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On Intuitionistic Fuzzy Graph Structures

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Abstract: *In this paper, we introduce the notion of intuitionistic fuzzy graph structure $\tilde{G}$ = (A, B_1, B_2,......,B_k) and discuss some analogs of fuzzy graph structure theoretical concepts. We also discuss some properties of intuitionistic fuzzy B_i- tree and intuitionistic fuzzy B_i-forests.*

Keywords: *Intuitionistic fuzzy graph structure (IFGS), tree, forest, edge, path.*
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I. Introduction

In 1965, the concept of fuzzy sets was introduced by L.A. Zadeh [9]. A. Rosenfeld [7] gave the idea of fuzzy relation and fuzzy graph and developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. E. Sampatkumar in [8] has generalized the notion of graph G = (V, E) to graph structure G = (V, R_1, R_2,......., R_k) and studied their properties. T. Dinesh and T. V. Ramakrishnan [2] gave the perception of fuzzy graph structure G = (μ, ρ_1, ρ_2,…..., ρ_k) and studied their properties.

One of the remarkable generalizations of fuzzy sets was intuitionistic fuzzy sets given by K.T. Atanassov's[1]. He gave the notion of intuitionistic fuzzy relation and discussed intuitionistic fuzzy graphs which were further studied in [6]. In this article, we introduce the notion of intuitionistic fuzzy graph structures and investigate some of their properties. We also discuss some properties of intuitionistic fuzzy B_i- trees and intuitionistic fuzzy B_i-forests.

II. Preliminaries

In this section, we review some definitions that are necessary in this paper which are mainly taken from [2], [5] and [8].

Definition (2.1) G = (V, R_1, R_2,..., R_k) is a graph structure if V is a non empty set and R_1,R_2,...,R_k are relations on V which are mutually disjoint such that each R_i , i=1,2,3,...,k, is symmetric and irreflexive.

Definition (2.2) A R_i-cycle is an alternating sequence v_0, e_1, v_1, e_2,...,v_{n-1},e_n,v_n= v_0 consisting of vertices and R_i –edges only.

Definition (2.3) A graph structure is a R_i-forest if the subgraph structure induced by R_i-edges is a forest, i.e., if it has no R_i-cycles.

Definition (2.4) An intuitionistic fuzzy graph is of the form G = (V, E) where

(i) V = {v_1, v_2, ..., v_n} such that $\mu_1 : V\to [0,1]$ and $\gamma_1 : V\to [0,1]$ denote the degree of membership and degree of non membership of the element $v_i \in$ V, respectively such that $0 \le \mu_1(v_i) + \gamma_1 (v_i) \le 1$, for every $v_i \in V$, (i = 1,2,….,n),

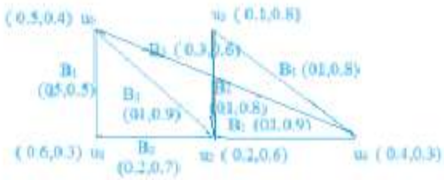
(ii) $E \subseteq V \times V$ where $\mu_2: V \times V \to [0,1]$ and $\gamma_2 : V \times V \to [0,1]$ are such that
$\mu_2(v_i,v_j) \le \min\{\mu_1(v_i),\mu_1(v_j)\}$ and $\gamma_2(v_i,v_j) \le \max\{\gamma_1(v_i),\gamma_1(v_j)\}$ and $0 \le \mu_2 (v_i , v_j) + \gamma_2 (v_i, v_j) \le 1$,
for every $(v_i ,v_j) \in E$, (i, j = 1,2,….,n).

III. Intuitionistic Fuzzy Graph Structure

Definition (3.1): Let G = (V, R_1, R_2,..., R_k) be a graph structure and let A be an intuitionistic fuzzy subset (IFS) on V and B_1, B_2 ,..., B_k are intuitionistic fuzzy relations (IFR) on V which are mutually disjoint, symmetric and irreflexive such that
$\mu_{B_i}(u,v) \le \mu_A(u) \wedge \mu_A(v)$ and $\nu_{B_i}(u,v) \le \nu_A(u) \vee \nu_A(v)$ $\forall$ *u*, *v* $\in$ V and *i* = 1, 2, ..., k. Then $\tilde{G}$ = (A, B_1, B_2,…..., B_k) is an intuitionistic fuzzy graph structure (IFGS) of G.

Note(3.2): Throughout this paper , unless otherwise specified $\tilde{G}$ = (A, B_1, B_2,...,B_k) will represent an intuitionistic fuzzy graph structure with respect to graph structure G = (V, R_1, R_2 ,..., R_k) and i =1, 2,..., k will refer to the number of intuitionistic fuzzy relations on V.

Example (3.3): Consider the graph structure G = (V, R_1, R_2, R_3), where V = { u_0, u_1, u_2, u_3, u_4} and R_1 ={ (u_0, u_1), (u_0, u_2), (u_3, u_4)}, R_2 ={(u_1, u_2), (u_2, u_4)}, R_3 = {(u_2, u_3),(u_0, u_4)} are the relations on V. Let A = {< u_0, 0.5,0.4 >, < u_1, 0.6,0.3>, < u_2 , 0.2,0.6 >, <u_3, 0.1,0.8> , <u_4, 0.4,0.3> } be an IFS on V and B_1 = { (u_0, u_1), 0.5,0.5 >, < (u_0, u_2), 0.1,0.9 >, < (u_3, u_4), 0.1,0.8 >}, B_2 = { < (u_1, u_2), 0.2,0.7 >,<(u_2, u_4),0.1,0.9 >}, B_3 ={ <(u_2, u_3), 0.1,0.8 >,<(u_0, u_4),0.3,0.6 >} are IFRs on V.



It can be easily verified that $\mu_{B_i}(u,v) \le \mu_A(u) \wedge \mu_A(v)$ and $\nu_{B_i}(u,v) \le \nu_A(u) \vee \nu_A(v)$ $\forall$ *u*, *v* ∈ V, i = 1, 2, 3. Thus, $\tilde{G}$ = (A, B_1, B_2, B_3) is an IFGS of G.

Definition (3.4): Let $\tilde{G}$ = (A, B_1, B_2, .., B_k) be an IFGS of a graph structure G = (V, R_1, R_2,...,R_k), then $\tilde{H}$ = (A, C_1,C_2,..., C_k) is called a partial intuitionistic fuzzy spanning subgraph structure of $\tilde{G}$. if

$\mu_{C_r}(u,v) \le \mu_{B_r}(u,v)$ and $\nu_{C_r}(u,v) \le \nu_{B_r}(u,v)$ for r =1,2,...,k and ∀ u,v ∈ V, (u,v) ∈ B_i and i = 1,2,..., k.

Note (3.5): Supp(A) = {u∈V: μ_A (u) > 0 , ν_A (u) < 1} and Supp(B_i) = {(u,v) ∈V×V : μ_{B_i} (u,v) > 0 , ν_{B_i} (u,v) < 1}.

Definition (3.6): Let $\tilde{G}$ be an IFGS of graph structure G, then (u,v) is called a B_i−edge of $\tilde{G}$ if (u,v) ∈ supp(B_i).

Definition (3.7): A B_i - path of an IFGS $\tilde{G}$ is a sequence of vertices,u_0,u_1,...,u_n which are distinct (except possibly $u_0 = u_n$) such that (u_{j-1},u_j) is a B_i-edge for all j = 1,2,...,n.

In example (3.3), u_1, u_0, u_2 is a B_1-path, u_1, u_2, u_4 is a B_2-path.

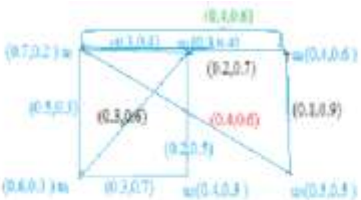
Definition (3.8): A path P in an IFGS $\tilde{G}$ is a sequence of vertices v_1, v_2 , ..., v_n (∈V) which are distinct (except possibly $v_1 = v_n$) such that (v_j, v_{j+1}) is a B_p-edge for some p∈{1,2,…, k}.

Definition (3.9): Two vertices of an IFGS $\tilde{G}$ joined by a B_i-path are said to be B_i-connected.

In example (3.3), u_1 and u_2 are B_1-connected and u_0 and u_5 are B_3-connected.

Definition (3.10): A B_i-cycle is an alternating sequence of vertices and edges u_0,e_1,u_1,e_2,...,u_{n-1},e_n, $u_n = u_0$ consisting only of B_i -edges.

Example (3.11): Consider an IFGS $\tilde{G}$ = (A, B_1, B_2, B_3, B_4) such that V = { u_0, u_1, u_2, u_3, u_4, u_5 }. Let R_1 = { (u_0, u_1), (u_1, u_2), (u_2, u_3), (u_0, u_3)}, R_2 ={ (u_0, u_5)}, R_3 = {(u_1, u_3) , (u_3, u_4), (u_4, u_5)}, R_4 ={ (u_0, u_4)}. A = {< u_0, 0.7,0.2 >, < u_1, 0.6,0.3>, < u_2 , 0.4,0.3 >, <u_3, 0.3,0.4>, <u_4, 0.4,0.6 >, <u_5, 0.5,0.5 > }, B_1 = { (u_0, u_1), 0.5,0.3 > , < (u_1, u_2), 0.3,0.7>, < (u_2, u_3), 0.2,0.5>, < (u_0, u_3), 0.3,0.4 >}, B_2 = { < (u_0, u_5), 0.4,0.6 > }, B_3 = { < (u_1, u_3), 0.3,0.6 >, <(u_3, u_4), 0.2,0.7>, <(u_4, u_5), 0.1,0.9 >, B_4 = {< (u_0, u_4),0.4,0.6 >} are terms as defined in definition (3.1).



Here, (u_0,u_1), (u_1,u_2), (u_2,u_3), (u_3 ,u_0) is a B_1-cycle, but (u_1,u_3), (u_3,u_4), (u_4,u_5) is not a B_3-cycle.

Definition (3.12) : $\tilde{G}$ is an intuitionistic fuzzy B_i–cycle iff (supp (A), supp (B_1), supp (B_2), ... , supp (B_k)) is a R_i –cycle and there exists no unique (x,y) in supp(B_i) such that

$\mu_{B_i}(x,y) = \wedge\{\mu_{B_i}(u,v) \mid (u,v) \in supp(B_i)\}$ and $\nu_{B_i}(x,y) = \vee\{\nu_{B_i}(u,v) \mid (u,v) \in supp(B_i)\}$.

Definition (3.13): An IFGS $\tilde{G}$ is a B_i-forest if the subgraph structure induced by B_i -edges is a forest, i.e., if it has no B_i -cycles.

Definition (3.14): $\tilde{G}$ is a B_i-forest if its B_i–edges form a R_i-forest.
In Example (3.11), In $\tilde{G}$, there is no B_i-forest for i =1, 2, 3, 4.

Definition (3.15): $\tilde{G}$ is B_i-connected if any two vertices in V are joined by a B_i - path.
In Example (3.11), $\tilde{G}$ is B_1–connected, B_2–connected, B_3–connected, B_4–connected.

Definition (3.16): An IFGS $\tilde{G}$ is connected if every two vertices are joined by a path.
In Example (3.11), $\tilde{G}$ is connected IFGS.

Definition (3.17): The μ_{B_i}-strength of a B_i-path $u_0,u_1,\ldots,u_n$ of an IFGS $\tilde{G}$ is $\min\{\mu_{B_i}(u_{j-1},u_j) : j = 1,2,., n\}$. It is denoted by $S_{\mu_{B_i}}$.
In example (3.3), the μ_{B_i}-strength of a B_1-path u_1, u_0, u_2 is 0.1 and μ_{B_2}-strength of a B_2-path u_1,u_2, u_4 is 0.1.
Definition (3.18): The ν_{B_i}-strength of a B_i-path $u_0,u_1,\ldots,u_n$ of an IFGS $\tilde{G}$ is $\max\{\nu_{B_i}(u_{j-1},u_j) : j =1,2,\ldots, n\}$. It is denoted by $S_{\nu_{B_i}}$.
In example (3.3) the ν_{B_1}-strength of a B_1-path u_1,u_0,u_2 is 0.9 and ν_{B_2}-strength of a B_2-path u_1,u_2, u_4 is 0.9

Definition (3.19): The strength of a B_i-path u_0, $u_1,\ldots,u_n$ of an IFGS $\tilde{G}$ is

$$S_{B_i} = (S_{\mu_{B_i}}, S_{\nu_{B_i}}) = \left(\bigwedge_{j=1}^{n} \mu_{B_i}(u_{j-1},u_j), \bigvee_{j=1}^{n} \nu_{B_i}(u_{j-1},u_j)\right).$$

In example (3.3), the strength of the B_1-path u_1, u_0, u_2 is (0.1, 0.9) and strength of the B_2-path u_1, u_2, u_4 is (0.1, 0.9).

Remark (3.20): The strength of a path in an IFGS $\tilde{G}$ is denoted by S, therefore $S = \left(\bigwedge_{i=1}^{k} S_{\mu_{B_i}}, \bigvee_{i=1}^{k} S_{\nu_{B_i}}\right)$.

Definition (3.21): In any IFGS $\tilde{G}$, we define the following:
$\mu_{B_i}^{2}(u,v) = \mu_{B_i} \circ \mu_{B_i}(u,v) = \mathrm{Max}\{\mu_{B_i}(u,w) \wedge \mu_{B_i}(w,v) : w \in V\}$and
$\mu_{B_i}^{j}(u,v) = (\mu_{B_i}^{j-1} \circ \mu_{B_i})(u,v)$, j = 2,3,..., m for any $m \geq 2$.

Also $\mu_{B_i}^{\infty}(u,v) = \vee\{\mu_{B_i}^{j}(u,v) : j =1,2,\ldots\}$ i.e., $\mu_{B_i}^{\infty}(u,v) = \bigvee_{j=1}^{\infty} \mu_{B_i}^{j}(u,v)$.

Definition (3.22): In any IFGS $\tilde{G}$, $\nu_{B_i}^{2}(u,v) = \nu_{B_i} \circ \nu_{B_i}(u,v) = \mathrm{Min}\{\nu_{B_i}(u,w) \vee \nu_{B_i}(w,v)\}$and
$\nu_{B_i}^{j}(u,v) = (\nu_{B_i}^{j-1} \circ \nu_{B_i})(u,v)$, j = 2, 3,...,m for any $m \geq 2$.

Also $\nu_{B_i}^{\infty}(u,v) = \mathrm{Min}\{\nu_{B_i}^{j}(u,v): j=1,2,\ldots\}$, i.e., $\nu_{B_i}^{\infty}(u,v) = \bigwedge_{j=1}^{\infty} \nu_{B_i}^{j}(u,v)$.

Definition (3.23): $\tilde{G}$ is a B_i-tree when it is B_i–connected and has no B_i–cycle.
In example (3.11), $\tilde{G}$ is a B_3-tree. However $\tilde{G}$ is neither B_1-tree nor B_2-tree nor B_4-tree.

Definition (3.24): $\tilde{G}$ is an intuitionistic fuzzy B_i-tree if it has a partial intuitionistic fuzzy spanning sub-graph structure $\tilde{H}_i = (A,C_1,C_2,\ldots,C_k)$ which is a C_i-tree where for all B_i-edges not in $\tilde{H}_i$, $\mu_{B_i}(u,v) < \mu_{C_i}^{\infty}(u,v)$ and $\nu_{B_i}(u,v) > \nu_{C_i}^{\infty}(u,v)$.

Definition (3.25): $\tilde{G}$ is an intuitionistic fuzzy B_i-forest if it has a partial intuitionistic fuzzy spanning sub-graph structure $\tilde{H}_i$= (A, C_1, C_2,...,C_k) which is a C_i-forest where for all B_i-edges not in $\tilde{H}_i$, μ_{B_i}(u,v) < $\mu_{C_i}^{\infty}$(u,v) and ν_{B_i}(u,v) > $\nu_{C_i}^{\infty}$(u,v).

IV. Intuitionistic fuzzy B_i-trees and intuitionistic fuzzy B_i-forests

In this section, we discuss some properties of intuitionistic fuzzy B_i-trees and intuitionistic fuzzy B_i-forests.

Theorem (4.1): $\tilde{G}$ is an intuitionistic fuzzy B_i-forest if and only if in any B_i-cycle, there exists a B_i-edge (u,v) such that μ_{B_i}(u,v) < $\mu'^{\infty}_{B_i}$(u,v) and ν_{B_i}(u,v) > $\nu'^{\infty}_{B_i}$(u,v) where (A, B'_1, B'_2,....,B'_k) is the partial intuitionistic fuzzy spanning subgraph structure obtained by deleting (u, v) from $\tilde{G}$ and prime denotes the deletion of the edge (u,v).

Proof: Let (u,v) be an edge belonging to B_i-cycle such that μ_{B_i}(u,v)< $\mu'^{\infty}_{B_i}$(u,v) and ν_{B_i}(u,v) >$\nu'^{\infty}_{B_i}$(u,v) and for which μ_{B_i}(u,v) is the smallest and ν_{B_i}(u,v) is the largest.

If $\tilde{G}$ does not have any B_i-cycle, then it is an intuitionistic fuzzy B_i-forest, so the result is true.

Let $\tilde{G}$ has a B_i-cycle. Consider a B_i-edge (u,v) of that B_i–cycle such that μ_{B_i}(u,v) is the smallest among all B_i-edges of that B_i-cycle satisfying μ_{B_i}(u,v) < $\mu'^{\infty}_{B_i}$(u,v) and ν_{B_i}(u,v) is the largest among all B_i-edges of that B_i-cycle satisfying ν_{B_i}(u,v) > $\nu'^{\infty}_{B_i}$(u,v).

Now delete the B_i-edge (u,v). If (u,v) edge is deleted, the resulting partial intuitionistic fuzzy subgraph satisfies the path property of an intuitionistic fuzzy forest. If there are still B_i-cycles present, repeat the above process to remove them. Note that at this stage, no previously deleted edge is stronger than the edge being currently deleted. So the strength of deleted B_i-edges in a B_i-cycle increases in every step.

After removing all B_i-cycles, the resultant partial intuitionistic fuzzy spanning subgraph structure is a B_i-forest, say $\tilde{H}_i$. Let (u ,v) be not an edge of $\tilde{H}_I$,so there exists a B_i-path from u to v stronger than (u,v).
Even if some of its B_i-edges were deleted, there will be stronger B_i-paths for diverting around.
Repeating the process, we get a B_i-path consisting only of B_i-edges of $\tilde{H}_i$.
$\therefore$ $\tilde{G}$ is an intuitionistic fuzzy B_i-forest.

Conversely, let $\tilde{G}$ be an intuitionistic fuzzy B_i-forest.

Consider a B_i -cycle T_i of $\tilde{G}$. Some B_i -edge (u,v) of T_i is not in the partial intuitionistic fuzzy spanning subgraph structure $\tilde{H}_i$ = (A,C_1,C_2,...,C_k) which is a C_i-forest and
μ_{B_i}(u,v) < $\mu_{C_r}^{\infty}$(u,v) and ν_{B_i}(u,v) > $\nu_{C_r}^{\infty}$(u,v).
But $\mu_{C_r}^{\infty}$(u,v) < $\mu'^{\infty}_{B_i}$(u,v) and $\nu_{C_r}^{\infty}$(u,v) > $\nu'^{\infty}_{B_i}$ (u,v) where (A,B'_1, B'_2,...,B'_k) is the partial intuitionistic fuzzy spanning sub-graph structure obtained by deleting (u, v) from $\tilde{G}$ since (u, v) is not in $\tilde{H}_i$.
$\therefore$ μ_{B_i}(u,v) < $\mu'^{\infty}_{B_i}$(u,v) and ν_{B_i}(u,v) > $\nu'^{\infty}_{B_i}$(u,v).

Theorem (4.2): Let $\tilde{G}$ be an intuitionistic fuzzy B_i-tree and $\tilde{G}$* = (supp(A), supp(B_1), supp(B_2), ..., supp(B_k)) be not a R_i-tree. Then there exists atleast one R_i-edge (u,v) in supp(B_i) for which μ_{B_i}(u,v) < $\mu_{B_i}^{\infty}$(u,v) and ν_{B_i}(u,v) > $\nu_{B_i}^{\infty}$(u,v).

Proof: Let $\tilde{G}$ be an intuitionistic fuzzy B_i-tree, then there exists a partial intuitionistic fuzzy spanning subgraph structure $\tilde{H}_i$ = (A, C_1, C_2,...,C_k) which is a C_i-tree and μ_{B_i}(u,v) < $\mu_{C_i}^{\infty}$(u,v) and ν_{B_i}(u,v) > $\nu_{C_i}^{\infty}$(u,v) for all (u,v) not in $\tilde{H}_i$. Clearly $\mu_{C_i}^{\infty}$(u,v) ≤ $\mu_{B_i}^{\infty}$(u,v) and $\nu_{C_i}^{\infty}$ (u,v) ≥$\nu_{B_i}^{\infty}$(u,v).
$\therefore$ μ_{B_i}(u,v) < $\mu_{B_i}^{\infty}$(u,v) and ν_{B_i}(u,v) > $\nu_{B_i}^{\infty}$(u,v) ∀(u,v) not in $\tilde{H}_i$.
$\tilde{G}$ be an intuitionistic fuzzy B_i-tree and $\tilde{G}$* is not a R_i –tree.

Hence there exists at least one B_i-edge (u, v) not in $\tilde{H}_i$. i.e., there exists at least one R_i -edge (u,v) in supp(B_i) with $\mu_{B_i}(u,v) < \mu_{B_i}^{\infty}(u,v)$ and $\nu_{B_i}(u,v) > \nu_{B_i}^{\infty}(u,v)$.

Theorem (4.3): Let $\tilde{G}$ be an IFGS. If there is atmost one strongest B_i-path between any two vertices, then $\tilde{G}$ must be an intuitionistic fuzzy B_i-forest.

Proof: Suppose there exists at most one strongest B_i -path between any two vertices of $\tilde{G}$.

If possible, let $\tilde{G}$ be not an intuitionistic fuzzy B_i -forest. Then there exists a B_i -cycle, say P_i in $\tilde{G}$ such that $\mu_{B_i}(u,v) \geq \mu_{B_i}'^{\infty}(u,v)$ and $\nu_{B_i}(u,v) \geq \nu_{B_i}'^{\infty}(u,v)$ $\forall$ u,v in P_i where (A, B'_1, B'_2,...,B'_k) is the partial intuitionistic fuzzy spanning sub-graph structure obtained by the deletion of (u, v) by theorem 4.1. i.e., (u,v) is the strongest B_i -path from u to v.

The strength of a B_i -path is the strength of the weakest B_i -edge of that B_i -path.

$\therefore$ (u,v) cannot be a weakest B_i -edge of P_i since in that case the remaining B_i -edges of P_i form a strongest B_i-path which is a contradiction to our assumption.

$\therefore$ $\tilde{G}$ is an intuitionistic fuzzy B_i -forest.

Theorem (4.4): If $\tilde{G}*$ is a R_i-cycle. Then $\tilde{G}$ is an intuitionistic fuzzy B_i-cycle if and only if $\tilde{G}$ is not an intuitionistic fuzzy B_i-tree.

Proof: If possible, let $\tilde{G}$ be an intuitionistic fuzzy B_i-tree.

$\therefore$ $\tilde{G}$ has a partial intuitionistic fuzzy spanning subgraph structure $\tilde{H}_i$ = (A,C_1,C_2,...,C_k) which is a C_i-tree.

Since $\tilde{G}*$ is a R_i –cycle, therefore, supp(B_i)−supp(C_i) ={(u,v), for some u,v ∈ V}.

By definition of an intuitionistic fuzzy B_i -cycle, there does not exist unique B_i –edge (x,y) with $\mu_{B_i}(x,y) = \wedge\{ \mu_{B_i}(u,v) \mid (u,v) \in \text{supp}(B_i)\}$ and $\nu_{B_i}(x,y) = \vee\{ \nu_{B_i}(u,v) \mid (u,v) \in \text{supp}(B_i)\}$. So there exists no C_i -path in $\tilde{H}_i$ from u to v having greater strength than $\mu_{B_i}(u,v)$ otherwise, $\tilde{G}$ will not be an intuitionistic fuzzy B_i-cycle.

$\therefore$ by the definition of an intuitionistic fuzzy B_i -tree, $\tilde{G}$ is not an intuitionistic fuzzy B_i-tree.

Conversely, let $\tilde{G}$ be not an intuitionistic fuzzy B_i -tree. Then it has no partial intuitionistic fuzzy spanning subgraph structure $\tilde{H}_i$ which is a C_i-tree.

Let us assume that $\tilde{G}$ is an intuitionistic fuzzy B_i-cycle. Let (A,C_1, C_2,...,C_k) be a partial intuitionistic fuzzy spanning subgraph structure of $\tilde{G}$ which is a C_i-tree. Then

$\mu_{C_i}^{\infty}(u,v) \leq \mu_{B_i}(u,v)$ and $\nu_{C_i}^{\infty}(u,v) \leq \nu_{B_i}(u,v)$ $\forall$ (u,v) ∈ supp(B_i) and $\mu_{C_i}(u,v) = 0$ and $\nu_{C_i}(u,v) = 1$.

$\mu_{C_i}(x,y) = \mu_{B_i}(x,y)$ and $\nu_{C_i}(x,y) = \nu_{B_i}(x,y)$ $\forall$ (x,y) ∈ supp (B_i)−{(u,v)}.

Thus B_i does not attain $\wedge\{ \mu_{B_i}(x,y) \mid (x,y) \in \text{supp}(B_i)\}$ and $\vee\{ \nu_{B_i}(x,y) \mid (x,y) \in \text{supp}(B_i)\}$ uniquely.

Lemma (4.5): Let $\tilde{G}$ be an IFGS with $\mu_{B_i}(u,v) = \mu_A(u) \wedge \mu_A(v)$ and $\nu_{B_i}(u,v) = \nu_A(u) \vee \nu_A(v)$ for some i and $\forall$ (u,v) ∈ supp(B_i) where supp(B_i) ≠ φ. Then $\mu_{B_i}^{\infty}(u,v) = \mu_{B_i}(u,v)$ and $\nu_{B_i}^{\infty}(u,v) = \nu_{B_i}(u,v)$ for that i.

Proof: Let $\mu_{B_i}(u,v) = \mu_A(u) \wedge \mu_A(v)$ and $\nu_{B_i}(u,v) = \nu_A(u) \vee \nu_A(v)$ for some i and $\forall$(u,v) ∈ supp(B_i)

$$\mu_{B_i}^{\infty}(u,v) = \bigvee_{j=1}^{\infty} \mu_{B_i}^{j}(u,v) = \mu_A(u) \wedge \mu_A(v) = \mu_{B_i}(u,v) \qquad (\text{since } \mu_{B_i}^{j}(u,v) \leq \mu_A(u) \wedge \mu_A(v) \;\; \forall j)$$

and $$\nu_{B_i}^{\infty}(u,v) = \bigwedge_{j=1}^{\infty} \nu_{B_i}^{j}(u,v) = \nu_A(u) \vee \nu_A(v) = \nu_{B_i}(u,v) \qquad (\text{since } \nu_{B_i}^{j}(u,v) \leq \nu_A(u) \vee \nu_A(v) \;\; \forall j)$$

Using the above result, we can prove the following property of intuitionistic fuzzy B_i-tree.

Theorem (4.6): Let $\tilde{G}$ be an intuitionistic fuzzy B_i-tree. Then $\mu_{B_i}(u,v) < \mu_A(u) \wedge \mu_A(v)$ and

ν_{B_i} (u,v) < $\nu_A(u) \vee \nu_A(v)$ for some (u,v) in supp(B_i).

Proof: If possible, let μ_{B_i} (u,v) = $\mu_A(u) \wedge \mu_A(v)$ and ν_{B_i} (u,v) = $\nu_A(u) \vee \nu_A(v)$ for some i and $\forall$(u,v) ∈ supp(B_i) , then by Lemma (4.5), $\mu_{B_i}{}^{\infty}$(u,v) = μ_{B_i} (u,v) and $\nu_{B_i}{}^{\infty}$(u,v) = ν_{B_i} (u,v) -----(1)

As $\widetilde{G}$ be an intuitionistic fuzzy B_i-tree, ∴ $\widetilde{G}$ has a partial intuitionistic fuzzy spanning subgraph structure $\widetilde{H}_i$ = (A,C_1,C_2,..,C_k) which is a C_i-tree and μ_{B_i} (u,v) < $\mu_{C_i}{}^{\infty}$(u,v) and ν_{B_i} (u,v) < $\nu_{C_i}{}^{\infty}$ (u,v).

∴ $\mu_{B_i}{}^{\infty}$(u,v) < $\mu_{C_i}{}^{\infty}$(u,v) and $\nu_{B_i}{}^{\infty}$(u,v) < $\nu_{C_i}{}^{\infty}$(u,v) (using (1)) which is not possible.

Thus μ_{B_i} (u,v) < $\mu_A(u) \wedge \mu_A(v)$ and ν_{B_i} (u,v) < $\nu_A(u) \vee \nu_A(v)$ for some (u,v) in supp(B_i).

∴ $\widetilde{G}$ = (A, C_1, C_2,...,C_k) is an intuitionistic fuzzy B_i -cycle.

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