

## Geometrical Representation of Euler's Number

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**Abstract:** One of the most famous mathematical constants is the Euler's number  $e$ . It is a beautiful number and pops up everywhere in nature. It is the natural language of growths and changes. Thus, it has many different graphical representations. However, today I'm going to show you a geometrical representation of  $e$  to help you visualize it better.

**Keywords:** Another third and divide it into a fourth, infinity, remaining half into a third

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### I. Introduction

The famous Euler's constant  $e$  is one of the most important mathematical constants. Several graphical representations have been given to help us understand this beautiful number. However, there are few or none geometrical interpretations of this number. Hence, I have provided one geometrical interpretation for the number.

**The interpretation:**

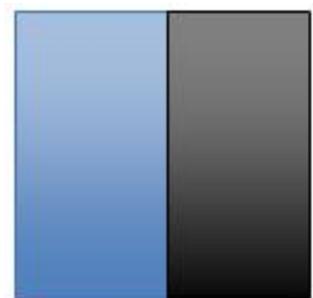
We start with Euler's formula for  $e$ :

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots \infty \quad \dots(1)$$

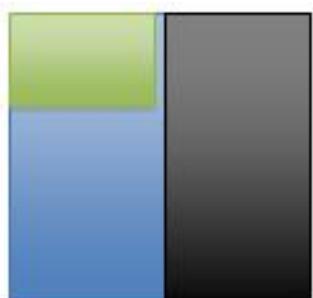
Now, we consider a square of side 1 unit.



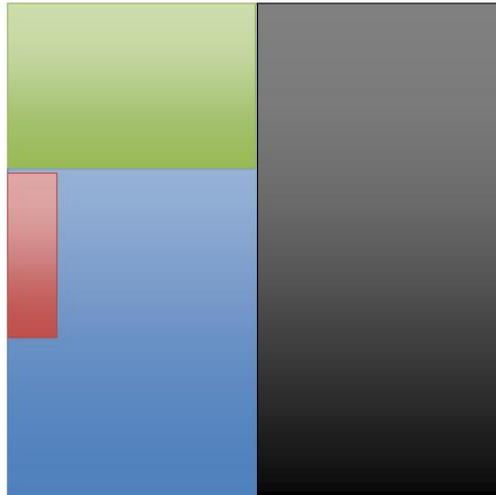
Now we divide this square into a half



Now we divide the remaining half into a third

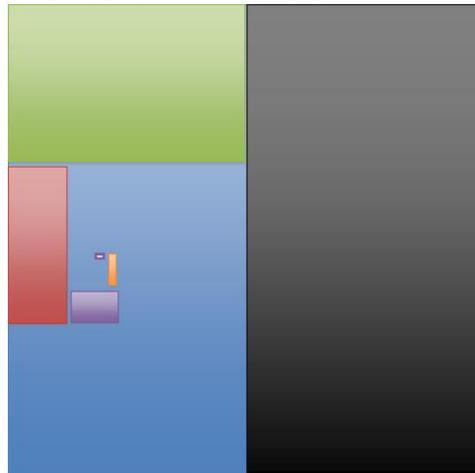


Now, we take another third and divide it into a fourth



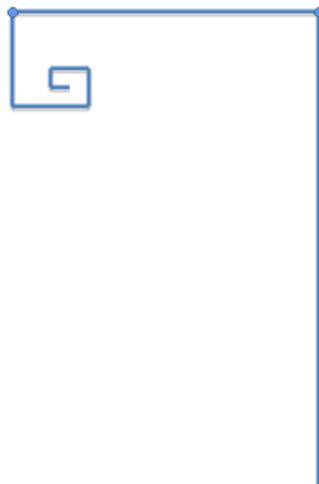
(NOTE: I have magnified the square)

You continue like this, dividing another fourth into a fifth, then another fifth into a sixth and so on to infinity. What you will get will resemble the figure given in the next page.



(I have shown six divisions)

However this is not the final thing. It only represents  $e$  from the third term onwards. For the first 2 terms we simply add 2 squares of unit area. Therefore the final structure looks like:



The extended line represents 2 extra squares. This is the geometrical representation of  $e$ .

**Explanation:** To understand why this works, let us look at the meaning of factorials. Factorial means the product of all natural numbers upto a given number. Therefore, when we divide a square into half, and then a half into a third and then another third into a fourth, we are multiplying a half, a third, and a fourth which gives us  $1/(4!)$ . Thus we are generating such terms to infinity as given by Euler's formula.

## **II. Conclusion**

This work is important because it gives us insight into an important number. If we look at a number in different dimensions, more properties of the number can come to light.