An EOQ Model with Polynomial of Nth Degree Demand Rate, Constant Deterioration, Linear Holding Cost and Without Shortages under Inflation

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Abstract: In existing literature the inventory models are developed under the assumption of demand rate to be constant or linear or quadratic. In this paper, an inventory model is developed where demand rate is generalised as a polynomial of n° degree with constant deterioration rate. The holding cost is a linear function of time. Shortages are not allowed. A numerical example has been illustrated using MATLAB to describe the model and the sensitivity analysis of various parameters is carried out with graphical and tabular data.

Keywords: Constant deterioration; Demand rate is a polynomial of n° degree; Inflation; Inventory; Time dependent holding cost

I. Introduction


The rest of this paper is organized as follows. Section II describes the assumptions used throughout this paper. In section III notations used in paper are listed. The mathematical formulation is given in section IV. Section V describes the mathematical solution of the problem formulated in section IV. The algorithm of the solution is provided in section VI. Concluding remarks and suggestions for future research are provided in Section VII while a numerical solution is mentioned in section VIII.

II. Assumptions

The following assumptions are made in developing the model.

- The inventory system considers a single item only.
- The demand rate is deterministic and a polynomial of n\textsuperscript{th} degree is time dependent.
- The deterioration rate is constant.
- The inventory system is considered over a finite time horizon.
- Holding cost is time dependent linear function \( (p + qt) \)
- Lead time is zero.
- Shortages are not allowed.

III. Notations

The following notations use for inventory model.

- \( A \) : Setup cost.
- \( D(t) \) : A polynomial of n\textsuperscript{th} degree in a period [0,T]
- \( \theta \) : Deteriorating cost is constant.
- \( Q_0 \) : Initial ordering quantity
- \( TD \) : Total demand in a cycle period [0, T]
- \( DU \) : Deteriorating unit in a cycle period [0, T]
- \( C_d \) : Deteriorating cost per unit
- \( DC \) : Deteriorating cost
- \( HC \) : Holding cost
- \( TC(T) \) : Total inventory cost
- \( T^* \) : Optimal length size
- \( Q_0^* \) : Optimal initial order quantity
- \( TC^*(T^*) \) : Optimal total cost in the period [0, T]

IV. Mathematical Formulation

Consider the inventory model of constant deteriorating items with demand rate is a polynomial of n\textsuperscript{th} degree. As the inventory reduces due to demand rate as well as deterioration rate during the interval, the differential representing the inventory status is governed by [0,T]

\[
\frac{dI(t)}{dt} + \theta I(t) = -D(t), \quad 0 \leq t \leq T
\]

\[
D(t) = \sum_{i=0}^{n} a_i t^i \quad \text{where } a_0, a_1, a_2, \ldots \ldots \ a_n \text{ are } n\textsuperscript{th} \text{ constants and } a_n \neq 0.
\]

V. Mathematical Solution

The solution with boundary condition \( I(T) = 0 \), of the Equation

\[
\frac{dI(t)}{dt} + \theta I(t) = -\sum_{i=0}^{n} a_i t^i, \quad 0 \leq t \leq T
\]
\[ I(t) = \sum_{i=0}^{n} a_i \left[ \frac{T^{i+1}}{i+1} - \frac{T^{i+2}}{i+2} + \frac{T^{i+3}}{i+3} \right] e^{-\theta t} \]  
\hspace{10cm} (3)

Where we use the expansion \( e^{-\alpha t} \approx 1 - \alpha t + \frac{(\alpha t)^2}{2} \), \( \alpha \) is small and positive.

So the initial order quantity is obtained by putting the boundary condition in Equation (3) \( I(0) = Q_0 \).

Therefore,

\[ Q_0 = \sum_{i=0}^{n} a_i \left[ \frac{T^{i+1}}{i+1} + \theta \frac{T^{i+2}}{i+2} + \theta^2 \frac{T^{i+3}}{i+3} \right] \]  
\hspace{10cm} (4)

The total demand during the cycle period \([0, T]\) is

\[ TD = \int_{0}^{T} D(t) \, dt \]
\[ = \int_{0}^{T} \sum_{i=0}^{n} a_i T^{i+1} \, dt \]
\[ = \sum_{i=0}^{n} a_i \frac{T^{i+1}}{i+1} \]  
\hspace{10cm} (5)

Then the number of deterioration units is

\[ DU = Q_0 - TD \]
\[ = -\sum_{i=0}^{n} a_i \left[ \theta \frac{T^{i+2}}{i+2} + \theta^2 \frac{T^{i+3}}{i+3} \right] \]  
\hspace{10cm} (6)

The deterioration cost for the cycle \([0, T]\)

\[ DC = C_d \cdot \text{(Number of deterioration units)} \]
\[ = -C_d \sum_{i=0}^{n} a_i \left[ \theta \frac{T^{i+2}}{i+2} + \theta^2 \frac{T^{i+3}}{i+3} \right] \]  
\hspace{10cm} (7)

Holding Cost for the cycle \([0,T]\) is

\[ HC = \int_{0}^{T} (p + qt) e^{-\alpha t} I(t) \, dt \]
\[ = \int_{0}^{T} (p + qt) e^{-\alpha t} \sum_{i=0}^{n} a_i \left[ \frac{T^{i+1}}{i+1} - \frac{T^{i+2}}{i+2} + \frac{T^{i+3}}{i+3} \right] \, dt \]
\[ = \sum_{i=0}^{n} a_i \int_{0}^{T} (p + qt) \left[ 1 - (r + \theta)t \right] \left[ \frac{T^{i+1}}{i+1} - \frac{T^{i+2}}{i+2} + \frac{T^{i+3}}{i+3} \right] \, dt \]
\[ = \sum_{i=0}^{n} a_i \int_{0}^{T} \left[ p - (pr + p\theta - q)t - q(r + \theta)t^2 \right] \left[ \frac{T^{i+1}}{i+1} + \theta \frac{T^{i+2}}{i+2} + \theta^2 \frac{T^{i+3}}{i+3} \right] \, dt \]  
\hspace{10cm} (8)
The total inventory cost TC (T)= Ordering cost (A) + Deterioration cost (DC) + Holding cost (HC).

Therefore the total variable cost per unit time

\[
TC(T) = \frac{A}{T} - C_J \sum_{i=0}^{n} a_i \left[ \theta \frac{T^{i+4}}{i+2} + \theta^2 \frac{T^{i+5}}{i+3} \right]
\]

\[
+ \sum_{i=0}^{n} a_i \left[ \frac{(pr + p\theta - q)}{T} + 2q(r + \theta) \right] \left[ \frac{(i+4)T^{i+1}}{2(i+2)(i+3)} + \theta \frac{(i+5)T^{i+2}}{2(i+3)(i+4)} + \theta^2 \frac{(i+6)T^{i+3}}{2(i+4)(i+5)} \right]
\]

\[+ \sum_{i=0}^{n} a_i \left[ \frac{(pr + p\theta - q)}{T} + 2q(r + \theta) \right] \left[ \frac{(i+4)T^{i+1}}{2(i+2)(i+3)} + \theta \frac{(i+5)T^{i+2}}{2(i+3)(i+4)} + \theta^2 \frac{(i+6)T^{i+3}}{2(i+4)(i+5)} \right]
\]

\[+ \left[ 2q(r + \theta) \right] \left[ \frac{(i+1)(i+6)T^{i+1}}{6(i+2)(i+4)} + \theta \frac{(i+1)(i+2)(i+7)T^{i+2}}{6(i+3)(i+5)} + \theta^2 \frac{(i+1)(i+2)(i+3)(i+8)T^{i+3}}{6(i+4)(i+6)} \right]
\]

The necessary and sufficient conditions for minimize cost a given value T are

\[
\frac{dTC(T)}{dT} = 0 \quad \text{And} \quad \frac{d^2TC(T)}{dT^2} > 0 \quad \text{then differentiation with respect to T of (9), we get}
\]

\[
\frac{dTC(T)}{dT} = \frac{-A}{T^2} - C_J \sum_{i=0}^{n} a_i \left[ \theta \frac{(i+1)T^{i+4}}{i+2} + \theta^2 \frac{(i+2)T^{i+5}}{i+3} \right]
\]

\[
- \left[ \frac{p}{T} - q(r + \theta) \right] \left[ \frac{T^{i+2}}{i+2} + \theta \frac{T^{i+3}}{i+3} + \theta^2 \frac{T^{i+4}}{i+4} \right]
\]

\[+ \left[ \frac{p}{T} - (pr + p\theta - q) - q(r + \theta) T \right] \left[ \frac{T^{i+1} + T^{i+2} + \theta T^{i+3} + \theta^2 T^{i+4}}{i+3} \right]
\]

\[+ \sum_{i=0}^{n} a_i \left[ \frac{(pr + p\theta - q)}{T} + 2q(r + \theta) \right] \left[ \frac{(i+1)(i+4)T^{i+1}}{2(i+2)(i+3)} + \theta \frac{(i+2)(i+5)T^{i+2}}{2(i+3)(i+4)} + \theta^2 \frac{(i+3)(i+6)T^{i+3}}{2(i+4)(i+5)} \right]
\]

\[+ \left[ 2q(r + \theta) \right] \left[ \frac{(i+1)(i+6)T^{i+1}}{6(i+2)(i+4)} + \theta \frac{(i+1)(i+2)(i+7)T^{i+2}}{6(i+3)(i+5)} + \theta^2 \frac{(i+1)(i+2)(i+3)(i+8)T^{i+3}}{6(i+4)(i+6)} \right]
\]

And the again differentiation with respect to T of (10), we get

\[
\frac{d^2TC(T)}{dT^2} = \frac{2A}{T^3} - C_J \sum_{i=0}^{n} a_i \left[ \theta \frac{(i+1)T^{i+4}}{i+2} + \theta^2 \frac{(i+1)(i+2)T^{i+5}}{i+3} \right]
\]

\[
- \left[ \frac{2p}{T^3} \left( \frac{T^{i+2}}{i+2} + \theta \frac{T^{i+3}}{i+3} + \theta^2 \frac{T^{i+4}}{i+4} \right) \right]
\]

\[+ \left[ \frac{p}{T^2} + q(r + \theta) T \right] \left[ \frac{T^{i+1} + T^{i+2} + \theta T^{i+3} + \theta^2 T^{i+4}}{i+3} \right]
\]

\[+ \sum_{i=0}^{n} a_i \left[ \frac{(pr + p\theta - q)}{T^3} \left( \frac{(i+1)(i+4)T^{i+1}}{2(i+2)(i+3)} + \theta \frac{(i+2)(i+5)T^{i+2}}{2(i+3)(i+4)} + \theta^2 \frac{(i+3)(i+6)T^{i+3}}{2(i+4)(i+5)} \right) \right]
\]

\[+ \left[ \frac{(pr + p\theta - q)}{T^2} T \right] \left[ \frac{(i+1)(i+6)T^{i+1}}{6(i+2)(i+4)} + \theta \frac{(i+1)(i+2)(i+7)T^{i+2}}{6(i+3)(i+5)} + \theta^2 \frac{(i+1)(i+2)(i+3)(i+8)T^{i+3}}{6(i+4)(i+6)} \right]
\]
VI. Algorithm

To find out the solution following algorithm used

Step1: Find derivative \( \frac{dTC(T)}{dT} \) and put \( \frac{dTC(T)}{dT} = 0 \)

Step2: Solve equation (10) for \( T \)

Step3: Find the derivative \( \frac{d^2TC(T)}{dT^2} \) and check \( \frac{d^2TC(T)}{dT^2} > 0 \) for \( T^* \) optimal length

Step4: Find optimal total cost \( TC(T^*) \) and initial order quantity \( Q^*(T^*) \)

VII. Conclusion

In the present paper, we developed an inventory model for variable deteriorating item with inflation, exponential declaring demand and without shortages give analytical solution, numerical solution and the effect of parameters of the model that minimize the total inventory cost. The deterioration factor taken into consideration in this model, as almost all items undergo either direct spoilage or physical decay in the course time, deterioration is natural feature in the inventory system. The model is very practical for the industries in which the demand rate is depending upon the time and holding cost is linear function with inflation. This model can further be extended by taking more realistic assumptions such as finite replenishment rate, probabilistic demand rate etc.

VIII. Numerical Solution

Consider an inventory system with the following parameter in proper units \( A = 100, \theta = 0.02, p = 0.5, q = 0.3, C_d = 5 \), and parameter \( r = 0.001 \), for consideration five constants are taken into account randomly with values \( a_1 = 1.2, a_2 = 0.1, a_3 = 0.5, a_4 = 0.3, a_5 = 0.4 \) The computer output of the program by using Mat lab software is \( T^* = 4.125 \) \( .Q_o^* = 5823.72 \) and \( TC^* = 10658.22 \). The analysis shows that as the value of constants \( a_1 \) increases then Total Cost \( TC^* \) and \( Q_o^* \) increases highly whereas if the inflation rate is increased then Total Cost \( TC^* \) and \( Q_o^* \) decreases.

References


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