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# Solution of the Non-Linear Third Order Partial Differential Equation of a Steady Hydromagnetic Flow through a Channel with Parallel Stationary Porous Plates

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***Abstract:*** *This paper seeks to find the solution of the non-linear third order partial differential equation of a steady hydromagnetic laminar flow of a conducting viscous incompressible fluid through a channel with two parallel porous plates. The two plates are stationary and there is magnetic field moving at right angle to the electric field. Due to the porous nature of the plates, the fluid is withdrawn through both walls of the channel at the same rate. The specific equations governing the flow are discussed, transformed using non-dimensionalization techniques into a third order partial differential equation, simplified using Taylor's series expansion and solved by the method of regular perturbation. Expressions for the velocity components are discussed and represented in form of graphs plotted by use of MATLAB programming application. The velocity profiles parallel (axial) and normal (radial) to the plates are investigated. The results indicates that the radial velocity decreases with increase in Reynolds number while the axial velocity is zero at the plates and increases to the maximum at the centre line depicting the normal free flow velocity of the stream when there is no magnetic field in the fluid flow. The axial velocity of the fluid decreases with increase in Hartmann number. The study has its application in hydromagnetic devices where the interaction between velocities profiles, magnetic and electric fields are utilized in the design of various machines, for instance removal of pollutants from plant discharge stream by absorption.*

***Keywords****: Porous medium, hydromagnetic flow, Regular perturbation, axial and radial velocity*

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## I. Introduction

A steady hydromagnetic flow which involves the interaction of electrically conducting fluid and moves past a magnetic field, exerts a force on the fluid due to the induced currents thus affects the original magnetic field and the velocity of the fluid flow. The forces generated in this way are of the same order of magnitude as the hydrodynamic forces and are taken into account when considering the fluid flow. Kearsley (1994) studied the problem of steady state couette flow with viscous heating and found an exact solution for non linear problem with thermal mechanical coupling. Das et al [2012] analyzed a three dimensional couette flow of a viscous incompressible electrically conducting fluid between two infinite horizontal parallel porous flat plates in the presence of a transverse magnetic field. They solved the governing equations by using the series expansion method and the expressions for the velocity field, temperature field, skin friction and heat flux in terms of Nusselt number were obtained. They found that magnetic parameter retards the main fluid velocity and accelerates radial velocity of the flow field. Bhargava and Takhar [2001] studied the numerical solution of free convection MHD micropolar fluid between two parallel porous vertical plates. The profiles for velocity, microrotation and temperature were presented for a wide range of Hartmann numbers and micropolar parameter. Israel-Cookey and Nwaigwe (2010) considered unsteady MHD flow of a rotating fluid over a vertical moving heated porous plate with time-dependent suction. In their study closed form analytical solutions were constructed for the problem, the results were discussed quantitatively with the aid of dimensionless parameters. Manyonge at el (2012) examined the motion of a two dimensional steady flow of a viscous, electrically conducting incompressible fluid flowing between two infinite parallel plates where the lower plate was porous and upper not. The parallel plates were under the influence of transverse magnetic field and constant pressure gradient. The resulting differential equations were solved using analytical method and solutions expressed in terms of Hartmann number and the effects of magnetic inclinations to velocity were discussed graphically. In this paper, we shall discuss the solution of the third order partial differential equation of an incompressible flow by the method of regular perturbation and the power series expansion. The solution to this problem has many applications in MHD power generators, electrostatic precipitation for air purification, oil reservoir engineering, lubrication of porous bearings, porous walled flow reactors and in polymer technology among others.

**1.1 Formulation of the problem**

We consider the steady laminar flow of an incompressible viscous conducting fluid with a small electrical conductivity between the two parallel stationary non conducting porous plates in the presence of a uniform transverse magnetic field, $B$. Both the two porous plates are taken to have equal porosity. We choose a Cartesian coordinate system $(x, y, z)$ where the $x$ and $y$ are parallel and perpendicular to the channel plates respectively and the origin is taken at the centre of the channel. We assume the length of the plates to be L and 2h is the distance between the two parallel plates. The two plates are of infinite length in $z$-direction, therefore all the physical quantities involved are independent of $z$ thus the problem is a two-dimensional. The upper and the lower plates are subjected to a constant suction, $V(>0)$. Denoting $u$,$v$ and $w$ to be the component of velocity in the directions of $x$,$y$ and $z$ increasing respectively as shown in figure 1 below.

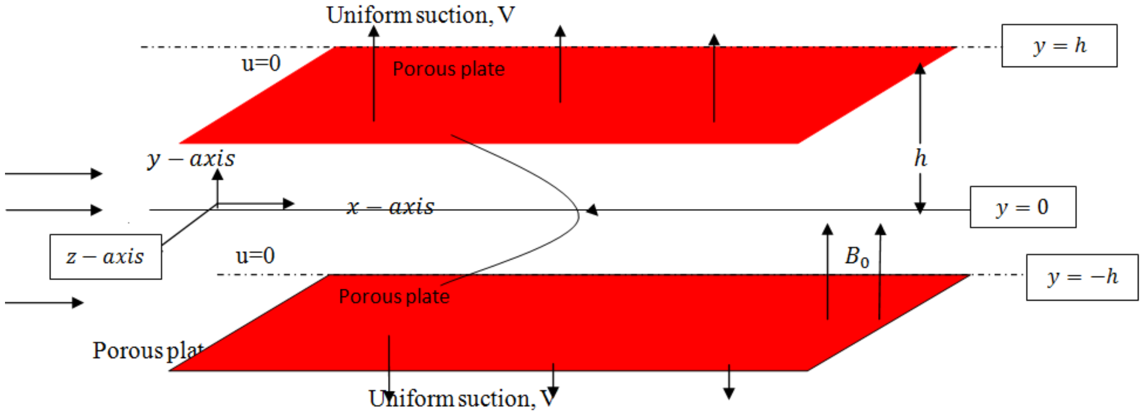



**Figure1**. The physical configuration of the problem

**Governing equations**

The general equations for hydromagnetic fluid flows are:

**1 Equation of continuity**

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \quad \dots \quad (1)$$

Where $\rho$ is the density of the fluid, since we are considering an incompressible flow, $\rho$ is a constant then equation (1) becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \dots \quad (2)$$

## II. 2 Momentum Equation

It is derived from Newton's second law of motion which requires that the sum of all forces acting on the control volume be equal to the rate of increase of the fluid momentum within the control volume.

Thus from Navier-Stokes equations we have

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = F_x + \frac{\partial}{\partial x}\delta_{xx} + \frac{\partial}{\partial y}\tau_{yx} \dots \quad (3)$$

and

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = F_y + \frac{\partial}{\partial y}\delta_{yy} + \frac{\partial}{\partial x}\tau_{xy} \dots \quad (4)$$

The stresses are related to the velocity components in the form (Mohanty, 2006)

$$\delta_{xx} = -p + 2\mu\frac{\partial u}{\partial x}, \delta_{yy} = -p + 2\mu\frac{\partial v}{\partial y} \quad \text{and } \tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \dots \quad (5)$$

Substituting (5) in (3) we have

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = F_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \mu\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \dots \quad (6)$$

Simplifying (6) using (2), the x-momentum equation (3) becomes

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = F_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \dots \quad (7a)$$

Similarly the y-momentum equation (4) becomes

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = F_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \dots \quad (7b)$$

Considering the external body force which is due to electromagnetic force and the force of gravity along x-axis which is zero, the x-component equation is

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B^2 u}{\rho}\ldots\ldots\ldots \quad (8)$$

And the y-component is

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)\ldots\ldots\ldots \quad (9)$$

## III. Equation of Conservation Of Energy

This law can be derived by applying the first law of thermodynamics to the differential control volume in the flow field and it states that energy can neither be created nor destroyed, but can be transformed from one state to another. In thermodynamics, energy and work of a system are related in this law which states that the rate of change of heat transferred into the system is equal to the total sum of the rate of internal energy and the work done on the system, that is

$$\frac{dQ}{dt}=\frac{dE}{dt}+\frac{dW}{dt}\ldots\ldots\ldots \quad (10)$$

The total rate of heat $dQ$ within the system for an adiabatic process is the negative partial sum of heat along $x$ and $y$ co-ordinates within the system and is given by

$$\frac{dQ}{dt}=-\left(\frac{\partial Q_x}{\partial x}dx+\frac{\partial Q_y}{\partial y}dy\right)\ldots\ldots\ldots \quad (11)$$

The internal energy $E$ in the fluid consists of the kinetic and potential energy and is described by

$$\frac{dE}{dt}=\rho\left[\frac{De}{Dt}+\frac{1}{2}\frac{D}{Dt}(u^2+v^2)\right]dxdy\ldots\ldots\ldots \quad (12)$$

The change in the internal energy of the system undergoing an adiabatic change is equal to negative work done. This is so since internal energy is directly proportional to temperature of the system. The expression for work done on the system is

$$\frac{dW}{dt}=\frac{-dW_f}{dt}-\frac{dW_B}{dt}\ldots\ldots\ldots \quad (13)$$

Substituting equations (11), (12) and (13) into (10) and simplifying we get

$$\rho\frac{De}{Dt}=K\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\mu\Phi\ldots\ldots\ldots \quad (14)$$

Where the viscous- energy dissipation term $\Phi$ is expressed in Cartesian co-ordinates as

$$\Phi=2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left[\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right]\ldots\ldots\ldots \quad (15)$$

## IV. Ohm's Law (Electrodynamics Equation)

Ohm's law characterizes the ability of material to transport electric charge under the influence of an applied electric field, so for a generalized Ohm's law is given by

$$\vec{J}=\sigma\left(\vec{E}+\vec{q}\times\vec{B}\right)+\rho_C\vec{q}\ldots\ldots\ldots \quad (16)$$

Where $\rho_c\vec{q}$ represents the displacement current which is usually negligible at the fluid velocity $\vec{q}$, then the law reduces to Lorentz force (force associated with motion across a magnetic field)

$$\vec{J}=\sigma\left(\vec{E}+\vec{q}\times\vec{B}\right)\ldots\ldots\ldots \quad (17)$$

## V. Maxwell's Equations

It's a set of four differential equations that describes the relationship between the electric and magnetic fields and their sources independent of the properties of matter. They are:

$\vec{\nabla}\times\vec{B}=\mu\vec{J}+\mu\varepsilon_0\frac{\partial\vec{E}}{\partial t}$ (Ampere law)……… (18)

$\vec{\nabla}.\vec{B}=0$ (Gauss' law for magnetism)……… (19)

$-\frac{\partial\vec{B}}{\partial t}=\vec{\nabla}\times\vec{E}$ (Faraday's law of induction)……… (20)

$\vec{\nabla}.\varepsilon_0\vec{E}=\rho_e$ (Gauss' law for electricity)……… (21)

**The boundary conditions**

The conditions for the hydromagnetic flow through a channel with parallel porous plates where the fluid is withdrawn from both walls of the channel at the same rate are as follows:

$$u(x,h)=0,\ u(x,-h)=0,\ v(x,h)=V,\ and\ \ v(x,-h)=-V\ldots\ldots\ldots \quad (22)$$

where $V$ is the suction velocity at the plates of the channel and $h$ is the channel width from the centre of the channel to the plates.

**Non-dimensionalization**

To non-dimensionalize the equations (2), (9) and (10) we let

$\eta = \frac{y}{h} \implies \partial\eta = \frac{\partial y}{h}$……………………………………………………………… (23)

Substituting the non-dimensional equation (23) into equations (2), (9) and (10) we get

$\frac{\partial u}{\partial x} + \frac{\partial v}{h\partial\eta} = 0$……………………………………………………………… (24)

$u\frac{\partial u}{\partial x} + \frac{v}{h}\frac{\partial v}{\partial\eta} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{1}{h^2}\frac{\partial^2 u}{\partial\eta^2}\right) - \frac{\sigma B^2 u}{\rho}$……………………………………… (25)

$u\frac{\partial v}{\partial x} + \frac{v}{h}\frac{\partial v}{\partial\eta} = \frac{1}{\rho h}\frac{\partial p}{\partial\eta} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{1}{h^2}\frac{\partial^2 u}{\partial\eta^2}\right)$……………………………………………… (26)

Equations (24), (25), and (26) give the dimensionless form of the governing equations.

The boundary conditions (22) then reduces to

$u\,(x,1) = 0,\ \ u(x,-1) = 0$ and $v(x,1) = V, v(x,-1) = -V$……………………….(27)

Introducing the stream function $\psi$ then we have

$u(x,\eta) = \frac{\partial\psi}{\partial y}$ and $v(x,\eta) = -\frac{\partial\psi}{\partial x}$

The dimensionless form becomes

$u = \frac{1}{h}\frac{\partial\psi}{\partial\eta}, v = -\frac{\partial\psi}{\partial x}$……………………………………… (28)

The equation of continuity can be satisfied by a stream function of the form

$\psi(x,y) = [hU(0) - Vx]f(\eta)$…………………………………………………………. (29)

where U (0) is the average entrance velocity at $x = 0$. Differentiating equation (29) with respect to $\eta$ and $x$ and substituting into equation (28) the velocity components becomes

$u = \frac{\partial\psi}{\partial y} = \frac{1}{h}[hU(0) - Vx]f_\eta(\eta)$……………………………………………………………… (30)

and

$v = -\frac{\partial\psi}{\partial x} = Vf(\eta)$……………………………………………………………… (31)

where $f_\eta(\eta)$ is the partial differentiation with respect to the dimensionless variable $\eta$.

Since we are considering a situation when the fluid is being withdrawn at constant rate from both the walls, then $V$ is independent of $x$ and using (30) and (31) in (25) and (26) the equation of momentum reduces to

$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \left[\left(U(0) - \frac{Vx}{h}\right)\left(\frac{V}{h}\left(ff_\eta - {f_\eta}^2\right) - \frac{v}{h^2}f_{\eta\eta\eta} + \frac{\sigma B^2}{\rho}f_\eta\right)\right]$……………………………… (32)

or

$-\frac{1}{h\rho}\frac{\partial p}{\partial\eta} = \frac{V^2}{h}ff_\eta - \frac{vV}{h^2}f_{\eta\eta}$ ……………………………………………………………… (33) Differentiating (32) with respect to $\eta$, we get

$\frac{\partial^2 p}{\partial x\partial\eta} = \left(U(0) - \frac{Vx}{h}\right)\frac{\partial}{\partial\eta}\left[\frac{V}{h}\left(ff_{\eta\eta} - {f_\eta}^2\right) - \frac{v}{h^2}f_{\eta\eta\eta} + \frac{\sigma B^2 f_\eta}{\rho}\right]$……….................. (34)

Also differentiating (36) with respect to $x$ and simplifying, we get

$\frac{\partial^2 p}{\partial x\partial\eta} = 0$……………………………………………………………… (35)

Substituting equation (35) into equation (34) and simplifying becomes

$\frac{\partial}{\partial\eta}\left[\frac{V}{h}\left(ff_{\eta\eta} - {f_\eta}^2\right) - \frac{v}{h^2}f_{\eta\eta\eta} + \frac{\sigma B^2 f_\eta}{\rho}\right] = 0$ …………………………………. (36)

Integrating (36) with respect to $\eta$ and substituting the non-dimensionless parameters (R and M) we get

$f_{\eta\eta\eta} + R\left({f_\eta}^2 - ff_{\eta\eta}\right) - \varepsilon Rf_\eta = K$ ……………………………. (37)

Where $\varepsilon = \frac{{H_0}^2{\mu_e}^2\sigma h}{\rho V}, R = \frac{\rho ul}{\mu} = \frac{ul}{v}$ and $K$ is an arbitrary constant to be determined.

The solution of the equations of motion and continuity is given by a non linear third order partial differential equation (37) which is to be solved by perturbation method when $R$ and $\varepsilon$ are small subject to the boundary conditions on $f(\eta)$ which are:

$f(1) = 1,\ f(-1) = -1, f_\eta(1) = 0,\ and\ f_\eta(-1) = 0$ ………………………… (38)

Let investigate equation (37)

$$f_{\eta\eta\eta} + R\left({f_\eta}^2 - ff_{\eta\eta}\right) - \varepsilon Rf_\eta = K$$

Where

$\varepsilon = \frac{{H_0}^2{\mu_e}^2\sigma h}{\rho V},\ \ R = \frac{\rho ul}{\mu} = \frac{ul}{v}$ (Suction Reynolds number) and $K$ is the arbitrary constants of integration.

The equation (37) is subject to the boundary conditions

$f(\eta) = f(1) = 1,\ f(-1) = -1, f_\eta(1) = 0,\ and\ f_\eta(-1) = 0$

and

$f_0(-1) = -1,\ f_{0_\eta}(-1) = 0,\ f_0(1) = 1, f_{0_\eta}(1) = 0$ ……………………………… (39)

Where $f_n(0) = f_{n_\eta}(0) = 0$ and $f_n(1) = f_n{}'(1) = 0$ when n>0 ………………………. (40)

$f = \sum_{n=0}^{\infty} R^n f_n(\eta) = f_0(\eta) + R^1 f_1(\eta) + R^2 f_2(\eta)$……………………………………. (41)

Or

$k = \sum_{n=0}^{\infty} k_n R^n = k_0 + R^1 k_1 + R^2 k_2 + \cdots$ ……………………………………… (42)

Differentiating equation (41) with respect to $\eta$ we get

$$\left.\begin{aligned} f_\eta &= f_{0_\eta} + Rf_{1_\eta} + R^2 f_{2_\eta} + \cdots \\ f_{\eta\eta} &= f_{0_{\eta\eta}} + Rf_{1_{\eta\eta}} + R^2 f_{2_{\eta\eta}} + \cdots \\ f_{\eta\eta\eta} &= f_{0_{\eta\eta\eta}} + Rf_{1_{\eta\eta\eta}} + R^2 f_{2_{\eta\eta\eta}} + \cdots \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (43)$$

Substituting equation (41) and (42) into (37) we get

$\left(f_{0_{\eta\eta\eta}} + Rf_{1_{\eta\eta\eta}} + R^2 f_{2_{\eta\eta\eta}} + \cdots\right) + R[\left(f_{0_\eta} + Rf_{1_\eta} + R^2 f_{2_\eta} + \cdots\right)^2 - \{(f_0 + Rf_1 + R^2 f_2 + \cdots)\left(f_{0_{\eta\eta}} + Rf1\eta\eta+R2f2\eta\eta+\ldots-\varepsilon f0\eta+Rf1\eta+R2f2\eta+\ldots\right]=k0+Rk1+R2k2$

Expanding the above expression we get

$$f_{0_{\eta\eta\eta}} + R\left[f_{0_{\eta\eta\eta}} + f_{0_\eta}{}^2 - f_0 f_{1_{\eta\eta}} - \varepsilon f_{0_\eta} + \cdots\right] + R^2\left[f_{2_{\eta\eta\eta}} + 2f_{0_\eta} f_{1_\eta} - f_0 f_{1_{\eta\eta}} - f_{0_{\eta\eta}} f_1 - \varepsilon f_{1_\eta} + \cdots\right] + R^3\left[f_{3_{\eta\eta\eta}} + f_{0_\eta} f_{2_\eta} + f_{1_\eta}{}^2 + \cdots\right] = k_0 + Rk_1 + R^2 k_2 + \cdots$$

Equating the coefficients of R we get

$f_{0_{\eta\eta\eta}} = k_0$ …………………………………………… (44)

$f_{1_{\eta\eta\eta}} + f_{0_\eta}{}^2 - f_0 f_{1_{\eta\eta}} - \varepsilon f_{0_\eta} = k_1$ ……………………………………… (45)

$f_{2_{\eta\eta\eta}} + 2f_{0_\eta} f_{1_\eta} - f_0 f_{1_{\eta\eta}} - f_{0_{\eta\eta}} f_1 - \varepsilon f_{1_\eta} = k_2$ …………………………… (46)

.
.
.

and so on.

Integrating equation (44) twice and solving subject to conditions (39) we get

$A = 0, B = \frac{3}{2}, C = 0,$ and $k_1 = -3$

Thus the solution to (44) becomes

$f_0(\eta) = \frac{\eta}{2}(3 - \eta^2)$……………………………………………………… (47)

Solving equation (45) using the derivatives of (47) we get

$f_{1_{\eta\eta\eta}} = k_1 + \frac{3}{2}\varepsilon - \frac{15}{4}\eta^4 - \left(\frac{3\varepsilon-9}{2}\right)\eta^2 - \frac{9}{4}$………………………………………… (48)

Integrating (48) twice and solving subject to the boundary conditions (37) we get

$f_1(\eta) = 0.010714285\eta^3 - 0.003571429\eta^7 - 0.025\varepsilon\eta^5 - 0.025\varepsilon\eta - 0.007142857\eta + 0.05\varepsilon\eta^3$ ……………………………………………………… (49)

Therefore the first order perturbation solutions for $f(\eta)$ is given by

$f^{(1)}(\eta) = f_0(\eta) + Rf_1(\eta)$

That is $f^{(1)}(\eta) = \frac{\eta}{2}(3 - \eta^2) + 0.010714285R\eta^3 - 0.003571429R\eta^7 - 0.025M^2\eta^5 - 0.025M^2\eta - 0.007142857R\eta + 0.05M^2\eta^3$ …………………………………………….. (50)

And for $k$ is given by

$k^{(1)} = k_0 + Rk_1$

That is $k^{(1)} = -3 - 1.2M^2 + 2.314285714R$ ………………………………… (51)

where $M^2 = \varepsilon R$ in the above equations

The first order expressions for the velocity components are

$u(x,\eta) = (U(0) - \frac{Vx}{h})f'(\eta)$ and $v(\eta) = Vf(\eta)$…………………………………….. (52)

From the equation

$f(\eta) = \frac{\eta}{2}(3 - \eta^2) + 0.010714285R\eta^3 - 0.003571429R\eta^7 - 0.025M^2\eta^5 - 0.025M^2\eta - 0.007142857R\eta + 0.05M^2\eta^3$ ……………………….. (53)

The derivative of (53) is

$f_\eta(\eta) = \frac{3}{2}(1 - \eta^2) + 0.032142855R\eta^2 - 0.021428574R\eta^6 - 0.1M^2\eta^4 - 0.025M^2 - 0.007142857R - 0.15M^2\eta^2$………………………………….. (54)

Substituting (53), (54) in (52) we obtain

$u(x,\eta) = \left(U(0) - \frac{Vx}{h}\right)\left[\frac{3}{2}(1-\eta^2) + 0.032142855R\eta^2 - 0.021428574R\eta^6 - 0.1M^2\eta^4 - 0.025M^2 - 0.007142857R - 0.15M2\eta2\right.$ ……………………… (55)

And

$v(\eta) = Vf(\eta) = V\left[\frac{\eta}{2}(3-\eta^2) + 0.010714285R\eta^3 - 0.003571429R\eta^7 - 0.025M^2\eta^5 - 0.025M^2\eta - 0.007142857R\eta + 0.05M2\eta3\right.$ ………………………… (56)

From (55) $U(0)$ is the average entrance velocity and $V$ is the suction velocity. Since the fluid is being withdrawn at the same rate from both porous walls, therefore $V$ is independent of $x$ hence $Vx$ can be fixed. This means that the flow along the vertical and the horizontal axes are constant and only depends on the Reynolds's number $R$ and Hartmann's number $M^2$.

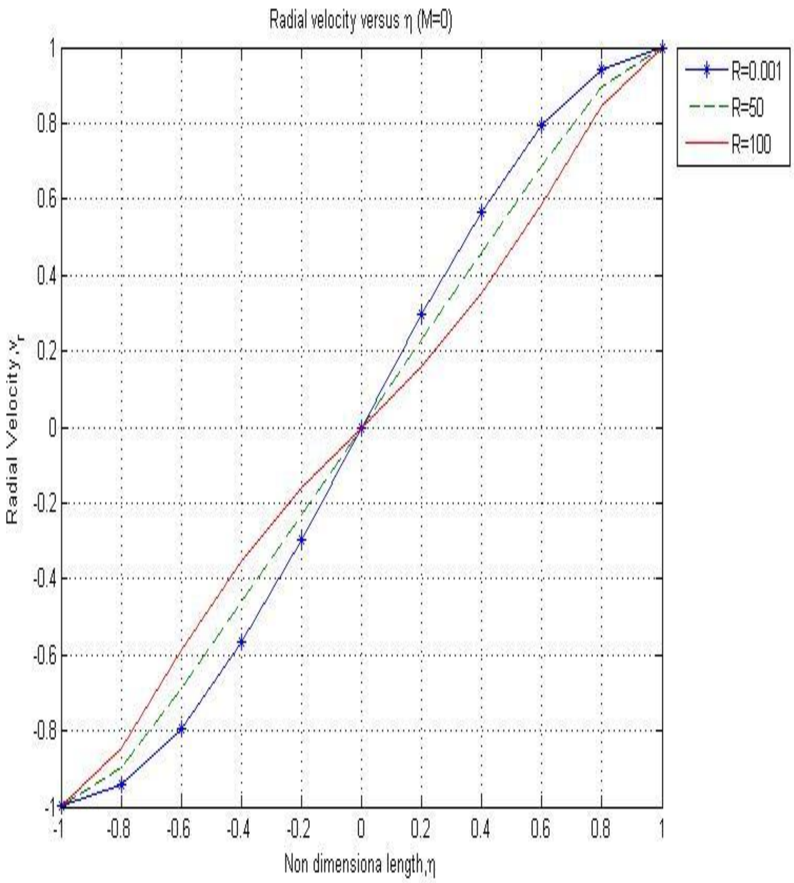
Therefore the radial velocity $v_r$ (parallel to y-axis) becomes

$f(\eta) = v_r = \frac{\eta}{2}(3-\eta^2) + 0.010714285R\eta^3 - 0.003571429R\eta^7 - 0.025M^2\eta^5 - 0.025M^2\eta - 0.007142857R\eta + 0.05M^2\eta^3$………………………………… (57)

And the axial velocity $v_a$ (parallel to x-axis) becomes

$f_\eta(\eta) = v_a = \frac{3}{2}(1-\eta^2) + 0.032142855R\eta^2 - 0.021428574R\eta^6 - 0.1M^2\eta^4 - 0.025M^2 - 0.007142857R - 0.15M^2\eta^2$………………………………. (58)

To investigate the velocity profiles in the hydromagnetic flow, we plot the values of radial velocity, $v_r$ and axial velocity, $v_a$ against non dimensional length, $\eta$ as we vary the values of suction Reynolds number, $R$ and Hartmann number, $M$. The following tables 1-4 and its corresponding figures 2-5 are discussed.



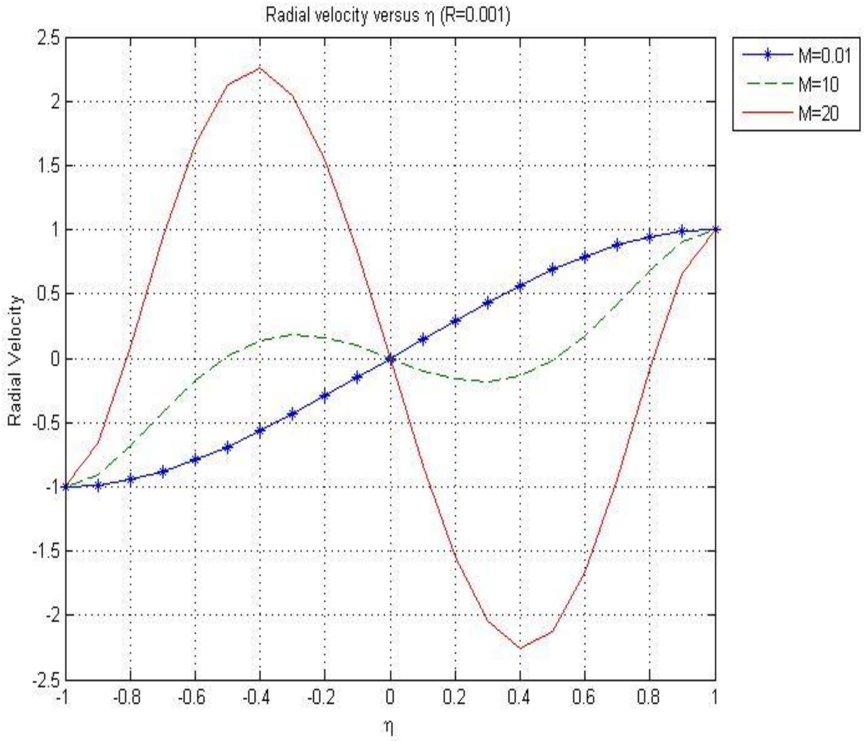



**Figure 3:** Radial velocity profiles as a function of $\eta$ for constant R=0.001 and varying M.

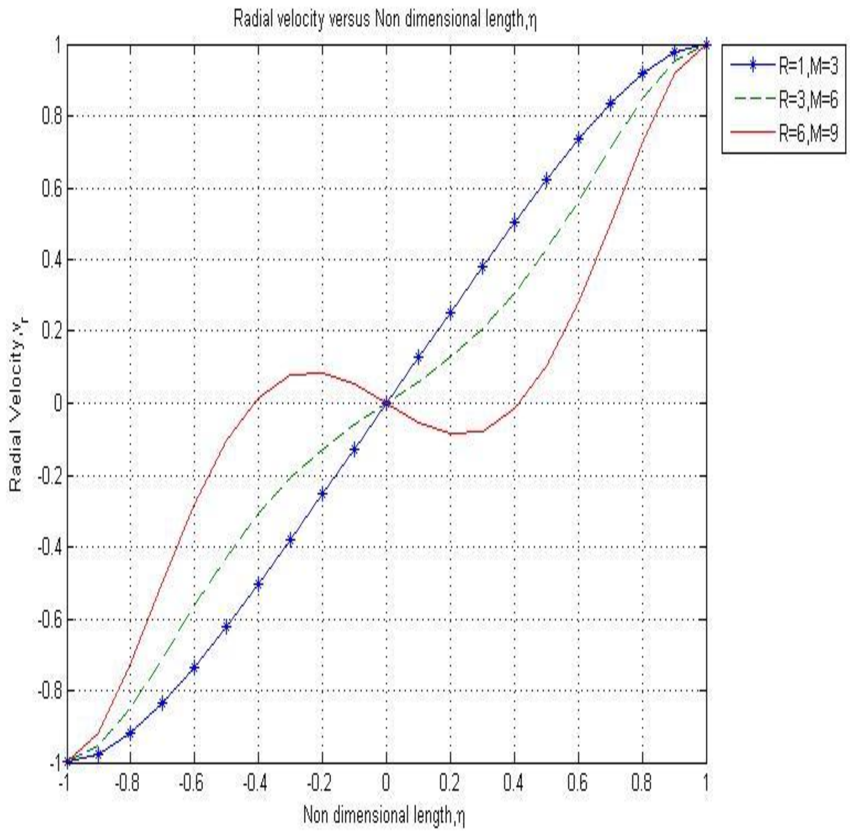



**Figure4**. Graph of radial velocity profiles as a function of $\eta$ for the range of R(1-6,) and M(3,6,9)

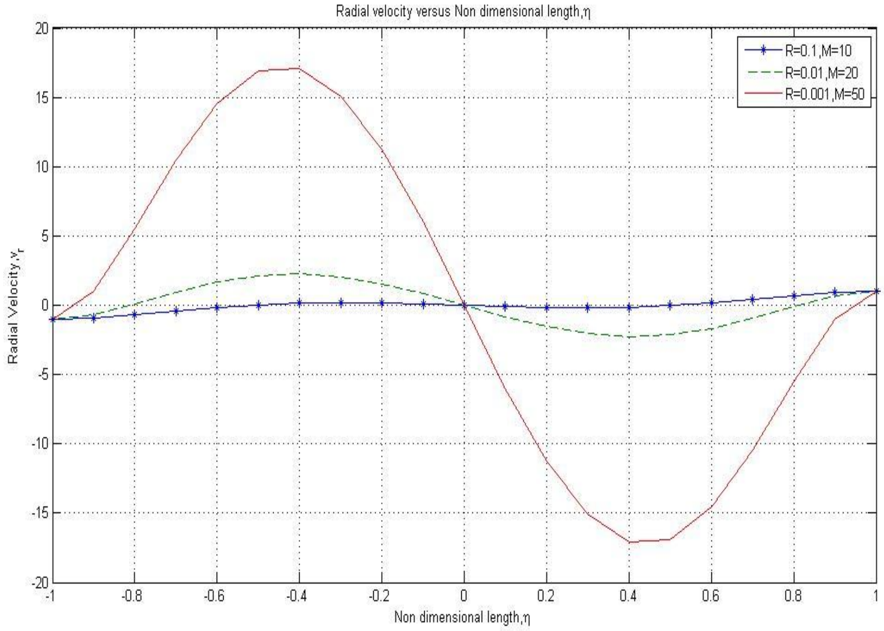



**Figure 5**: Graph of radial velocity profiles as a function of $\eta$ for small R and large M

**4.2 Axial velocity, $v_a$**

The axial velocity $v_a$ (parallel to x-axis) is the derivative of the radial velocity (parallel to y-axis) given by equation (67).

$$f^{'}(\eta) = v_a = \frac{3}{2}(1-\eta^2) + 0.032142855R\eta^2 - 0.021428574R\eta^6 - 0.1M^2\eta^4 - 0.025M^2 - 0.007142857R - 0.15M^2\eta^2$$

We plot the axial velocity $v_a$ profiles against the non dimensional length $\eta$ as shown by figures 6,7 and 8

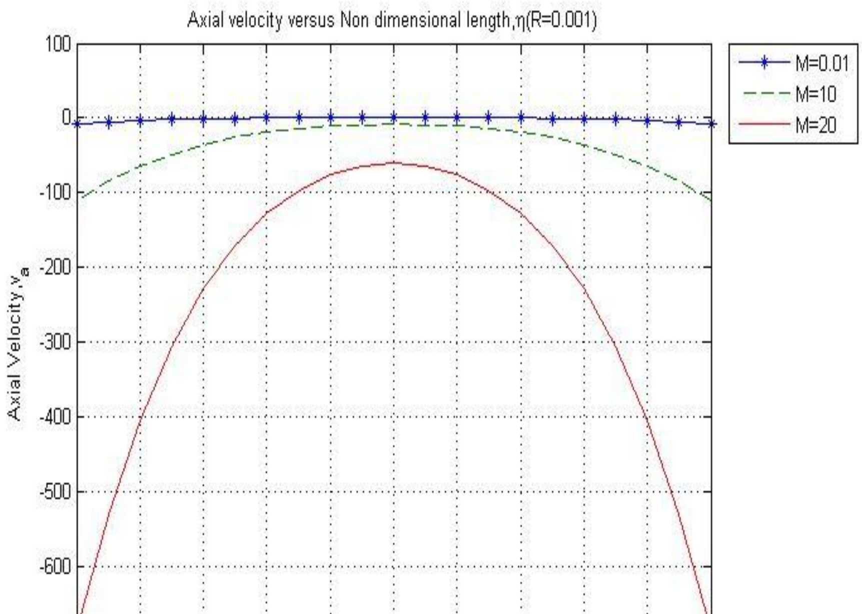



**Figure 6**: Graph of radial velocity profiles as a function of $\eta$ for constant R=0.001 and varying M (0.01, 10, 20)

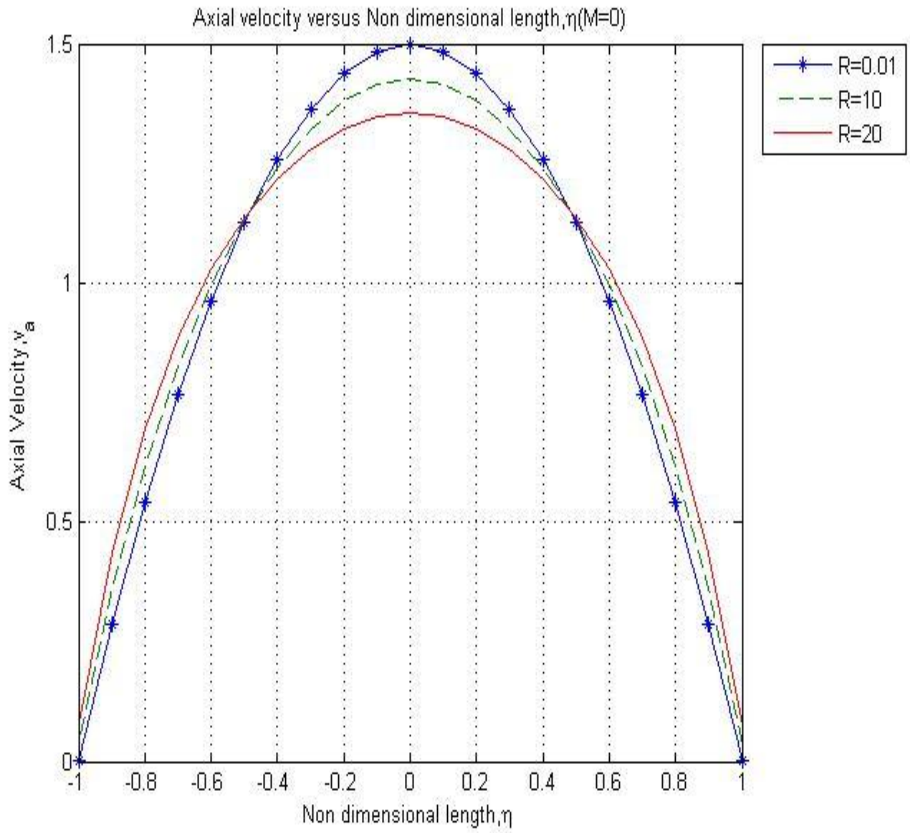



**Figure 7**: Axial velocity profiles as a function of $\eta$ for Constant M=0 and varying R(0.01,10,20)

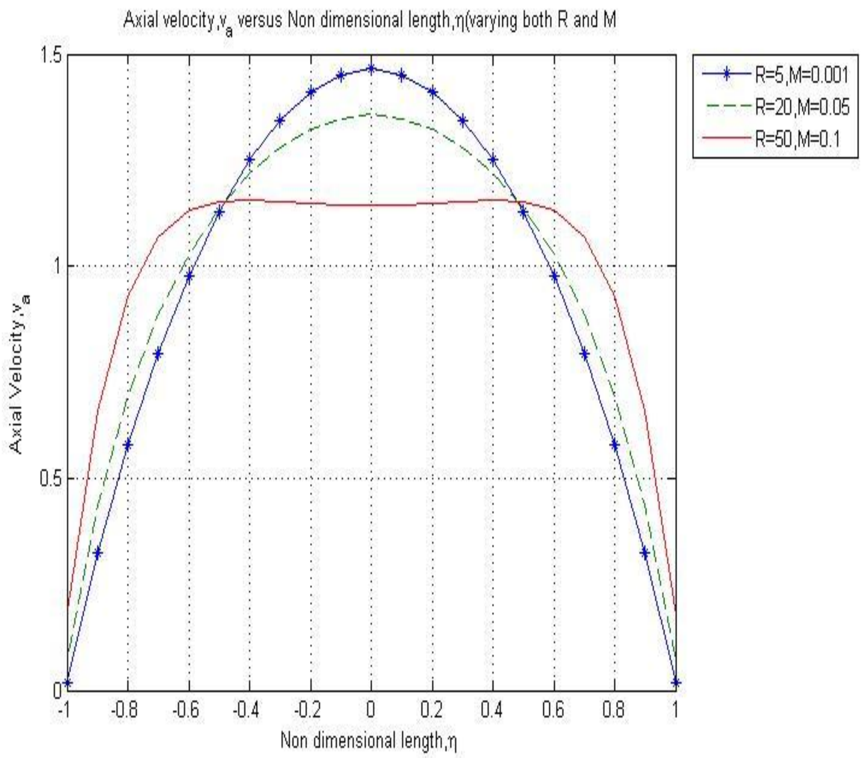



**Figure 8:** Graph of axial velocity profiles as a function of $\eta$ for large R and small M.

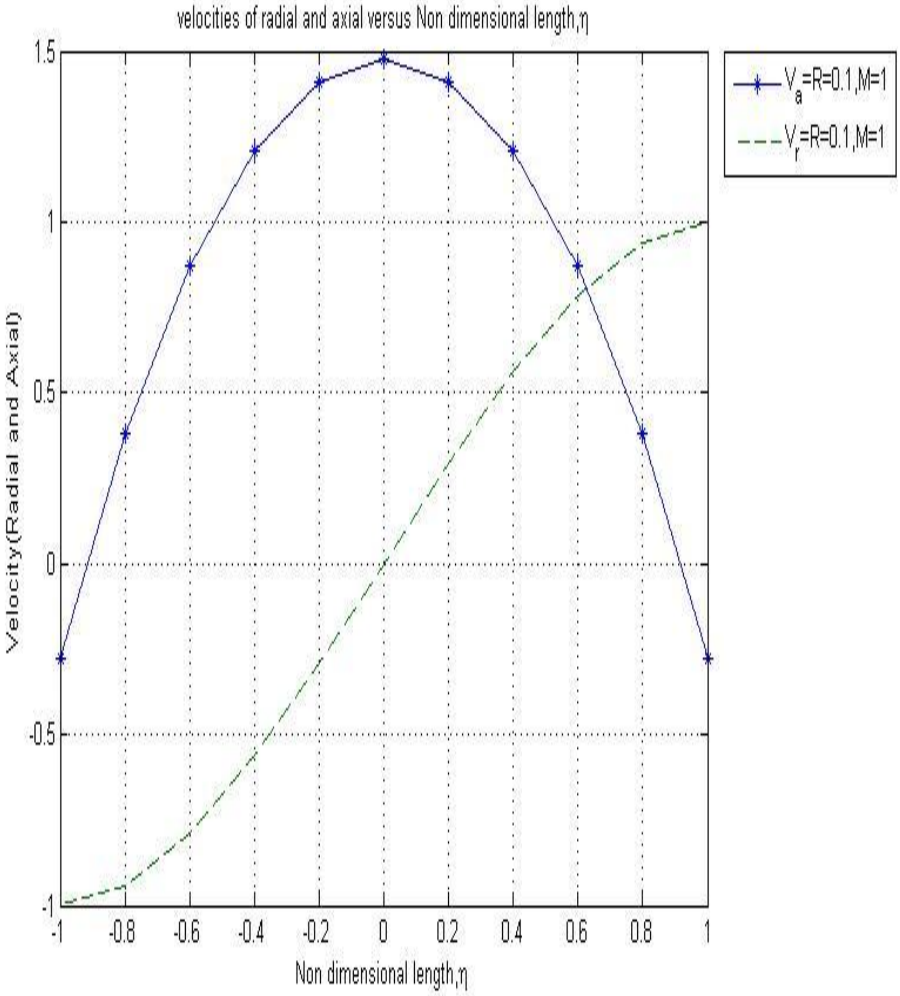



**Figure 9:** Graph of radial and axial velocity profiles as a function of $\eta$ for R=0.1 and M=1

## VI. Discussion

In the figure 2 it is observed that the radial velocity decreases as Reynolds number increases when Hartman number is zero, but increases from the central region of the flow towards the plates as the non dimensional length increases. It is observed that when the fluid is non MHD (M=0) there is no magnetic field existing in the flow thus reduces the radial velocity. When the viscous forces in the flow becomes very small the inertia forces dominates thus increases the radial velocity towards the porous plates.

In figure 3, it's observed that for different values of increasing M in the region $-1 \leq \eta \leq 0, v_r$ increases with small value of R because inertia forces are negligible while in the region $0 \leq \eta \leq 1, V_r$ decreases with the increase of M significantly. When M and R are significantly small the radial velocity is less sinusoidal about the centre of the flow and increases as the non dimensional numbers increases.

In figure 4 as M and R increases tremendously the radial velocity profile steepen for the range $-1 \leq \eta \leq 0$ and reduces in the range $0 \leq \eta \leq 1$. This is because viscous forces are minimal and the flow is dominated by inertia forces and electric conductivity is high .

In figure 5 as the increase of Reynolds number from 0.1 to 0.001and as M increases (10 to 50) the radial velocity profile becomes more sinusoidal about the central position and flat near the plates. The curve has a minimum turning point between $0 \leq \eta \leq 1$ due to the high presence of magnetic field which reduces the radial velocity towards the plates. In figure 6 it is found that the effect of decreasing M increases velocity field when Reynolds number is kept minimal at 0.001.The fluid velocity profile is parabolic with maximum magnitude along the channel centerline and minimum at the plates and it stretches outwards as Hartmann number reduces. Figure 7 shows the fluid velocity profile when there is no magnetic field that is when Hartmann number is zero. It is observed that the axial velocity is zero at the plates and increases to the maximum at the central region thus forming a curve with maximum turning point depicting the normal free flow velocity of the stream with very small Reynolds number. As Reynolds number increases the axial velocity decreases meaning that the viscous forces are minimal thus inertia forces dominates the flow.

In figure 8, the increase significantly in Reynolds number and insignificantly in Hartmann number reduces the axial velocity. For instance, when R=50 and M=0.1 there is almost the same velocity in the range -0.6 to 0.6 this indicates that when there is less presence of magnetic field the flow is dominated by inertia forces. It is interesting to note that in figures 6,7 and 8 the fluid velocity decreases with increasing in magnetic field.

Figure 9 compares the radial and axial velocities at low Reynolds and Hartmann numbers. It is observed that axial velocity (parallel to x-axis) forms a parabolic curve which shows that the fluid velocity retards at the plates and maximum at the centre of the plates while the radial velocity (parallel to y-axis) increases with increase in non dimensional length. This indicates the presence of viscous forces and magnetic field in the fluid flow.

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