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On Decomposition of Nano Continuity

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Abstract: *The aim of this paper is to obtain the decomposition of nano continuity in nano topological spaces.*
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I. Introduction and Preliminaries

Continuity and its decomposition have been intensively studied in the field of topology and other several branches of mathematics. Levine.N in [10] introduced the notion and decomposition of continuity in topological spaces. Jingcheng Tong in [6] introduced the notion of A-sets and A-continuity and established a decomposition of continuity. Further, Jingcheng Tong in [5] introduced the notion of B-sets and B-continuity and established a decomposition of continuity. Ganster.M and Reilly.I.L in [2] improved Tong's decomposition result. Jingcheng Tong in [4] generalized Levine's [10] decomposition theorem by introducing the notions of expansion of open sets in topological spaces. In recent years various classes of near to continuous functions were defined by various authors. Recently, Maria Luisa Colasante in [13] studied almost continuity and the notions of expansion of open sets in topological spaces. Lellis Thivagar.M and Carmel Richard in [8] introduced the notion of Nano topology which was defined in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it. Lellis Thivagar.M and Carmel Richard in [9] studied a new class of functions called nano continuous functions and their characterizations in nano topological spaces. In this paper, we study the notions of expansion of nano-open sets and obtain decomposition of nano continuity in nano topological spaces. In this connection, we refer [1], [3], [7], [11], [12], [14], [15] and [16].

Definition 1.1[8] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.
2. That is, $L_R(X) = \cup \{R(x): R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by $x \in U$.
3. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \cup \{R(x): R(x) \cap X \neq \varphi\}$.
4. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Property 1.2[8] If (U, R) is an approximation space and $X, Y \subseteq U$, then
(1) $L_R(X) \subseteq X \subseteq U_R(X)$.
(2) $L_R(\varphi) = U_R(\varphi) = \varphi$ and $L_R(U) = U_R(U) = U$.
(3) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$.
(4) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$.
(5) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.
(6) $L_R(X \cap Y) = U_R(X) \cap U_R(Y)$.
(7) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$.
(8) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$.
(9) $U_R U_R(X) = L_R U_R(X) = U_R(X)$.
(10) $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 1.3[8] Let U be the universe, R be an equivalence relation on U and
$\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 1.2, $\tau_R(X)$ satisfies the following axioms:
(1) U and $\phi \in \tau_R(X)$.
(2) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
(3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano-open sets. If $(U, \tau_R(X))$ is a nano topological space[8] where $X \subseteq U$ and if $A \subseteq U$, then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by $NInt(A)$. $NInt(A)$ is the largest nano-open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by $NCl(A)$. That is, $NCl(A)$ is the smallest nano closed set containing A.

Definition 1.4[9] Let $(U, \tau_R(X))$ and $(V, \tau_R^{'}(Y))$ be two nano topological spaces. Then a mapping $f: (U, \tau_R(X)) \to (V, \tau_R^{'}(Y))$ is nano continuous on U if the inverse image of every nano-open set in V is nano-open in U.

II. Expansion of nano-open sets.

Definition 2.1. Let $(U, \tau_R(X))$ be a nano topological space, 2^U be the set of all subsets of U. A mapping $\mathcal{A}: \tau_R(X) \to 2^U$ is said to be an expansion on $(U, \tau_R(X))$ if $D \subseteq \mathcal{A}D$ for each $D \in \tau_R(X)$.

Remark 2.2. Let us study the expansion of nano-open sets in nano topological spaces. Let $(U, \tau_R(X))$ be an nano topological space,

1. Define $\mathcal{CL}: \tau_R(X) \to 2^U$ by $\mathcal{CL}(D) = NCl(D)$ for each $D \in \tau_R(X)$.Then $\mathcal{CL}$ is an expansion on $(U, \tau_R(X))$, because $D \subseteq \mathcal{CL}(D)$ for each $D \in \tau_R(X)$.
2. Since for each $D \in \tau_R(X)$, D is nano-open and hence $NInt(D) = D$, $\mathcal{F}(D)$ can be defined as $\mathcal{F}: \tau_R(X) \to 2^U$ by $\mathcal{F}(D) = (NCl(D) - D)^c$. Then $\mathcal{F}$ is an expansion on $(U, \tau_R(X))$.
   Here, $\mathcal{F}(D) = (NCl(D) - D)^c = (NCl(D) \cap D^c)^c = (NCl(D))^c \cup D \supseteq D$ for each $D \in \tau_R(X)$.
3. Define $\mathcal{N}Int\mathcal{CL}: \tau_R(X) \to 2^U$ by $\mathcal{N}Int\mathcal{CL}(D) = \mathcal{N}IntNCl(D)$ for each $D \in \tau_R(X)$. Then $\mathcal{N}Int\mathcal{CL}$ is an expansion on $(U, \tau_R(X))$, because $D \subseteq \mathcal{N}Int\mathcal{CL}(D)$ for each $D \in \tau_R(X)$.
4. Define $\mathcal{F}_s: \tau_R(X) \to 2^U$ by $\mathcal{F}_s(D) = D \cup (\mathcal{N}IntNCl(D))^C$ for each $D \in \tau_R(X)$. Then $\mathcal{F}_s$ is an expansion on $(U, \tau_R(X))$.

Definition 2.3. Let $(U, \tau_R(X))$ be an nano topological space. A pair of expansion $\mathcal{A}, \mathcal{B}$ on$(U, \tau_R(X))$ is said to be mutually dual if $\mathcal{A}D \cap \mathcal{B}D = D$ for each $D \in \tau_R(X)\cdot$

Example 2.4. Let $U = \{a, b, c, d\}$, $X = \{a, b\}$ and $\frac{U}{R} = \{\{a\}, \{c\}, \{b, d\}\}$ with nano topology $\tau_R(X) = \{U, \emptyset, \{a\}, \{a, b, d\}, \{b, d\}\}$ then $\mathcal{CL}\{a\} = \{a, c\}$, $\mathcal{CL}\{b, d\} = \{b, c, d\}$, $\mathcal{CL}\{U\} = \{U\}$, $\mathcal{F}\{a\} = \{a, b, d\}$, $\mathcal{F}\{b, d\} = \{a, b, d\}$, $\mathcal{F}\{U\} = \{U\}$ .Here $\mathcal{CL}$ and $\mathcal{F}$ are both mutually dual to $\mathcal{A}$.

Proposition 2.5. Let $(U, \tau_R(X))$ be a nano topological space. Then the expansions $\mathcal{CL}$ and $\mathcal{F}$ are mutually dual.
Proof: Let $D \in \tau_R(X)$.
Now,
$\mathcal{CL}(D) \cap \mathcal{F}(D) = NCl(D) \cap (NCl(D) - D)^c = NCl(D) \cap (NCl(D) \cap D^c)^c$
$= NCl(D) \cap \left((NCl(D))^c \cup D\right) = \left(NCl(D) \cap (NCl(D))^c\right) \cup \left((NCl(D)) \cap D\right) = \phi \cup D = D$
That is, $\mathcal{CL}(D) \cap \mathcal{F}(D) = D$, for each $D \in \tau_R(X)$. Therefore the expansions $\mathcal{CL}$ and $\mathcal{F}$ are mutually dual.

Remark 2.6. The identity expansion $\mathcal{A}D = D$ is mutually dual to any expansion $\mathcal{B}$. The pair of expansions $\mathcal{CL}, \mathcal{F}$ and $NInt\mathcal{CL}$, $\mathcal{F}_s$ are easily seen to be mutually dual.

Definition 2.7. A function $f: (U, \tau_R(X)) \to (V, \tau_R^{'}(Y))$ is said to be nano almost continuous if for each nano-open set E in V containing $f(x)$, there exists an nano-open set D in U containing x such that $f(D) \subseteq NInt(NCl(E))$.

Theorem 2.8. A function $f: (U, \tau_R(X)) \to (V, \tau_R^{'}(Y))$ is nano almost continuous if and only if $f^{-1}(E) \subseteq NInt(f^{-1}(NInt(NCl(E))))$ for any nano-open set E in V.

Proof. Necessity: Let E be an arbitrary nano-open set in V and let $x \in f^{-1}(E)$ then $f(x) \in E$. Since E is nano-open, it is a neighborhood of $f(x)$ in V. Since f is nano almost continuous at x, there exists a nano-open neighbourhood D of x in V such that $f(D) \subseteq NInt\big(NCl(E)\big)$. This implies that $D \subseteq f^{-1}(NInt(NCl(E)))$, thus $x \in D \subseteq f^{-1}(NInt(NCl(E)))$.
Thus $f^{-1}(E) \subseteq NInt(f^{-1}(NInt(NCl(E))))$.

Sufficiency : Let E be an arbitrary nano-open set in V such that $f\ (x) \in E$.
Then, $x \in f^{-1}(E) \subseteq NInt(f^{-1}(NInt(NCl(E))))$. Take $D = NInt(f^{-1}(NInt(NCl(E))))$, then $f\ (D) \subseteq f\ (f^{-1}(NInt(NCl(E)))) \subseteq NInt(NCl(E))$ such that $f\ (D) \subseteq NInt(NCl(E))$. By Definition 2.7, f is nano almost continuous.

Proposition 2.9. If a function $f: (U, \tau_R(X)) \to (V, \tau_R^{'}(Y))$ is nano continuous, then f is nano almost continuous.

Proof: Let E be a nano-open set in V, then $E \subseteq NInt(NCl(E))$. Since f is nano continuous, $f^{-1}(E)$ is nano-open in U such that $f\ (E) \subseteq f\ (f^{-1}(NInt(NCl(E))))$.
Since $f^{-1}(E) = NInt(f^{-1}(E))$ in U,
$f^{-1}(E) = NInt(f^{-1}(E)) \subseteq NInt(f^{-1}(NInt(NCl(E))))$.
By Theorem 2.8, f is nano almost continuous.

III. Decomposition of Nano continuity.

Definition 3.1. Let $(U, \tau_R(X))$ and $(V, \tau_R^{'}(Y))$ be two nano topological spaces. A mapping $f: (U, \tau_R(X)) \to (V, \tau_R^{'}(Y))$ is said to be $\mathcal{A} -$ expansion nano continuous if $f^{-1}(E) \subseteq NInt f^{-1}(AE)$, for each $E \in \tau_R^{'}(Y)$.

Theorem 3.2. Let $(U, \tau_R(X))$ and $(V, \tau_R^{'}(Y))$ be two nano topological spaces. Let $\mathcal{A}$ and $\mathcal{B}$ be two mutually dual expansion on V. Then a mapping $f: (U, \tau_R(X)) \to (V, \tau_R^{'}(Y))$ is nano continuous if and only if f is $\mathcal{A} -$expansion nano continuous and $\mathcal{B} -$expansion nano continuous.

Proof. Necessity: Since $\mathcal{A}$ and $\mathcal{B}$ are mutually dual on V, $\mathcal{A}E \cap \mathcal{B}E = E$ for each $E \in \tau_R^{'}(Y)$.
Let $E \in \tau_R^{'}(Y)$ then $f^{-1}(E) = f^{-1}(AE) \cap f^{-1}(BE)$. Since f is nano continuous, $f^{-1}(E) = NInt f^{-1}(E)$. So, $f^{-1}(E) = NInt\big(f^{-1}(\mathcal{A}E) \cap f^{-1}(\mathcal{B}E)\big) = NInt f^{-1}(\mathcal{A}E) \cap NInt f^{-1}(\mathcal{B}E)$. Thus $f^{-1}(E) \subseteq NInt f^{-1}(\mathcal{A}E)$ and $f^{-1}(E) \subseteq NInt f^{-1}(\mathcal{B}E)$, for each $E \in \tau_R^{'}(Y)$.
Hence f is $\mathcal{A} -$ expansion nano continuous and $\mathcal{B} -$ expansion nano continuous.

Sufficiency: Let $\mathcal{A}$ and $\mathcal{B}$ be two mutually dual expansion on $(V, \tau_R^{'}(Y))$. Since f is $\mathcal{A} -$expansion nano continuous, $f^{-1}(E) \subseteq NInt f^{-1}(\mathcal{A}E)$, for each $E \in \tau_R^{'}(Y)$. Since f is $\mathcal{B} -$ expansion nano continuous, $f^{-1}(E) \subseteq NInt f^{-1}(\mathcal{B}E)$, for each $E \in \tau_R^{'}(Y)$. Also $\mathcal{A}E \cap \mathcal{B}E = E$ for each $E \in \tau_R^{'}(Y)$.
Therefore, $f^{-1}(\mathcal{A}E) \cap f^{-1}(\mathcal{B}E) = f^{-1}(E)$.
Hence, $NInt f^{-1}(E) = (NInt f^{-1}(\mathcal{A}E) \cap NInt f^{-1}(\mathcal{B}E)) \supseteq f^{-1}(E) \cap f^{-1}(E) = f^{-1}(E)$.
So, $NInt f^{-1}(E) \supseteq f^{-1}(E)$. But, $NInt f^{-1}(E) \subseteq f^{-1}(E)$ always. Therefore, $f^{-1}(E) = NInt f^{-1}(E)$. This implies that $f^{-1}(E)$ is an open set in $(U, \tau_R(X))$ for each $E \in \tau_R^{'}(Y)$. Therefore f is nano continuous.

Corollary 3.3. A mapping $f: (U, \tau_R(X)) \to (V, \tau_R^{'}(Y))$ is nano continuous if and only if f is nano almost continuous and $\mathcal{F}_s -$ expansion nano continuous.

Proof: We have that the condition f is nano almost continuous is equivalent to f is $NInt\mathcal{CL} -$expansion nano continuous, and the condition f is $\mathcal{F}_s -$expansion nano continuous is equivalent to $f^{-1}(E) \subset NInt f^{-1}(E \cup (NIntNCl(E)^c))$, for each nano-open set E in V. Since $NInt\mathcal{CL}$ and $\mathcal{F}_s$ are mutually dual, the result follows from theorem 3.2.

Theorem 3.4. Let $\mathcal{A}$ be any expansion on $(V, \tau_R^{'}(Y))$. Then the expansion $\mathcal{B}E = E \cup (\mathcal{A}E)^c$ is the maximal expansion on $(V, \tau_R^{'}(Y))$ which is mutually dual to $\mathcal{A}$.

Proof: Let $\mathcal{B}_A$ be the set of all expansions on $(V, \tau_R^{'}(Y))$ which are mutually dual to $\mathcal{A}$. Since $E \subset \mathcal{A}E$, for any $E \in \tau_R^{'}(Y)$, $\mathcal{A}E$ can be written as $\mathcal{A}E = E \cup (\mathcal{A}E \backslash E)$. Let $\mathcal{B}E = E \cup (\mathcal{A}E)^c = (\mathcal{A}E \backslash E)^c$. It is obvious that $\mathcal{B}$ is an expansion on $(V, \tau_R^{'}(Y))$ and $\mathcal{A}E \cap \mathcal{B}E = E$ for any $E \in \tau_R^{'}(Y)$. Thus $\mathcal{B} \in \mathcal{B}_A$.Given any expansion $\mathcal{B}'$ on $(V, \tau_R^{'}(Y))$, write $\mathcal{B}'E = E \cup (\mathcal{B}'E \backslash E)$. If $\mathcal{B}' \in \mathcal{B}_A$, then $(\mathcal{A}E \backslash E) \cap (\mathcal{B}'E \backslash E) = \phi$, thus $(\mathcal{B}'E \backslash E) \subset (\mathcal{A}E \backslash E)^c$. Therefore $\mathcal{B}'E \subset \mathcal{B}E$ and we have that $\mathcal{B}' < B$, that is, $\mathcal{B}$ is the maximal element of $\mathcal{B}_A$.

Definition 3.5. Let $(U, \tau_R(X))$ and $(V, \tau_R^{'}(Y))$ be two nano topological spaces. Let $\mathcal{B}$ be an expansion on $(V, \tau_R^{'}(Y))$. A mapping $f: (U, \tau_R(X)) \to (V, \tau_R^{'}(Y))$ is said to be closed $\mathcal{B} -$ nano continuous if $f^{-1}((\mathcal{B}E)^c)$ is a nano closed set in $(U, \tau_R(X))$ for each $E \in \tau_R^{'}(Y)$.

Proposition 3.6. A closed $\mathcal{B}$ −nano continuous mapping $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ is $\mathcal{B}-$ expansion nano continuous.

Proof: We first prove $(f^{-1}((\mathcal{B}E)^c))^c = f^{-1}(\mathcal{B}E)$. Let $x \in (f^{-1}((\mathcal{B}E)^c))^c$.
Then x$\notin f^{-1}((\mathcal{B}E)^c))$. Hence $f(x) \notin (\mathcal{B}E)^c$, this implies $f(x) \in (\mathcal{B}E)$ and $x \in f^{-1}(\mathcal{B}E)$.
So, $(f^{-1}((\mathcal{B}E)^c))^c \subseteq f^{-1}(\mathcal{B}E)$.
Conversely, let $x \in f^{-1}(\mathcal{B}E)$. Then $f(x) \in (\mathcal{B}E)$. Hence, $f(x) \notin (\mathcal{B}E)^c$, $x \notin f^{-1}((\mathcal{B}E)^c))$, this implies $x \in (f^{-1}((\mathcal{B}E)^c))^c$. So, $f^{-1}(\mathcal{B}E) \subseteq (f^{-1}((\mathcal{B}E)^c))^c$. Therefore, $(f^{-1}((\mathcal{B}E)^c))^c = f^{-1}(\mathcal{B}E)$.
Since $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ is a $\mathcal{B}$ −nano continuous mapping, $f^{-1}((\mathcal{B}E)^c))$ is a nano closed set in $(U,\tau_R(X))$. Hence $f^{-1}(\mathcal{B}E)$ is nano-open in $(U,\tau_R(X))$ and so $f^{-1}(\mathcal{B}E) = NIntf^{-1}(\mathcal{B}E)$. Also, and this implies $f^{-1}(E) \subseteq f^{-1}(\mathcal{B}E) = NIntf^{-1}(\mathcal{B}E)$. Therefore $f^{-1}(E) \subseteq NIntf^{-1}(\mathcal{B}E)$ for each $E \in \tau_R^{'}(Y)$. Hence f is $\mathcal{B}$ −expansion nano continuous.

Example 3.7. Consider the identity mapping $I:(\mathcal{R},\tau_R(X)) \to (\mathcal{R},\tau_R(X))$. Let $U = \mathcal{R}$ and $X \subseteq U$, where $\mathcal{R}$ is the set of real numbers. Define $\mathcal{B}:\tau_R(X) \to 2^R$ such that $\mathcal{B}D = \mathcal{CL}(D)$ for all $D \in \tau_R(X)$. Let $X = \{[0,2],(2,3),[3,5]\}$ then $\tau_R(X) = \{U,\phi,[0,4],[0,5],(4,5]\}$. When $D = (4,5]$, then $I^{-1}((BD)^c)$ is not nano closed. Therefore I is not closed $\mathcal{B}$ −nano continuous even though it is nano continuous.

Definition 3.8. An expansion $\mathcal{A}$ on $(U,\tau_R(X))$ is said to be nano-open if $\mathcal{A}V \in \tau_R(X)$ for all $V \in \tau_R(X)$.

Definition 3.9. An nano-open expansion $\mathcal{A}$ on $(U,\tau_R(X))$ is said to be idempotent.

Example 3.10. The expansion $\mathcal{F}D = (NCl(D) - NInt(D))^c$ for each $D \in \tau_R(X)$ is idempotent.
In fact the expansion $\mathcal{F}$ is nano-open,
$\mathcal{F}(\mathcal{F}D) = \mathcal{F}(NCl(D) - NInt(D))^c = \mathcal{F}((NCl(D) \cap D^c)^c)$
$= \mathcal{F}((NCl(D))^c \cup D) = (NCl((NCl(D))^c \cup D))^c \cup ((NCl(D))^c \cup D)$
$= \left(NCl\left(NCl(D)\right)^c \cup NCl(D)\right)^c \cup (\left(NCl(D)\right)^c \cup D$
$= (NCl(NCl(D))^c)^c \cap (NCl(D))^c \cup ((NCl(D))^c \cup D)$
$= ((NCl(D))^c \cup D) = \mathcal{F}D$

Remark 3.11. From the Example 3.7, we conclude that nano continuity does not imply closed $\mathcal{B}$ −nano continuity. Since f is nano continuous it is $\mathcal{B}$ −expansion nano continuous, but it is not closed $\mathcal{B}$ −nano continuous. The proposition gives a condition under which an $\mathcal{B}$ −expansion nano continuous function is closed $\mathcal{B}$ −nano continuous and vice versa.

Proposition 3.12. Let $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ and $\mathcal{B}$ be idempotent, then f is $\mathcal{B}$ −expansion nano continuous if and only if f is closed $\mathcal{B}$ −nano continuous.
Proof: The sufficiency follows from Proposition 3.6.

Necessity: Let f be $\mathcal{B}$ −expansion nano continuous,where $\mathcal{B}$ is idempotent and E an nano- open subset of $(V,\tau_R^{'}(Y))$. Since $\mathcal{B}E$ is nano-open on $(V,\tau_R^{'}(Y))$ and $\mathcal{B}(\mathcal{B}E) = \mathcal{B}E$, then $f^{-1}(\mathcal{B}E) \subseteq NIntf^{-1}(\mathcal{B}(\mathcal{B}E)) = NIntf^{-1}(\mathcal{B}E)$. Thus $f^{-1}(\mathcal{B}E)$ is nano open in $(U,\tau_R(X))$ and therefore f is closed $\mathcal{B}$ −nano continuous.

Corollary 3.13: Let $\mathcal{A}$ and $\mathcal{B}$ be two mutually dual expansion on $(V,\tau_R^{'}(Y))$. If $\mathcal{B}$ is idempotent, then $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ is nano continuous if and only if f is $\mathcal{A}$ −expansion nano continuous and closed $\mathcal{B}$ −nano continuous.

IV. Conclusion.

In this paper, the notions of expansion of nano-open sets and decomposition of nano continuity in nano topological spaces are studied. The theory of expansions and decomposition in nano topological spaces has a wide variety of applications in real life. The decomposition of nano topological space can be applied in the study of independence of real time problems and in defining its attributes.

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