On Some Bilateral Generating Relations Involving I-Function

Gauri Shankar Pandey
Govt. Science College, Rewa (m. P.)

Abstract: The aim of this research paper is to establish some bilateral generating relations involving I-function of two variables.

I. Introduction

The I–function of two variables introduced by Sharma & Mishra [2], will be defined and represented as follows:

\[ I_2^m = \left| \begin{array}{l}
\phi_1(\xi, \eta) \\
\phi_2(\xi) \\
\phi_3(\eta)
\end{array} \right| \]

where

\[ \phi_1(\xi, \eta) = \frac{G(1-b_j+\eta(j+A\eta))}{\sum_{i=1}^{j} G(1-b_j+\eta(i+A\eta))} \]
\[ \phi_2(\xi) = \frac{G(1-e_j+\xi(j+E\xi))}{\sum_{i=1}^{j} G(1-e_j+\xi(i+E\xi))} \]
\[ \phi_3(\eta) = \frac{G(1-f_j+\eta(j+F\eta))}{\sum_{i=1}^{j} G(1-f_j+\eta(i+F\eta))} \]

\( \xi \) and \( \eta \) are not equal to zero, and an empty product is interpreted as unity \( p_1 \), \( p_1 \), \( p_\infty \), \( q_1 \), \( q_\infty \), \( n \), \( n_1 \), \( n_2 \), \( m_0 \) and \( m_1 \) are non negative integers such that \( p_i \geq n \geq 0 \), \( p_i \geq n_1 \geq n_2 \geq 0 \), \( q_i \geq 0 \), \( q_i \geq 0 \), \( q_i \geq 0 \), \( i = 1, ..., r \), \( i' = 1, ..., r' \), \( i'' = 1, ..., r'' \); \( k = 1, 2 \) also all the A’s, α’s, B’s, β’s, γ’s, δ’s, E’s and F’s are assumed to be positive quantities for standardization purpose; the definition of I-function of two variables given above will however, have a meaning even if some of these quantities are zero. The contour \( L_1 \) is in the \( \xi \)-plane and runs from \( -\infty \) to \( +\infty \), with loops, if necessary, to ensure that the poles of \( G(1-c_i+\gamma\xi) \) \( (j = 1, ..., n_1) \) lie to the right, and the poles of \( G(1-a_1+\alpha\xi+A\eta) \) \( (j = 1, ..., n) \) to the left of the contour. The contour \( L_2 \) is in the \( \eta \)-plane and runs from \( -\infty \) to \( +\infty \), with loops, if necessary, to ensure that the poles of \( G(1-e_1+\xi\eta) \) \( (j = 1, ..., n_2) \) lie to the right, and the poles of \( G(1-f_1+E\eta) \) \( (j = 1, ..., n) \) to the left of the contour. Also

\[ R' = \sum_{j=1}^{p_1} a_j + \sum_{j=1}^{p_1} \eta' j_1' - \sum_{j=1}^{q_1} b_j - \sum_{j=1}^{q_1} \delta_j' < 0, \]
\[ S' = \sum_{j=1}^{p_1} A_j + \sum_{j=1}^{p_1} E_j - \sum_{j=1}^{q_1} B_j - \sum_{j=1}^{q_1} F_j < 0, \]
\[ U' = \sum_{j=n+1}^{p_1} a_j - \sum_{j=1}^{p_1} \beta_j + \sum_{j=1}^{m_0} d_j - \sum_{j=m_1}^{m_0+1} \beta_j' + \sum_{j=1}^{q_1} y_j - \sum_{j=n_1+1}^{q_1} y_j' > 0, \]
\[ V' = -\sum_{j=n+1}^{p_1} a_j - \sum_{j=1}^{p_1} B_j - \sum_{j=m_1}^{m_0} F_j - \sum_{j=m_2+1}^{m_0} F_j' + \sum_{j=n_1}^{q_1} E_j - \sum_{j=n_2+1}^{q_1} E_j' > 0, \]

and \( |\arg x| < \frac{\pi}{2} U', \arg y | < \frac{\pi}{2} V' \).

In the present investigation we require the following formulae:

From Rainville [1, p.93]:

\[ z F_1 \left[ 1+\alpha+n_1 - 1 \right] = \frac{(1+n_0)}{(1+n_2/2)}. \]
From Shrivastava and Manocha [3, p.37 (10), 34, 44],

\[
(a)_n = (a, n) = \frac{\Gamma(a+n)}{\Gamma(a)} \tag{5}
\]

\[
(1-z)^{-a} = \sum_{n=0}^{\infty} (a)_n \frac{z^n}{n!}. \tag{6}
\]

### II. Bilateral Generating Relations

In this section we establish the following bilateral Generating Relations:

\[
\sum_{n=0}^{\infty} \frac{t^n}{n!} 2F_1[-n; \frac{a}{2} - 1] \tag{7}
\]

\[
\int_{1}^{\infty} \int_{1}^{\infty} \Phi_1(\xi, \eta) \Phi_2(\xi) \Phi_3(\eta) \pi(1 - \frac{a}{2} - n) + 0\xi x^\xi y^\eta d\xi d\eta
\]

On expressing I-function in contour integral form as given in (1) and using (4), we get

\[
\Delta = \sum_{n=0}^{\infty} \frac{t^n}{n!} \frac{(1+a)_n}{(1+a/2)_n}
\]

\[
\frac{1}{(2\pi)^2} \int_{1}^{\infty} \int_{1}^{\infty} \Phi_1(\xi, \eta) \Phi_2(\xi) \Phi_3(\eta) \Gamma(1 + a/2) x^\xi y^\eta d\xi d\eta
\]

In the view of (5) and (6), we arrive at R.H.S. of (7) as follows:

\[
\Delta = \sum_{n=0}^{\infty} \frac{t^n}{n!} \frac{(1+a)_n}{(1+a/2)_n}
\]

\[
\frac{1}{(2\pi)^2} \int_{1}^{\infty} \int_{1}^{\infty} \Phi_1(\xi, \eta) \Phi_2(\xi) \Phi_3(\eta) \Gamma(1 + a/2) x^\xi y^\eta d\xi d\eta
\]

\[
= \sum_{n=0}^{\infty} \frac{t^n}{n!} (1+a)_n x^\xi y^\eta d\xi d\eta
\]

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\[
= \frac{1}{(2\pi a)^2} \int_{L_1} \int_{L_2} \phi_1(\xi, \eta) \theta_2(\xi) \theta_3(\eta) \\
\Gamma(1 + a/2)(1 - t)^{-(a+1)x^t y^t} d\xi \eta
\]

Proceeding on similar lines as above, the results (8) can be derived easily.

### III. Particular Cases

On choosing \( r = 1, \ r' = 1 \) and \( r'' = 1 \) in main integrals, we get following integrals in terms of \( H \)-function of two variables:

\[
\sum_{n=0}^{\infty} \frac{t^n}{n!} \binom{-n; a;}{1+2+n; -1} \\
\binom{0n_1m_2n_2+1m_3n_3}{p_1q_1r_1s_1+t_1r_2s_2+t_1t_2} \frac{y}{y}^{(a/2,0)}
\]

\[
= (1 - t)^{-(a+1)} \binom{0n_1m_2n_2+1m_3n_3}{p_1q_1r_1s_1+t_1r_2s_2+t_1t_2} \frac{y}{y}^{(a/2,0)}.
\]  

provided that \( U > 0, V > 0, |\arg x| < \frac{1}{2} U \pi, |\arg y| < \frac{1}{2} V \pi \) where \( U \) and \( V \) are given by:

\[
U = -\sum_{j=1}^{p_1} a_j - \sum_{j=1}^{q_1} b_j + m_2 - \sum_{j=1}^{q_2} \delta_j + \sum_{j=1}^{n_2} \gamma_j > 0,
\]

\[
V = -\sum_{j=1}^{p_1} a_j - \sum_{j=1}^{q_1} b_j + m_3 - \sum_{j=1}^{q_3} \delta_j + \sum_{j=1}^{n_3} \gamma_j > 0.
\]

\[
\sum_{n=0}^{\infty} \frac{t^n}{n!} \binom{-n; a;}{1+2+n; -1} \\
\binom{0n_1m_2+1n_2m_3n_3}{p_1q_1r_1s_1+t_1r_2s_2+t_1t_2} \frac{y}{y}^{(1+a/2,0)}
\]

\[
= (1 - t)^{-(a+1)} \binom{0n_1m_2+1n_2m_3n_3}{p_1q_1r_1s_1+t_1r_2s_2+t_1t_2} \frac{y}{y}^{(1+a/2,0)}.
\]  

provided that \( U > 0, V > 0, |\arg x| < \frac{1}{2} U \pi, |\arg y| < \frac{1}{2} V \pi \) where \( U \) and \( V \) are given in (10) and (11).

### References

