IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 13, Issue 1 Ver. I (Jan. - Feb. 2017), PP 23-25
www.iosrjournals.org

On Some New Linear Generating Relations Involving I-Function of Two Variables

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Abstract: *The aim of this research paper is to establish some linear generating relations involving I-function of two variables.*

I. Introduction

The I–function of two variables introduced by Sharma & Mishra [2], will be defined and represented as follows:

$$I\begin{bmatrix}x\\y\end{bmatrix} = I^{0,n:m_1,n_1:m_2,n_2}_{p_i,q_i:r:p_i',q_i':r':p_i'',q_i'':r''}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}[(a_j:\alpha_j,A_j)_{1,n}],[(a_{ji}:\alpha_{ji},A_{ji})_{n+1,p_i}]\\ [(b_{ji}:\beta_{ji},B_{ji})_{1,q_i}]\end{matrix}\right.$$

$$\left.\begin{matrix}:[(c_j;\gamma_j)_{1,n_1}],[(c_{ji'};\gamma_{ji'})_{n_1+1,p_{i'}}];[(e_j;E_j)_{1,n_2}],[(e_{ji''};E_{ji''})_{n_2+1,p_{i''}}]\\ :[(d_j;\delta_j)_{1,m_1}],[(d_{ji'};\delta\gamma_{ji'})_{m_1+1,q_{i'}}];[(f_j;F_j)_{1,m_2}],[(f_{ji''};F_{ji''})_{m_2+1,q_{i''}}]\end{matrix}\right]$$

$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta)\,x^{\xi}y^{\eta}\,d\xi d\eta, \qquad (1)$$

where

$$\phi_1(\xi,\eta) = \frac{\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\xi+A_j\eta)}{\sum_{i=1}^{r}[\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}\xi-A_{ji}\eta)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}\xi+B_{ji}\eta)},$$

$$\theta_2(\xi) = \frac{\prod_{j=1}^{m_1}\Gamma(d_j-\delta_j\xi)\prod_{j=1}^{n_1}\Gamma(1-c_j+\gamma_j\xi)}{\sum_{i'=1}^{r'}[\prod_{j=m_1+1}^{q_{i'}}\Gamma(1-d_{ji'}+\delta_{ji'}\xi)\prod_{j=n_1+1}^{p_{i'}}\Gamma(c_{ji'}-\gamma_{ji'}\xi)},$$

$$\theta_3(\eta) = \frac{\prod_{j=1}^{m_2}\Gamma(f_j-F_j\eta)\prod_{j=1}^{n_2}\Gamma(1-e_j+E_j\eta)}{\sum_{i''=1}^{r''}[\prod_{j=m_2+1}^{q_{i''}}\Gamma(1-f_{ji''}+F_{ji''}\eta)\prod_{j=n_2+1}^{p_{i''}}\Gamma(e_{ji''}-E_{ji''}\eta)},$$

x and y are not equal to zero, and an empty product is interpreted as unity p_i, $p_{i'}$, $p_{i''}$, q_i, $q_{i'}$, $q_{i''}$, n, n_1, n_2, n_j and m_k are non negative integers such that $p_i \geq n \geq 0$, $p_{i'} \geq n_1 \geq 0$, $p_{i''} \geq n_2 \geq 0$, $q_i > 0$, $q_{i'} \geq 0$, $q_{i''} \geq 0$, (i = 1, …, r; i′ = 1, …, r′; i′′ = 1, …, r′′; k = 1, 2) also all the A's, α's, B's, β's, γ's, δ's, E's and F's are assumed to be positive quantities for standardization purpose; the definition of I-function of two variables given above will however, have a meaning even if some of these quantities are zero. The contour L_1 is in the ξ–plane and runs from $-\omega\infty$ to $+\omega\infty$, with loops, if necessary, to ensure that the poles of $\Gamma(d_j-\delta_j\xi)$ (j = 1, ..., m_1) lie to the right, and the poles of $\Gamma(1-c_j+\gamma_j\xi)$ (j = 1, ..., n_1), $\Gamma(1-a_j+\alpha_j\xi+A_j\eta)$ (j = 1, ..., n) to the left of the contour.

The contour L_2 is in the η–plane and runs from $-\omega\infty$ to $+\omega\infty$, with loops, if necessary, to ensure that the poles of $\Gamma(f_j-F_j\eta)$ (j=1,....., n_2) lie to the right, and the poles of $\Gamma(1-e_j+E_j\eta)$ (j = 1, ..., m_2), $\Gamma(1-a_j+\alpha_j\xi+A_j\eta)$ (j = 1, ..., n) to the left of the contour. Also

$$R' = \sum_{j=1}^{p_i}\alpha_{ji} + \sum_{j=1}^{p_{i'}}\gamma_{ji'} - \sum_{j=1}^{q_i}\beta_{ji} - \sum_{j=1}^{q_{i'}}\delta_{ji'} < 0,$$

$$S' = \sum_{j=1}^{p_i}A_{ji} + \sum_{j=1}^{p_{i''}}E_{ji''} - \sum_{j=1}^{q_i}B_{ji} - \sum_{j=1}^{q_{i''}}F\delta_{ji'} < 0,$$

$$U = \sum_{j=n+1}^{p_i}\alpha_{ji} - \sum_{j=1}^{q_i}\beta_{ji} + \sum_{j=1}^{m_1}\delta_j - \sum_{j=m_1+1}^{q_{i'}}\delta_{ji'} + \sum_{j=1}^{n_1}\gamma_j - \sum_{j=n_1+1}^{p_{i'}}\gamma_{ji'} > 0, \qquad (2)$$

$$V = -\sum_{j=n+1}^{p_i}A_{ji} - \sum_{j=1}^{q_i}B_{ji} - \sum_{j=1}^{m_2}F_j - \sum_{j=m_{21}+1}^{q_{i''}}F_{ji''} + \sum_{j=1}^{n_2}E_j - \sum_{j=n_2+1}^{p_{i''}}E_{ji''} > 0, \qquad (3)$$

and $|\arg x| < ½\,U\pi$, $|\arg y| < ½\,V\pi$.

II. Linear Generating Relations

In this section we establish the following linear generating relations:

$$\sum_{l=0}^{\infty}\frac{t^l}{l!}\, I^{0,\,n\;:m_1,n_1\;:m_2,n_2}_{p_i,q_i:r\;:p_i'+1,q_i':r'\;:p_i'',q_i'':r''}\left[\begin{matrix}x\\ y\end{matrix}\middle|\begin{matrix}\ldots,\ldots:\ldots\ldots,(\lambda-l;\alpha):\ldots\ldots,\ldots\ldots\\ \ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots\end{matrix}\right]$$

$$= (1+t)^{(\lambda-1)}\, I^{0,\,n\;:m_1,n_1\;:m_2,n_2}_{p_i,q_i:r\;:p_i'+1,q_i':r'\;:p_i'',q_i'':r''}\left[\begin{matrix}x(1+t)^{\alpha}\\ y\end{matrix}\middle|\begin{matrix}\ldots,\ldots:\ldots\ldots,(\lambda;\alpha):\ldots\ldots,\ldots\ldots\\ \ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots\end{matrix}\right], \qquad (4)$$

$|\arg x| < \frac{1}{2}U\pi$, $|\arg y| < \frac{1}{2}V\pi$, where U and V is given in (2) and (3) respectively;

$$\sum_{l=0}^{\infty}\frac{t^l}{l!}\, I^{0,\,n\;:m_1,n_1+1\;:m_2,n_2}_{p_i,q_i:r\;:p_i'+1,q_i':r'\;:p_i'',q_i'':r''}\left[\begin{matrix}x\\ y\end{matrix}\middle|\begin{matrix}\ldots,\ldots:(-\lambda-l;\alpha),\ldots\ldots:\ldots\ldots,\ldots\ldots\\ \ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots\end{matrix}\right]$$

$$= (1-t)^{-(\lambda+1)}\, I^{0,\,n\;:m_1,n_1+1\;:m_2,n_2}_{p_i,q_i:r\;:p_i'+1,q_i':r'\;:p_i'',q_i'':r''}\left[\begin{matrix}x(1-t)^{-\alpha}\\ y\end{matrix}\middle|\begin{matrix}\ldots,\ldots:(-\lambda;\alpha),\ldots\ldots:\ldots\ldots,\ldots\ldots\\ \ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots\end{matrix}\right], \qquad (5)$$

$|\arg x| < \frac{1}{2}U\pi$, $|\arg y| < \frac{1}{2}V\pi$, where U and V is given in (2) and (3) respectively.

Proof:
To prove (4), consider

$$\Delta = \sum_{l=0}^{\infty}\frac{t^l}{l!}\, I^{0,\,n\;:m_1,n_1\;:m_2,n_2}_{p_i,q_i:r\;:p_i'+1,q_i':r'\;:p_i'',q_i'':r''}\left[\begin{matrix}x\\ y\end{matrix}\middle|\begin{matrix}\ldots,\ldots:\ldots\ldots,(\lambda-l;\alpha):\ldots\ldots,\ldots\ldots\\ \ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots\end{matrix}\right]$$

On expressing I-function in contour integral form as given in (1), we get

$$\Delta = \sum_{l=0}^{\infty}\frac{t^l}{l!}\Big[\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\,\theta_2(\xi)\theta_3(\eta)$$

$$\times\ \frac{1}{\Gamma\{\lambda-l-\alpha\xi\}}x^{\xi}y^{\eta}\,d\xi d\eta\Big]$$

$$= \sum_{l=0}^{\infty}\frac{(-t)^l}{l!}\Big[\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\,\theta_2(\xi)\theta_3(\eta)$$

$$\times\ \frac{\{1-\lambda-\alpha\xi\}_l}{\Gamma\{\lambda-\alpha\xi\}}x^{\xi}y^{\eta}\,d\xi d\eta\Big].$$

On changing the order of summation and integration, we have

$$\Delta = \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\,\theta_2(\xi)\theta_3(\eta)x^{\xi}y^{\eta}$$

$$\times\frac{1}{\Gamma\{\lambda-\alpha\xi\}}\times\Big[\sum_{l=0}^{\infty}\frac{(-t)^l}{l!}\{1-\lambda-\alpha\xi\}_l\Big]\,d\xi d\eta$$

$$= (1+t)^{\lambda-1}\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\,\theta_2(\xi)\theta_3(\eta)x^{\xi}y^{\eta}\,\frac{(1+t)^{-\alpha\xi}}{\Gamma\{\lambda-\alpha\xi\}}\,d\xi d\eta$$

which in view of (1), provides (4).

Proceeding on similar lines as above, the results (5) can be derived.

III. Particular Cases

I. On specializing the parameters in main formulae, we get following generating relations in terms of I-function of one variable, which are the results given by Khare [1, p.21-23, (2.1) and (2.2)]:

$$\sum_{l=0}^{\infty}\frac{t^l}{l!}\, I^{m,n}_{p_i+1,q_i:r}\left[z\Big|^{\ldots,(\lambda-l,\alpha)}_{\ldots,\ldots}\right]$$

$$= (1+t)^{(\lambda-1)} I^{m,n}_{p_i+1,q_i:r}\left[z(1+t)^{-\alpha}\Big|^{\ldots,(\lambda,\alpha)}_{\ldots,\ldots}\right], \qquad (6)$$

provided that |argz| < ½ Bπ, where B is given by $B=\sum_{j=1}^{n}\alpha_j-\sum_{j=n+1}^{p_i}\alpha_{ji}+\sum_{j=1}^{m}\beta_j-\sum_{j=m+1}^{q_i}\beta_{ji}>0$;

$$\sum_{l=0}^{\infty}\frac{t^l}{l!}\, I^{m,n+1}_{p_i+1,q_i:r}\left[z\Big|^{(-\lambda-l,\alpha),\ldots}_{\ldots\ldots,}\right]$$

$$= (1-t)^{-(1+\lambda)} I^{m,n+1}_{p_i+1,q_i:r}\left[z(1-t)^{-\alpha}\Big|^{(-\lambda,\alpha),\ldots}_{\ldots\ldots,}\right], \qquad (7)$$

provided that |argz| < ½ Bπ, where B is given by $B=\sum_{j=1}^{n}\alpha_j-\sum_{j=n+1}^{p_i}\alpha_{ji}+\sum_{j=1}^{m}\beta_j-\sum_{j=m+1}^{q_i}\beta_{ji}>0$.

II. On choosing r = 1 in (6) and (7), we get following generating relations in terms of H-function of one variable, which are the results given by Shrivastava & Shrivastava [3, p.65, (2.1) and (2.2)]:

$$\sum_{l=0}^{\infty}\frac{(t)^l}{l!}\, H^{m,n}_{p+1,q}\left[z\Big|^{(a_j,\alpha_j)_{1,p},(\lambda-l,\alpha)}_{(b_j,\beta_j)_{1,q}}\right]$$

$$= (1+t)^{(\lambda-1)} H^{m,n}_{p+1,q}\left[z(1+t)^{-\alpha}\Big|^{(a_j,\alpha_j)_{1,p},(\lambda,\alpha)}_{(b_j,\beta_j)_{1,q}}\right], \qquad (8)$$

provided that |argz| < ½ Aπ, where A is given by $A=\sum_{j=1}^{n}\alpha_j-\sum_{j=n+1}^{p}\alpha_j+\sum_{j=1}^{m}\beta_j-\sum_{j=m+1}^{q}\beta_j>0$;

$$\sum_{l=0}^{\infty}\frac{(t)^l}{l!}\, H^{m,n+1}_{p+1,q}\left[z\Big|^{(-\lambda-l,\alpha),(a_j,\alpha_j)_{1,p}}_{(b_j,\beta_j)_{1,q}}\right]$$

$$= (1-t)^{-(1+\lambda)} H^{m,n+1}_{p+1,q}\left[z(1-t)^{-\alpha}\Big|^{(-\lambda,\alpha),(a_j,\alpha_j)_{1,p}}_{(b_j,\beta_j)_{1,q}}\right], \qquad (9)$$

provided that |argz| < ½ Aπ, where A is given by $A=\sum_{j=1}^{n}\alpha_j-\sum_{j=n+1}^{p}\alpha_j+\sum_{j=1}^{m}\beta_j-\sum_{j=m+1}^{q}\beta_j>0$.

References

[1]. Khare, Sandeep: An Advanced Study of I-Function of one Variable with Some Applications, Ph.D. Thesis, A.P.S. University, Rewa (M.P.), 2009.

[2]. Sharma C. K. and Mishra, P. L.: On the I-function of two variables and its certain properties, ACI, 17 (1991), 1-4.

[3]. Shrivastava, Shweta and Shrivastava, B. M. L.: Some new generating relations and identities for H-function, Vijnana Parishad Anusandhan Patrika, Vol. 49, No.1, January, 2006, p. 63-77.