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Time to Recruitment for a Single Grade Manpower System with Two Thresholds, Different Epochs for Inter-Decisions and Exits Having Correlated Wastages

G. Ravichandran[1]; A. Srinivasan[2]
[1] Assistant Professor in Mathematics, TRP Engineering College (SRM GROUP), Irungalur, Trichy-621 105, Tamil Nadu, India.
[2] Professor Emeritus, PG & Research Department of Mathematics, Bishop Heber College, Trichy-620 017, Tamil Nadu, India.
Corresponding Author: G. Ravichandran

Abstract: *Attrition takes place whenever a marketing organization takes policy decisions such as revision of targets and pay structure. It is not wise to make recruitment frequently due to cost factor. Since decision and its consequences are probabilistic in nature, a well planned recruitment policy is the need of the hour. The objective of the present paper is to study this problem of time to recruitment for a single grade manpower system using an univariate recruitment policy based on shock model approach. In this paper the variance of time to recruitment is obtained when (i) the system has optional and mandatory thresholds for cumulative loss in manpower (ii) wastage due to attrition form a sequence of exchangeable and constantly correlated exponential random variables (iii) the inter- exit times are independent and identically distributed exponential random variables (iv) the inter-policy decision times form either a geometric process or an order statistics. A numerical study of the analytical results for the performance measures shows that mean and variance of time to recruitment either increase or decrease according as the average loss of manpower increases or the average inter – decision times decreases.*

Keywords*: Single grade manpower system; different decision and exit epochs; correlated wastages; geometric process; order statistics; univariate policy of recruitment involving two thresholds and variance of time to recruitment.*

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I. Introduction

Attrition is a common phenomenon in many organizations. This leads to the depletion of manpower. Recruitment on every occasion of depletion of manpower is not advisable since every recruitment involves cost. Hence the cumulative depletion of manpower is permitted till it reaches a level, called the threshold. If the total loss of manpower exceeds this threshold, the activities in the organization will be affected and hence recruitment becomes necessary. In [1, 2, 3] the authors have discussed the manpower planning models by Markovian and renewal theoretic approach. In [4] the author has studied the problem of time to recruitment for a single grade manpower system when the loss of manpower forms a sequence of exchangeable and constantly correlated exponentially distributed random variables. In [5] the author has initiated the study of the problem of time to recruitment for a single grade manpower system by incorporating alertness in the event of cumulative loss of manpower due to attrition crossing the threshold, by considering optional and mandatory thresholds for the cumulative loss of manpower in this manpower system. In all the above cited work, it is assumed that attrition takes place instantaneously at decision epochs. This assumption is not realistic as the actual attrition will take place only at exit points which may or may not coincide with decision points. This aspect is taken into account for the first time in [6] and the variance of time to recruitment is obtained when the inter-decision times and exit times are independent and identically distributed exponential random variables using univariate policy for recruitment and Laplace transform in the analysis. In [8, 9, 10] the authors have studied the problem when inter-decision times form a sequence of exchangeable and constantly correlated exponential random variables, a geometric process and an order statistics respectively. Recently, in [11, 12, 13, 14] the authors have studied the work in [6, 8, 9, 10] by considering optional and mandatory thresholds which is a variation from the work of [5] in the context of considering non-instantaneous exits at decision epochs. In the present paper, for a single grade manpower system, a mathematical model is constructed in which attrition due to policy decisions take place at exit points and there are optional and mandatory thresholds as control limits for the cumulative loss of manpower. A univariate policy of recruitment based on shock model approach is used to determine the variance of time to recruitment when the system has different epochs for policy decisions and exits. The present paper extends the research work in [11, 13, 14] when loss of manpower forms a sequence of exchangeable and constantly correlated exponentially distributed random variables.

II. Model Description

Consider an organization taking decisions at random epochs in (0, ∞) and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. It is assumed that the loss of manpower is linear and cumulative. Let X_i be the continuous random variable representing the amount of depletion of manpower (loss of man hours) caused at the i^{th} exit point and S_k be the total loss of manpower up to the first k exit points. It is assumed that X_i's are exchangeable and constantly correlated exponential random variables with mean $\frac{1}{\alpha}$(α>0). Let $M_k(.)$ be the distribution of S_k, k=1,2,3,….Let ρ be the correlation between X_i and X_j where $i \neq j$ and $\phi(n,x) = \int_0^x e^{-z} z^{n-1} dz$ .Let $b = a(1-\rho)$ where a is the mean of X_i ,i =1,2,3,… Let U_k be the continuous random variable representing the time between the (k-1)th and kth policy decisions. Let W_i be the continuous random variable representing the time between the (i-1)th and ith exit times. It is assumed that W_i's are independent and identically distributed exponential random variables with probability density function g(.), probability distribution function G(.) and mean $\frac{1}{\delta}, (\delta > 0)$. Let $N_e(t)$ be the number of exit points lying in (0,t]. Let Y be the optional threshold level for the cumulative depletion of manpower in the organization with probability density function h_1(.), following an exponential distribution with mean $\frac{1}{\theta_1}, (\theta_1 > 0)$. Let Z (Y < Z) be the mandatory threshold level for the cumulative depletion of manpower in the organization with probability density function h_2(.), following an exponential distribution with mean $\frac{1}{\theta_2}, (\theta_2 > 0)$ .Let p be the probability that the organization is not going for recruitment when optional threshold is exceeded by the cumulative loss of manpower. Let q be the probability that every policy decision has exit of personnel. As q = 0 corresponds to the case where exits are impossible, it is assumed that $q \neq$ 0. Let T be the random variable denoting the time to recruitment with probability distribution function L(.), density function $l(.)$, mean E(T) and variance V(T). Let $\bar{a}(.)$ be the Laplace transform of a(.). The univariate CUM policy of recruitment employed in this paper is stated as follows:

Recruitment is done whenever the cumulative loss of manpower in the organization exceeds the mandatory threshold. The organization may or may not go for recruitment if the cumulative loss of manpower exceeds the optional threshold.

III. Main Result

From the recruitment policy, we note that

$$P(T > t) = \sum_{k=0}^{\infty} P[N_e(t)=k] P(S_k \le Y) + p \sum_{k=0}^{\infty} P[N_e(t)=k] P(S_k > Y) P(S_k \le Z) \qquad (1)$$

By the law of total probability, we have

$$P(S_k \le Y) = \int_0^{\infty} P[S_k \le Y / Y \le y] dP[Y \le y] \qquad (2)$$

Since Y and S_k are independent with probability density function $h_1(.)$ and distribution $M_k(.)$ respectively, from (2) we get

$$P(S_k \le Y) = \int_0^{\infty} M_k(y) h_1(y) dy \qquad (3)$$

Using the result of Gurland [4], it can be shown that

$$M_k(y) = (1-\rho)\sum_{i=0}^{\infty} \frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}} \frac{\phi(k+i,\frac{y}{b})}{(k+i-1)!}, \text{ where } \phi(k+i,\frac{y}{b}) = \int_0^{\frac{y}{b}} e^{-z} z^{k+i-1} dz \quad (4)$$

Using (4) in (3), we get

$$P(S_k \le Y) = (1-\rho)A_k, \text{ where } A_k = \frac{(b\theta_1+1)}{(b\theta_1+1)^k[(1-\rho+k\rho)(b\theta_1+1)-k\rho]} \quad (5)$$

Similarly

$$P(S_k \le Z) = (1-\rho)B_k, \text{ where } B_k = \frac{(b\theta_2+1)}{(b\theta_2+1)^k[(1-\rho+k\rho)(b\theta_2+1)-k\rho]} \quad (6)$$

From Renewal theory [7], we have $P[N_e(t)=k] = G_k(t) - G_{k+1}(t)$ and $G_0(t) = 1$ (7)

Using (5), (6) and (7) in (1), we have

$$P(T > t) = (1-\rho)\sum_{k=0}^{\infty} [G_k(t)-G_{k+1}(t)]A_k + p\sum_{k=0}^{\infty}[G_k(t)-G_{k+1}(t)][(1-\rho)B_k - (1-\rho)^2 A_k B_k] \quad (8)$$

$$L(t) = 1 - \{ (1-\rho)\sum_{k=0}^{\infty} [G_k(t)-G_{k+1}(t)]A_k + p\sum_{k=0}^{\infty}[G_k(t)-G_{k+1}(t)][(1-\rho)B_k - (1-\rho)^2 A_k B_k] \} \quad (9)$$

Taking Laplace transform, we get

$$\bar{l}(s) = 1-(1-\rho)[\sum_{k=0}^{\infty}\overline{g_k}(s)A_k - \sum_{k=0}^{\infty}\overline{g_{k+1}}(s)A_k] - p(1-\rho)[\sum_{k=0}^{\infty}\overline{g_k}(s)B_k - \sum_{k=0}^{\infty}\overline{g_{k+1}}(s)B_k]$$
$$+ p(1-\rho)^2[\sum_{k=0}^{\infty}\overline{g_k}(s)A_kB_k - \sum_{k=0}^{\infty}\overline{g_{k+1}}(s)A_kB_k] \quad (10)$$

$$\bar{l}(s) = 1-(1-\rho)[\sum_{k=0}^{\infty}\bar{g}(s)^k A_k - \sum_{k=0}^{\infty}\bar{g}(s)^{k+1}A_k] - p(1-\rho)[\sum_{k=0}^{\infty}\bar{g}(s)^k B_k - \sum_{k=0}^{\infty}\bar{g}(s)^{k+1}B_k]$$
$$+ p(1-\rho)^2[\sum_{k=0}^{\infty}\bar{g}(s)^k A_kB_k - \sum_{k=0}^{\infty}\bar{g}(s)^{k+1}A_kB_k] \quad (11)$$

$$\bar{l}(s) = 1-(1-\rho)[1-\bar{g}(s)]\sum_{k=0}^{\infty}\bar{g}(s)^k A_k - p(1-\rho)[1-\bar{g}(s)]\sum_{k=0}^{\infty}\bar{g}(s)^k B_k + p(1-\rho)^2[1-\bar{g}(s)]\sum_{k=0}^{\infty}\bar{g}(s)^k A_kB_k \quad (12)$$

It is known that $E(T^r) = (-1)^r[\frac{d^r}{ds^r}\bar{l}(s)]_{s=0}, \quad r = 1,2,3,...$ (13)

From (12) and (13), we get

$$E(T) = -\bar{g}'(0)(1-\rho)\left\{\sum_{k=0}^{\infty}A_k + p\sum_{k=0}^{\infty}B_k - p(1-\rho)\sum_{k=0}^{\infty}A_kB_k\right\} \quad (14)$$

$$E(T^2) = (1-\rho)\left\{2[\bar{g}'(0)]^2\left[\sum_{k=0}^{\infty}kA_k + p\sum_{k=0}^{\infty}kB_k - p(1-\rho)\sum_{k=0}^{\infty}kA_kB_k\right] + \bar{g}''(0)\left[\sum_{k=0}^{\infty}A_k + p\sum_{k=0}^{\infty}B_k - p(1-\rho)\sum_{k=0}^{\infty}A_kB_k\right]\right\} \quad (15)$$

Variance of time to recruitment can be computed from (14) and (15).

We now determine variance of time to recruitment for two different cases on inter-policy decision times.

Case(i): $\{U_k\}_{k=1}^{\infty}$ form a geometric process with rate $c, (c > 0)$. The distribution F(.) of U_1 is $F(t) = 1 - e^{-\lambda t}, \ \lambda > 0$.

From (14) and (15), we get

$$E(T) = -\bar{g}'(0)(1-\rho)\left\{\sum_{k=0}^{\infty}A_k + p\sum_{k=0}^{\infty}B_k - p(1-\rho)\sum_{k=0}^{\infty}A_kB_k\right\} \quad (16)$$

and

$$E(T^2) = (1-\rho)\left\{2[\bar{g}'(0)]^2\left[\sum_{k=0}^{\infty}kA_k + p\sum_{k=0}^{\infty}kB_k - p(1-\rho)\sum_{k=0}^{\infty}kA_kB_k\right] + \bar{g}''(0)\left[\sum_{k=0}^{\infty}A_k + p\sum_{k=0}^{\infty}B_k - p(1-\rho)\sum_{k=0}^{\infty}A_kB_k\right]\right\} \quad (17)$$

where $\overline{g}'(0) = \frac{c}{(c-1+q)}\overline{f}'(0)$, $\overline{g}''(0) = \frac{c^2}{(c^2-1+q)}\overline{f}''(0) + \frac{2c^2(1-q)}{(c^2-1+q)(c-1+q)}(\overline{f}'(0))^2$,

$$\overline{f}'(0) = -\frac{1}{\lambda} \text{ and } \overline{f}''(0) = \frac{2}{\lambda^2}. \tag{18}$$

Variance of time to recruitment for case (i) can be computed from (16), (17) and (18).

Case(ii): $\{U_k\}_{k=1}^{\infty}$ form an order statistics where the sample of size r associated with this order statistics is selected from an exponential population of independent and identically distributed inter-policy decision times, where the common distribution F(.) is given as in case(i).

Let $F_{u(j)}(.)$ and $f_{u(j)}(.)$ be the distribution and the probability density function of the j^{th} order statistics selected from the sample of size r from the exponential population $\{U_k\}_{k=1}^{\infty}$. From the theory of order statistics [14], it is known that

$$f_{u(j)}(t) = j\binom{r}{j}[F(t)]^{j-1} f(t)[1-F(t)]^{r-j},\ j = 1,2,..,r. \tag{19}$$

Suppose $f(t) = f_{u(1)}(t)$.

From (14), (15) and (19), we get

$$E(T) = -\overline{g}'(0)(1-\rho)\left\{\sum_{k=0}^{\infty} A_k + p\sum_{k=0}^{\infty} B_k - p(1-\rho)\sum_{k=0}^{\infty} A_k B_k\right\} \tag{20}$$

and

$$E(T^2) = (1-\rho)\left\{2[\overline{g}'(0)]^2\left[\sum_{k=0}^{\infty} kA_k + p\sum_{k=0}^{\infty} kB_k - p(1-\rho)\sum_{k=0}^{\infty} kA_k B_k\right] + \overline{g}''(0)\left[\sum_{k=0}^{\infty} A_k + p\sum_{k=0}^{\infty} B_k - p(1-\rho)\sum_{k=0}^{\infty} A_k B_k\right]\right\} \tag{21}$$

$$\text{where } \overline{g}'(0) = \frac{-1}{qr\lambda} \text{ and } \overline{g}''(0) = \frac{2}{(qr\lambda)^2} \tag{22}$$

Variance of time to recruitment when $f(t) = f_{u(1)}(t)$ can be computed from (20), (21) and (22).

Suppose $f(t) = f_{u(r)}(t)$.

From (14), (15) and (19), we get

$$E(T) = -\overline{g}'(0)(1-\rho)\left\{\sum_{k=0}^{\infty} A_k + p\sum_{k=0}^{\infty} B_k - p(1-\rho)\sum_{k=0}^{\infty} A_k B_k\right\} \tag{23}$$

and

$$E(T^2) = (1-\rho)\left\{2[\overline{g}'(0)]^2\left[\sum_{k=0}^{\infty} kA_k + p\sum_{k=0}^{\infty} kB_k - p(1-\rho)\sum_{k=0}^{\infty} kA_k B_k\right] + \overline{g}''(0)\left[\sum_{k=0}^{\infty} A_k + p\sum_{k=0}^{\infty} B_k - p(1-\rho)\sum_{k=0}^{\infty} A_k B_k\right]\right\} \tag{24}$$

$$\text{where } \overline{g}'(0) = \frac{-1}{q\lambda}\sum_{j=1}^{r}\frac{1}{j} \tag{25}$$

and

$$\overline{g}''(0) = \frac{-1}{(q\lambda)^2}\left\{(-2+q)\left[\sum_{j=1}^{r}\frac{1}{j}\right]^2 - q\sum_{j=1}^{r}\frac{1}{j^2}\right\} \tag{26}$$

Variance of time to recruitment when $f(t) = f_{u(r)}(t)$ can be computed from (23), (24), (25) and (26).

Remark:

Variance on time to recruitment for the case when $\{U_k\}_{k=1}^{\infty}$ is a sequence of independent and identically distributed exponential random variables can be obtained by taking $c = 1$ in case (i).

Note:

(i) When $p = 0$, our results for cases (i) and (ii) agree with the results in [9, 10] for the manpower system having only one threshold which is the mandatory threshold.

(ii) When $p = 0$ and $c = 1$, our results in case (i) agree with the results in [6] for the manpower system having only one threshold which is the mandatory threshold.

(iii) When $\rho = 0$ and $c = 1$, our results for case (i) agree with the results in [11] for the manpower system with two thresholds having loss of manpower as independent and identically distributed exponential random variables.

IV. Findings

From the above results, the following observations are presented which agree with reality.

1. When α increases and keeping all the other parameters fixed, the average loss of manpower increases. Therefore the mean and variance of time to recruitment increase.
2. As λ increases, on the average, the inter-decision time decreases and consequently the mean and variance of time to recruitment decrease when the other parameters are fixed.
3. The mean and variance of the time to recruitment decrease or increase according as *c>1* or *c<1*, since the geometric process of inter-policy decision times is stochastically decreasing when *c>1* and increasing when *c<1*.

V. Conclusion

The present work contributes to the existing literature in the sense that the models discussed in this paper are new in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs (ii) associating a probability for any decision to have exit points and (iii) provision of optional and mandatory thresholds. From the organization's point of view, our models are more suitable than the corresponding models with instantaneous attrition at decision epochs, as the provision of exit points at which attrition actually takes place, postpone the time to recruitment. It is proposed to study the present work when the loss of manpower process is a geometric process and the system has a back up resource.

References

[1] D. J. Bartholomew, Stochastic model for social processes, (New York, John Wiley and Sons, 1973).

[2] D. J. Bartholomew, F. Andrew Forbes, Statistical techniques for manpower planning, (New York, John Wiley and Sons, 1979).

[3] R. C. Grinold, K. T. Marshall, Manpower planning models, (New York, North-Holland, 1977).

[4] J. Gurland, Distribution of the maximum of arithmetic mean of correlated random variables, Ann. Mathematical Statistics, 26, 1995, 294-300.

[5] J. B. Esther Clara, Contributions to the study on some stochastic models in manpower planning, doctoral diss., Bharathidasan University, Tiruchirappalli, 2012.

[6] A. Devi, A. Srinivasan, Variance of time to recruitment for single grade manpower system with different epochs for decisions and exits, International Journal of Research in Mathematics and Computations, 2(1), 2014, 23-27.

[7] J. Medhi, Stochastic processes, (Wiley Eastern, second edition, New Delhi, 1994).

[8] A. Devi, A. Srinivasan, Expected time to recruitment in a single grade manpower system with different epochs for decisions and exits having correlated inter-decision times using univariate policy of recruitment, Asian Academic Research Journal of Multidisciplinary, 1(23), 2014, 571-580.

[9] A. Devi, A. Srinivasan, A stochastic model for time to recruitment in a single grade manpower system with different epochs for decisions and exits having inter- decision times as geometric process, Second International Conference on Business Analytic and Intelligence, (ICBAI), 2014.

[10] A. Devi, A. Srinivasan, Expected time to recruitment for a single grade manpower system with different epochs for decisions and exits with inter- decision times as an order statistics, International Journal of Innovative Research in Science, Engineering and Technology, 4(8), 2015, 7517-7520.

[11] G. Ravichandran, A. Srinivasan, Variance of time to recruitment for a single grade manpower system with two thresholds having different epochs for decisions and exits, Indian Journal of Applied Research, 5(1), 2015, 60-64.

[12] G. Ravichandran, A. Srinivasan, Time to recruitment for a single grade manpower system with two thresholds, different epochs for exits and correlated inter-decisions, International Research Journal of Natural and Applied Sciences, 2(2), 2015, 129-138.

[13] G. Ravichandran, A. Srinivasan, Time to recruitment for a single grade manpower system with two thresholds, different epochs for exits and geometric inter-decisions, IOSR Journal of Mathematics, 11(2), Ver. III, 2015, 29-32.

[14] G. Ravichandran, A. Srinivasan, Time to recruitment for a single grade manpower system with two thresholds, different epochs for inter-decisions as an order statistics and exits, International Journal Of Engineering And Science, 5(6), 2015, 24-28.

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