of interior of U_A with respect to k in H_A .Thus, U_A is strongly soft g^{**}-closed relative to H_A.

Theorem 3.20. Let (F_A, k) be a soft Čech closure space. If U_A be a soft subset of F_A is both soft open and strongly soft g^{**}-closed, then U_A is soft closed.

Proof. Suppose U_A is both soft open and strongly soft g^{**}-closed. Since, U_A is strongly soft g^{**}-closed. Then $k[int(U_A)] \subseteq V_A$. That is, $[U_A] = k[int(U_A)] \subseteq U_A$. Since, $U_A \subseteq k[U_A]$. Hence, U_A is soft closed.

Corollary 3.21. If U_A is both soft open and strongly soft g^{**}-closed in F_A, then U_A is both soft regular open and soft regular closed in F_A .

Proof. Since, U_A is soft open, then $U_A = int(U_A)$. Since, U_A is soft closed, then $U_A = int(U_A) = int(k[U_A])$ Hence, U_A is soft regular open. Also, $[int(U_A)] = k[U_A]$. Since, $U_A$is soft closed, then $k[int(U_A)] = U_A$. Hence, U_A is soft regular closed.

Corollary 3.22. If U_A is both soft open and strongly soft g^{**}-closed, then U_A is soft rg-closed.

Theorem 3.23. Let U_A be a soft subset of soft Čech closure space (F_A, k) is both strongly soft g^{**}-closed and soft semi open, then U_A is soft g^{**}-closed.

Proof. Suppose that, U_A is both strongly soft g^{**}-closed and soft semi open. Then, $k[int(U_A)] \subseteq G_A$, whenever $U_A \subseteq G_A$ and G_A is soft ∂-open subset of F_A . Since, U_A is soft semi open, then $U_A \subseteq k[int(U_A)]$. Then, $k[U_A] \subseteq k[int(U_A)] \subseteq G_A$. Thus, U_A is soft g^{**}-closed subset in F_A .

Corollary 3.24. If U_A is soft subset of a soft Čech closure space (F_A, k) is both strongly soft g^{**}-closed and soft open then U_A is soft g^{**}-closed subset in F_A .

Proof. Since, every soft open set is soft semi open. Then by the above theorem 3.23, result follows.

Definition 3.25. A soft subset U_A of a soft Čech closure space (F_A, k) is said to be soft g^{**}-closed set if $k[U_A] \subseteq G_A$, whenever $U_A \subseteq G_A$ and G_A is soft ∂-open in F_A .

Theorem 3.26. In a soft Čech closure space (F_A, k), every soft g^{**}-closed set is strongly soft g^{**}-closed , but not converse.

Proof. The proof is obvious from the definition 3.25 of soft g^{**}-closed set .

Example 3.27. In example 3.9, $F_{2A} = \{(x_1, \{u_2\})\}$ is strongly soft g^{**}-closed set but not soft g^{**}-closed.

Theorem 3.28. In a soft Čech closure space (F_A, k), every soft g^{**}-closed set is strongly soft g-closed.

Proof . The proof is obvious.

Result 3.29. The following example shows that the converse of the above theorem (3.28) is not true.

Example 3.30. In example 3.9, here $F_{2A} = \{(x_1, \{u_2\})\}$ is strongly soft g-closed set but not soft g^{**}-closed.

Result 3.31. The following examples shows that the strongly soft ∂-closed sets and soft g^{**}-closed sets in a soft Čech closure space (F_A, k) are independent to each other.

Example 3.32. In example 3.9, here $F_{2A} = \{(x_1, \{u_2\})\}$ is strongly soft ∂-closed set but not soft g^{**}-closed.

Example 3.33. In example 3.7, here $F_{7A} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}$ is soft g^{**}-closed set but not strongly soft ∂-closed.

IV. Strongly Soft g^{**}-Open Sets

Definition 4.1. Let (F_A, k) be a soft Čech closure space. A soft subset $V_A \subseteq F_A$ is called a strongly soft g^{**}-open set if its complement ${V_A}^C$ is strongly soft g^{**}-closed in F_A .

Example 4.2. In example 3.9, here $\emptyset_A, F_{2A}, F_{5A}, F_{8A}, F_{9A}, F_{10A}, F_{12A}, F_{14A}, F_A$ are strongly soft g^{**}-open sets in F_A .

Theorem 4.3. In a soft Čech closure space (F_A, k), every soft open set is strongly soft g^{**}-open.

Proof. Let V_A be a soft open set in F_A . Let $H_A \subseteq V_A$ and H_A is soft ∂-closed in F_A . Since, V_A is soft open, $H_A \subseteq V_A = int(V_A) \subseteq int\ (k[V_A])$. Therefore, V_A is strongly soft g^{**}-open.

Result 4.4. The converse of the above theorem (4.3) is not true as shown in the following example.

Example 4.5. In example 3.2, here $F_{3A} = \{(x_1, \{u_1, u_2\})\}$ is strongly soft g^{**}-open subset of F_A but not soft open.

Theorem 4.6. In a soft Čech closure space (F_A, k), every soft g^{**}-open set is strongly soft g^{**}-open.

Proof. Let V_A be a soft g^{**}-open set in F_A . Let $H_A \subseteq V_A$ and H_A is soft ∂-closed in F_A . Since, V_A is soft g^{**}-open, $H_A \subseteq V_A = int(V_A) \subseteq int(k[V_A])$. Therefore, V_A is strongly soft g^{**}-open.

Result 4.7. The converse of the above theorem (4.6) is not true as shown in the following example.

Example 4.8. In example 3.2, here, $F_{12A} = \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\}$ is strongly soft g^{**}-open in F_A but not soft g^{**}-open.

Result 4.9. Let U_A and V_A are two non-empty strongly soft g^{**}-open subsets of a soft Čech closure space (F_A, k), then the following example shows that, $U_A \cap V_A$ and $U_A \cup V_A$ need not be strongly soft g^{**}-open.

Example 4.10. In example 3.9, take, $U_A = F_{3A}$ and $V_A = F_{11A}$.
Then, $U_A \cap V_A = F_{3A} \cap F_{11A} = \{(x_1, \{u_1, u_2\})\} \cap \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}$.
$= \{(x_1, \{u_1\})\}$
$= F_{1A}$, which is not a strongly soft g^{**}-open subset in F_A .
Example 4.11. In example 3.7, take, $U_A = F_{5A}$ and $V_A = F_{9A}$.
Then, $U_A \cup V_A = F_{5A} \cup F_{9A}$
$= \{(x_2, \{u_2\})\} \cup \{(x_1, \{u_2\}), (x_2, \{u_1\})\}$.
$= \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\}$
$= F_{12A}$, which is not a strongly soft g^{**}-open subset in F_A .

Theorem 4.12. Let (F_A, k) be a soft Čech closure space. If V_A be a soft subset of F_A is both soft closed and strongly soft g^{**}-open, then V_A is soft open.

Proof. Suppose, V_A is both soft closed and strongly soft g^{**}-open. Since, V_A is strongly soft g^{**}-open. Then, $V_A \subseteq int(k[V_A])$. Since, V_A is soft closed, $V_A \subseteq int(k[V_A]) = int(V_A)$. Therefore, V_A is soft open.

Corollary 4.13. If V_A is both soft closed and strongly soft g^{**}-open in F_A, then V_A is both soft regular open and soft regular closed in F_A.

Proof. Since, V_A is soft closed and strongly soft g^{**}-open in F_A. By theorem 4.12, V_A is soft open. Since, V_A is soft closed. Then, $int(k[V_A]) = int(V_A) = V_A$. Therefore, V_A is soft regular open. Since, V_A is soft closed. Then, $V_A = k[V_A]$. Also, V_A is soft open. Then, $V_A = int(V_A)$. Then, $V_A = k[V_A] = k[int(V_A)]$. Thus, V_A is soft regular closed.

Corollary 4.14. If V_A is both soft closed and strongly soft g^{**}-open in F_A, then V_A is soft rg-open.

Theorem 4.15. Let (F_A, k) be a soft Čech closure space and let $V_A \subseteq F_A$. If V_A is strongly soft g^{**}-open, then $V_A - int(k[V_A])$ contains no non-empty soft closed set.

Proof. Suppose that U_A is a soft closed subset of $V_A - int(k[V_A])$. Then, $U_A \subseteq V_A \cap [int(k[V_A])]^C$.Therefore, $U_A \subseteq V_A$ and U_A is soft closed. Since, V_A is strongly soft g^{**}-open in F_A. This implies, $U_A \subseteq int(k[V_A])$. Also, $U_A \subseteq [int(k[V_A])]^C$. Hence, $U_A \subseteq int(k[V_A]) \cap [int(k[V_A])]^C = \emptyset_A$. Therefore, $V_A - int(k[V_A])$ contains no non-empty soft closed set.

Result 4.16. The converse of the above theorem (4.15) is not true as shown in the following example.

Example 4.17. In example 3.2, take, $V_A = F_{7A}$.
Then,$V_A - int(k[V_A]) = F_{7A} - int(k[F_{7A}]) = \{(x_1, \{u_1\}), (x_2, \{u_1\})\} - \{(x_2, \{u_1\})\} = \{(x_1, \{u_1\})\} = F_{1A}$, which does not contains non-empty soft closed sets. But, $V_A = F_{7A}$ is not a strongly soft g^{**}-open in F_A.

Corollary 4.18. Let (F_A, k) be a soft Čech closure space and let $V_A \subseteq F_A$. If a strongly soft g^{**}-open set V_A is soft regular open if and only if $V_A - int(k[V_A])$ is soft closed and $V_A \subseteq int(k[V_A])$.

Proof. Since, V_A is soft regular open. Then, $V_A = int(k[V_A])$. Thus, $V_A - int(k[V_A]) = \emptyset_A$ is soft closed. Conversely, suppose that $V_A - int(k[V_A])$ is soft closed. Since, V_A is strongly soft g^{**}-open. Then, by theorem 4.15, $V_A - int(k[V_A])$ contains no non-empty soft closed set. Since, $V_A - int(k[V_A])$ is itself soft closed, $V_A = int(k[V_A])$. Hence, V_A is soft regular open.

Theorem 4.19. In a soft $\check{C}ech$ closure space (F_A, k), V_A is strongly soft g^{**}-open if and only if $H_A \subseteq int(k[V_A])$, whenever $H_A \subseteq V_A$ and H_A is soft ∂-closed in F_A.

Proof. Let $H_A \subseteq V_A$ and H_A is soft ∂-closed in F_A. Then, ${V_A}^C \subseteq {H_A}^C$ and ${H_A}^C$ is soft ∂-open in F_A. Since, ${V_A}^C$ is strongly soft g^{**}-closed. Then, $k[int({V_A}^C)] \subseteq {H_A}^C$. Consequently, $H_A \subseteq (k[int({V_A}^C)])^C = int[int({V_A}^C)]^C \subseteq int(k[V_A])$. Conversely, let ${V_A}^C \subseteq U_A$ and U_A is soft ∂-open in F_A. Then, ${U_A}^C \subseteq V_A$ and ${U_A}^C$ is soft ∂-closed in F_A. By our assumption, ${U_A}^C \subseteq int(k[V_A])$. This implies, $k[int({V_A}^C)] \subseteq U_A$. Then, ${V_A}^C$ is strongly soft g^{**}-closed in F_A. Hence, V_A is strongly soft g^{**}-open in F_A.

V. Conclusion

In the present work, we have introduced strongly soft g^{**}-closed sets and strongly soft g^{**}-open sets in soft $\check{C}ech$ closure spaces, which are defined over an initial universe with a fixed set of parameters. We studied the behavior relative to union, intersection of strongly soft g^{**}-closed sets and strongly soft g^{**}-open sets. Also, we proved that every strongly soft ∂-closed set is strongly soft g^{**}-closed. In future, findings of this paper will contribute to find a new types of soft generalized closed sets and separation axioms in soft $\check{C}ech$ closure spaces.

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