IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 13, Issue 6 Ver. I (Nov. - Dec. 2017), PP 06-12
www.iosrjournals.org

Fuzzy Transportation problem of Trapezoidal Fuzzy numbers with New Ranking Technique

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Abstract: *In this paper we shall study fuzzy transportation problem, and we introduce an approach for solving a wide range of such problem by using a method which apply it for ranking of the fuzzy numbers. Some of the quantities in a fuzzy transportation problem may be fuzzy or crisp quantities. In many fuzzy decision problems, the quantities are represented in terms of fuzzy numbers may be triangular or trapezoidal. Thus, some fuzzy numbers are not directly comparable. First, we transform the fuzzy quantities as the cost, supply and demands, into crisp quantities by using Centroid ranking method [9] and then by using the VAM algorithm to solve and obtain the solution of the problem.*

Keywords: *Fuzzy set, Fuzzy transportation problem, Trapezoidal Fuzzy number, Ranking Technique.*

Date of Submission: 25-10-2017 Date of acceptance: 07-11-2017

I. Introduction

A fuzzy transportation problem is a transportation problem in which the transportation cost, supply and demand quantities are fuzzy quantities. The objective of the fuzzy transportation problem is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying fuzzy supply and demand limits. Very first basic transportation problem was developed by Hitchcock [1]. In 1965, Zadeh [2] introduced the notation of fuzziness that was reinforced by Bellman and Zadeh [3]. Zimmermann [4,5] has discussed about the effective solutions of fuzzy set theory, Fuzzy linear programming with several objective functions. In 1981 R.R. Yager [6] procedure for ordering fuzzy subsets of the unit interval, S.H. Chen [7] (1985) Ranking fuzzy numbers with maximizing set and minimizing set. S. Chanas, D. Kuchta [8] (1996) solved Fuzzy integer transportation problem. Defuzzification is a process that converts a fuzzy set or fuzzy number into a crisp value or number. On the centorids of fuzzy numbers by Wang [9]. P. Fortemps and M. Roubens [10] (1996) work on Ranking and defuzzification methods based on area compensation. S. Abbasbandy and T. Hajjari [11], A new approach for ranking of trapezoidal Fuzzy numbers (2009). A. Nagoor Gani and K. Abdul Razak [12] (2006) have solved fuzzy transportation problem in two stages. Then P. Pandian and G. Natrajan [13] (2010) has solved fuzzy transportation problem of trapezoidal numbers with algorithms and zero point method. A new method on ranking generalized trapezoidal fuzzy numbers based on centroid point and standard deviations by Chen and Chen [14] was derived. A new approach for ranking fuzzy numbers by distance method by C.H.Cheng[15].

In this paper we investigate more realistic problems, namely the transportation problem with fuzzy costs. Since the objective is to minimize the total cost or to maximize the total profit, subject to some fuzzy constraints, the objective function is also considered as a fuzzy number. First we transform the fuzzy quantities as the cost, supply and demands, into crisp quantities by centroid ranking method, and then by classical algorithms, obtain the optimum solution of the problem. This method is a systematic procedure, easy to apply and can be utilized for the all the type of transportation problem.

This paper is organized as follows: In section 2 deals with some basic definitions. In section 3 new ranking functions are discussed. In section 4 provides the mathematical formulation of fuzzy transportation problem and MODI methods is adopted to solve Fuzzy transportation problems , illustrate the proposed ranking method with numerical example . In section 5 the paper ends with a conclusion.

II. Preliminaries

2.1 Definition Let U be a universe of discourse.A fuzzy set $\tilde{A}$ of U is defined by a membership function $f_{\tilde{A}}: U \rightarrow [0,1]$,

2.2 Definition

A fuzzy number is a convex fuzzy subset of the real line R and is completely defined by its membership function. Let $\tilde{A}$ be a fuzzy number, whose membership function $f_{\tilde{A}}(x)$ can be defined as [9]

$$f_{\tilde{A}}(x)=\begin{cases} f_{\tilde{A}}^{L}(x) & \text{if } a_1 \le x \le a_2 \\ \omega & \text{if } a_2 \le x \le a_3 \\ f_{\tilde{A}}^{R}(x) & \text{if } a_3 \le x \le a_4 \\ 0 & \text{othewise} \end{cases}$$

Where $0 < \omega \le 1$ is a constant, $f_{\tilde{A}}^{L}:[a_1,a_2]\rightarrow[0,\omega]$ and $f_{\tilde{A}}^{R}:[a_3,a_4]\rightarrow[0,\omega]$ are two strictly monotonically and continuous mapping R to closed interval $[0,\omega]$. If $\omega = 1$,then $\tilde{A}$ is a normal fuzzy number; otherwise it is said to be a non normal fuzzy number. If the membership function $f_{\tilde{A}}(x)$ is piecewise linear, then $\tilde{A}$ is referred to as a trapezoidal fuzzy number and is usually denoted by $\tilde{A}=(a_1,a_2,a_3,a_4;\omega)$ .In particular, when $a_2 = a_3$,the trapezoidal fuzzy number is reduced to a triangular fuzzy number denoted by $\tilde{A}=(a_1,a_3,a_4;\omega)$ .So, triangular fuzzy numbers are special cases of trapezoidal fuzzy numbers.

Since $f_{\tilde{A}}^{L}(x)$ and $f_{\tilde{A}}^{R}(x)$ are both strictly monotonically and continuous functions, their inverse functions exists and should also be continuous and strictly monotonical .Let $g_{\tilde{A}}^{L}:[0,\omega]\rightarrow[a_1,a_2]$ and $g_{\tilde{A}}^{R}:[0,\omega]\rightarrow[a_3,a_4]$ be the inverse functions of $f_{\tilde{A}}^{L}(x)$ and $f_{\tilde{A}}^{R}(x)$ respectively. Then $g_{\tilde{A}}^{L}(y)$ and $g_{\tilde{A}}^{R}(y)$ should be integrable on the closed interval $[0,\omega]$ .In other words, both $\int_0^{\omega} g_{\tilde{A}}^{L}(y)dy$ and $\int_0^{\omega} g_{\tilde{A}}^{R}(y)dy$ should exists. In the case of trapezoidal fuzzy number the inverse functions $g_{\tilde{A}}^{L}(y)$ and $g_{\tilde{A}}^{R}(y)$ can be analytically expressed as

$$g_{\tilde{A}}^{L}(y)=a_1+\frac{(a_2-a_1)y}{\omega} \quad ,0\le y\le\omega$$

$$g_{\tilde{A}}^{R}(y)=a_4-\frac{(a_4-a_3)y}{\omega}, \quad 0\le y\le\omega$$

Consider a generalised fuzzy number $\tilde{A}=(a_1,a_2,a_3,a_4;\omega)$ whose membership function is defined as

$$f_{\tilde{A}}(x)=\begin{cases} \dfrac{\omega(x-a_1)}{(a_2-a_1)} & \text{if } a_1 \le x \le a_2 \\ \omega & \text{if } a_2 \le x \le a_3 \\ \dfrac{\omega(a_4-x)}{(a_4-a_3)} & \text{if } a_3 \le x \le a_4 \\ 0 & \text{othewise} \end{cases}$$

In order to determine centriod point $(\tilde{x}_0(\tilde{A}),\tilde{y}_0(\tilde{A}))$ of a fuzzy number $\tilde{A}$,and Wang[9] provided following centroid formulae:

$$\tilde{x}_0(\tilde{A}) = \frac{\int_{-\infty}^{\infty} x f_{\tilde{A}}(x)dx}{\int_{-\infty}^{\infty} f_{\tilde{A}}(x)dx}$$

$$= \frac{\int_{a_1}^{a_2} x f_{\tilde{A}}^{L}(x)dx + \int_{a_2}^{a_3} x\omega dx + \int_{a_3}^{a_4} x f_{\tilde{A}}^{R}(x)dx}{\int_{a_1}^{a_2} f_{\tilde{A}}^{L}(x)dx + \int_{a_2}^{a_3} \omega dx + \int_{a_3}^{a_4} f_{\tilde{A}}^{R}(x)dx}$$

$$= \frac{\int_{a_1}^{a_2} x \frac{\omega(x-a_1)}{(a_2-a_1)} dx + \int_{a_2}^{a_3} x\omega dx + \int_{a_3}^{a_4} x \frac{\omega(a_4-x)}{(a_4-a_3)} dx}{\int_{a_1}^{a_2} \frac{\omega(x-a_1)}{(a_2-a_1)} dx + \int_{a_2}^{a_3} \omega dx + \int_{a_3}^{a_4} \frac{\omega(a_4-x)}{(a_4-a_3)} dx}$$

$$\tilde{x}_0(\tilde{A}) = \frac{1}{3}\left[(a_1+a_2+a_3+a_4) - \frac{(a_4a_3-a_1a_2)}{(a_4+a_3)-(a_1+a_2)}\right]$$

$$\tilde{y}_0(\tilde{A}) = \frac{\int_0^{\omega} y[g_{\tilde{A}}^{R}(y) - g_{\tilde{A}}^{L}(y)]dy}{\int_0^{\omega} [g_{\tilde{A}}^{R}(y) - g_{\tilde{A}}^{L}(y)]dy}$$

$$= \frac{\int_0^{\omega} y\left([a_4-(a_4-a_3)\frac{y}{\omega}] - [a_1+(a_2-a_1)\frac{y}{\omega}]\right)dy}{\int_0^{\omega} \left([a_4-(a_4-a_3)\frac{y}{\omega}] - [a_1+(a_2-a_1)\frac{y}{\omega}]\right)dy}$$

$$\tilde{y}_0(\tilde{A}) = \frac{\omega}{3}\left[1 + \frac{a_3-a_2}{(a_4+a_3)-(a_1+a_2)}\right]$$

Where $\tilde{x}_0(\tilde{A})$ and $\tilde{y}_0(\tilde{A})$ is the centroid of the general trapezoidal fuzzy number.

2.3 Properties of Trapezoidal fuzzy numbers

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$, $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers, then the fuzzy numbers addition, fuzzy numbers subtraction and fuzzy members multiplication are defined as follows.

(i) $\tilde{A} + \tilde{B} = (a_1, a_2, a_3, a_4) + (b_1, b_2, b_3, b_4) = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)$

(ii) $\tilde{A} - \tilde{B} = (a_1, a_2, a_3, a_4) - (b_1, b_2, b_3, b_4) = (a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1)$

(iii) $\tilde{A} \otimes \tilde{B} = (a_1, a_2, a_3, a_4) \otimes (b_1, b_2, b_3, b_4) = (t_1, t_2, t_3, t_4)$

Where $t_1 = \min(a_1b_1, a_1b_4, a_4b_1, a_4b_4)$

$t_2 = \min(a_2b_2, a_2b_3, a_3b_2, a_3b_3)$

$t_3 = \max(a_2b_2, a_2b_3, a_3b_2, a_3b_3)$

$t_4 = \max(a_1b_1, a_1b_4, a_4b_1, a_4b_4)$

III. Proposed Ranking Method

An efficient approach for comparing the fuzzy numbers is by use of a ranking function $R : F(R) \to R,$ where $F(R)$ is a fuzzy numbers defined on set of real numbers, which maps each fuzzy number into a real number, where natural order exists. Wang [9] used a centroid based distance approach to rank fuzzy numbers.

For trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4; \omega)$, the ranking function is defined as

$$\Re(\tilde{A}) = \sqrt{\tilde{x}_0(\tilde{A}) + \tilde{y}_0(\tilde{A})}$$

Where $\tilde{x}_0(\tilde{A}) = \frac{1}{3}\left[(a_1 + a_2 + a_3 + a_4) - \frac{(a_4 a_3 - a_1 a_2)}{(a_4 + a_3) - (a_1 + a_2)}\right]$

$$\tilde{y}_0(\tilde{A}) = \frac{\omega}{3}\left[1 + \frac{a_3 - a_2}{(a_4 + a_3) - (a_1 + a_2)}\right].$$

For any two trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$, $\tilde{B} = (b_1, b_2, b_3, b_4)$ then we have

(i) $\tilde{A} \le \tilde{B} \Leftrightarrow \Re(\tilde{A}) \le \Re(\tilde{B})$

(ii) $\tilde{A} \ge \tilde{B} \Leftrightarrow \Re(\tilde{A}) \ge \Re(\tilde{B})$

(iii) $\tilde{A} = \tilde{B} \Leftrightarrow \Re(\tilde{A}) = \Re(\tilde{B})$

IV. Mathematical Formulation Of Fuzzy Transformation Problem

The fuzzy transportation problems, in which a decision maker is uncertain about the precise value of transportation cost, availability and demand, can be formulated as follows

$$\text{minimize} \quad \tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij}$$

$$\text{Subject to} \quad \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i, \quad i = 1,2,3,\ldots,m.$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j, \quad j = 1,2,3,\ldots,n.$$

$$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j, \quad i = 1,2,3,\ldots,n, \ j = 1,2,3,\ldots,n \quad \text{and} \quad \tilde{x}_{ij} \ge 0.$$

Where m = total number of sources
n = total number of destinations

$\tilde{a}_i$ = the fuzzy availability of the product at ith source

$\tilde{b}_i$ = the fuzzy demand of the product at jth destination

$\tilde{c}_{ij}$ = the fuzzy transportation cost for unit quantity of the product from ith source to jth destination

$\tilde{x}_{ij}$ = the fuzzy quantity of the product that should be transported from ith source to jth destination to minimize the total fuzzy transportation cost

$\sum_{i=1}^{m} \tilde{a}_i$ = total fuzzy availability of the product

$\sum_{j=1}^{n} \tilde{b}_j$ = total fuzzy demand of the product

$\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij}$ = total fuzzy transportation cost

If $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$ then the fuzzy transportation problem is said to be balanced fuzzy transportation problem, otherwise it is called unbalanced fuzzy transportation problem.

4.1. Algorithm for Vogel Approximation method

Step 1. Find the crisp value of the given Fuzzy Transportation problem by using centroid ranking method.

Step 2. Balance the given fuzzy transportation problem if either (total supply > total demand) or (total supply < total demand).
Step 3. Determine the penalty cost for each row and column by subtracting the lowest cell cost in the row or column from the next cell cost in the same row or column.
Step 4. Select the row or column with the highest penalty cost (breaking tiles arbitrarily or choosing the lowest cost cell).
Step 5. Allocate as much as possible to the feasible cell with the lowest transportation cost in the row or column with the highest penalty cost.
Step 6. Repeat 3 and 4 until all requirements have been meet.
Step 7. Compute total transportation cost for the feasible allocations.

4.2 Numerical Examples

Consider the fuzzy transportation problem,

The following table shows all the necessary information on the availability of supply to each warehouse, the following requirement of each market and unit transportation cost (in Rs) from each warehouse to each market. Here cost value, supplies and demands are trapezoidal fuzzy numbers. Here FAi and FRi are fuzzy supply and fuzzy demand. The given problem is balanced transportation problem. There exists fuzzy initial basic feasible solution.

Table 1

	FR1	FR2	FR3	FR4	Fuzzy Supply
FA1	(1, 2, 3, 4)	(1, 3, 4, 6)	(9, 11, 12, 14)	(5, 7, 8, 11)	(1, 6, 7, 12)
FA2	(0, 1, 2, 4)	(–1, 0, 1, 2)	(5, 6, 7, 8)	(0, 1, 2, 3)	(0, 1, 2, 3)
FA3	(3, 5, 6, 8)	(5, 8, 9, 12)	(12, 15, 16, 19)	(7, 9, 10, 12)	(5, 10, 12, 17)
Fuzzy Demand	(5, 7, 8, 10)	(1, 5, 6, 10)	(1, 3, 4, 6)	(1, 2, 3, 4)	

By using new ranking method of the trapezoidal fuzzy numbers,

$$\Re(\tilde{A}) = \sqrt{\tilde{x}_0(\tilde{A}) + \tilde{y}_0(\tilde{A})}$$

Where $\tilde{x}_0(\tilde{A}) = \frac{1}{3}\left[(a_1+a_2+a_3+a_4) - \frac{(a_4a_3 - a_1a_2)}{(a_4+a_3)-(a_1+a_2)}\right]$

$$\tilde{y}_0(\tilde{A}) = \frac{\omega}{3}\left[1 + \frac{a_3 - a_2}{(a_4+a_3)-(a_1+a_2)}\right].$$

For taking $\omega = 1$,we have

$\Re$(1, 2, 3, 4) = 2.54 $\Re$(0, 1, 2, 4) = 1.84 $\Re$(3, 5, 6, 8) = 5.51
$\Re$(1, 3, 4, 6) = 3.52 $\Re$(–1, 0, 1, 2) = 0.65 $\Re$(5, 8, 9, 12) = 8.51
$\Re$(9, 11, 12, 14) = 11.51 $\Re$(5, 6, 7, 8) = 6.51 $\Re$(12, 15, 16, 19) = 15.51
$\Re$(5, 7, 8, 11) = 7.82 $\Re$(0, 1, 2, 3) = 1.56 $\Re$(7, 9, 10, 12) = 9.51
Rank of all Supply: $\Re$(1, 6, 7, 12) = 6.51, $\Re$(0, 1, 2, 3) = 1.56, $\Re$(5, 10, 12, 17) = 11.01
Rank of all fuzzy Demand: $\Re$(5, 7, 8, 10) = 7.51, $\Re$(1, 5, 6, 10) = 5.51, $\Re$(1, 3, 4, 6) = 3.52, $\Re$(1, 2, 3, 4) = 2.54.

Substitute these values in fuzzy transportation problem; we get the crisp transportation problem which is shown following table.

Table 2

	FR1	FR2	FR3	FR4	Fuzzy available
FA1	2.54	3.52	11.51	7.82	6.51
FA2	1.84	0.65	6.51	1.56	1.56
FA3	5.51	8.51	15.51	9.51	11.01
Fuzzy requirement	7.51	5.51	3.52	2.54	19.08

The fuzzy transportation problem is balanced. After applying the VAM procedure for Initial Basic Feasible solution, the allocations are as follows

Table 3

	FR1	FR2	FR3	FR4	Fuzzy available
FA1	[1] 2.54	[5.51] 3.52	11.51	7.82	6.51
FA2	1.84	0.65	6.51	[1.56] 1.56	1.56
FA3	[6.51] 5.51	8.51	[3.52] 15.51	[0.98] 9.51	11.01
Fuzzy requirement	7.51	5.51	3.52	2.54	19.08

.

Minimum Transportation cost = $(2.54 \times 1) + (3.52 \times 5.51) + (1.56 \times 1.56) + (5.51 \times 6.51) + (15.51 \times 3.52) + (9.51 \times 0.98) = 124.1539$.

which is not optimal solution.

Using MODI method, the optimal solution is given by

Table 4

	FR1	FR2	FR3	FR4	
FA1	[1] 2.54	[5.51] 3.52	12.54 11.51 –1.03	6.54 7.82 1.28	$u_1 = -2.97$
FA2	–2.44 1.84 4.28	–1.46 0.65 2.11	7.56 6.51 –1.05	[1.56] 1.56	$u_2 = -7.95$
FA3	[6.51] 5.51	6.49 8.51 2.02	[3.52] 15.51	[0.98] 9.51	$u_3 = 0$
	$v_1 = 5.51$	$v_2 = 6.49$	$v_3 = 15.51$	$v_4 = 9.51$	

Table 5

	FR1	FR2	FR3	FR4
FA1	2.54	[5.51] 3.52	[1] 11.51	7.82
FA2	1.84	0.65	[1.56] 6.51	1.56
FA3	[7.51] 5.51	8.51	[0.96] 15.51	[2.54] 9.51

The above table satisfies the rim conditions with (m+n–1) non negative allocations at independent positions.
Thus the optimal allocation is

$$x_{12} = 5.51,\quad x_{13} = 1,\quad x_{23} = 1.56,\quad x_{31} = 7.51,\quad x_{33} = 0.96,\quad x_{34} = 2.54$$

The crisp value of the fuzzy transportation problem is:

Total cost = $(3.52 \times 5.51) + (11.51 \times 1) + (6.51 \times 1.56) + (5.51 \times 7.51) + (15.51 \times 0.96) + (9.51 \times 2.54)$
$= 121.4859$.

V. Conclusion

In this paper, the transportation costs are considered as imprecise numbers by fuzzy numbers which are more realistic and general in nature. More over fuzzy transportation problem of trapezoidal fuzzy number has been transformed into crisp transportation problem using ranking method. A numerical example shows that by this method we can have the optimal solution.

References

[1] Hictchcock FL. The distribution of a product from several sources to numerous localities. Journal of Mathematical Physics. 1941; 224–230.
[2] Zadeh LA. Fuzzy sets, information and control. 1965; 8: 338–353
[3] Bellman RE, Zadeh LA. Decision making in a fuzzy environment. Management Sci. 1970; 17: 141–164.
[4] Zimmermann HJ. Fuzzy set theory and its applications. Fourth Edition ISBN 0792374355; 1934.
[5] Zimmermann HJ. Fuzzy programming and linear programming with several objective functions. Fuzzy Sets and Systems. 1978; 45–55.
[6] Yager R.R. "A procedure for ordering fuzzy subsets of the unit interval", Information Sciences, 24, 143–161 (1981
[7] Chen, S. H. 1985. Ranking fuzzy number with maximizing set and minimizing set, Fuzzy Sets and Systems, 17(1): 113–129.
[8] Chanas S, Kuchta D. A concept of the optimal solution of the transportation problem with fuzzy cost coefficients. Fuzzy Sets and Systems. 1996; 82: 299–305.
[9] Ying-Ming Wang,Jian-Bo Yang,Dong-Ling Xu,Kwai-Sang Chin .On the centroids of fuzzy number .Fuzzy set and systems 2006 ; 919-926
[10] Fortemps, P. and Roubens, M. 1996. Ranking and defuzzification methods based on area compensation, Fuzzy Sets and Systems, 82: 319–330.
[11] Abbasbandy, S. and Hajjari, T. 2009. A new approach for ranking of trapezoidal fuzzy numbers. Computers and Mathematics with Applications, 57(3): 413–419.
[12] Gani A, Razak KA. Two stage fuzzy transportation problem. JournaSl of Physical Sciences. 2006; 63–69.
[13] Pandian P, Natarajan G. A new algorithm for finding a fuzzy optimal solution for fuzzy transportationproblem. Applied Mathematical Science. 2010; 4: 79–90.
[14] Chen SJ, Chen SM. Fuzzy risk analysis based on the ranking of generalized trapezoidal fuzzy numbers. Applied Intelligence. 2007; 26: 1–11.
[15] C.H.Chen .A new approach for ranking fuzzy numbers by distance method,Fuzzy sets and systems 95 (1998) 307-317.

M.K.Purushothkumar Fuzzy Transportation problem of Trapezoidal Fuzzy numbers with New Ranking Technique. IOSR Journal of Mathematics (IOSR-JM) , vol. 13, no. 6, 2017, pp. 06-12.