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Bianchi type III string cosmological models with the help of cosmological constant and bulk viscosity in general relativity

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Abstract: *We consider Bianchi type III string cosmological model in the presence of viscous fluid and* Λ. *To solve the Eainstein's field equations for Bianchi type III space time, time has been obtained by assuming the condition the coefficient of the viscosity is proportional to the expansion scalar,* $\eta \propto \theta$, *expansion scalar is proportional to shear scalar,* $\theta \propto \sigma$ *and cosmological constant is proportional to the Hubble parameter* $\Lambda \propto \mathrm{H}$. *The physical and geometrical behaviour of cosmological model are discussed.*

Keywords: *Bulk viscosity, cosmological constant* Λ, *Bianchi-III space time, expansion scalar, shear, Hubble parameter.*

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I. Introduction

In recent years, the problem of string cosmology has attracted wide attention. Comic string are topologically stable objects, which might be formed during the phase transition and before during the phase transition and before the creation of particles in the early universe. Nightingale [1] has investigated the role of viscosity in cosmology. Thus we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models. The effect of bulk viscosity on the cosmological evolution has been studied by a number of authors in the frame work general theory of reactivity. Pavon et. al. [2], Maatens [3], Kalyani and Singh [4], Singh et. al. [5]. On other hand one outstanding problem in cosmology is the cosmological constant problem [6, 7]. Bali and Dave [8] have presented Bianchi type III string cosmological model with bulk viscosity . Wang [9] have discussed the solutions of Bianchi types II, VI, VIII and IX for a cloud string. Misner [10, 11] have studied the effect of viscosity on the evolution of cosmological models. Heller and Klimek [12] emphasized that the introduction of bulk viscosity effectively removes the initial singularities within a certain class of cosmological models. Since, its introduction and its significance has been studied from time to time by various worker [13-15]. In modern cosmological theories the cosmological constant remains a focal point of internet.

Viscous fluid cosmological models in the early universe have been widely discussed in the literatures [15, 16]. Recently Singh et. al. [17] investigated Bianchi type III cosmological models with gravitational constant G and the cosmological constant Λ.

In this paper Bianchi type III string cosmological models with the help cosmological constant and Bulk viscosity. To obtain a determine cosmological model we assume that the coefficient of the viscosity is proportional to the expansion scalar $\eta \propto \theta$ and the expansion scalar is proportional to the shear scalar $\theta \propto \sigma$, $\Lambda \propto \mathrm{H}$.

Metric and field equations :

We consider the Bianchi type III metric of the from

$$ds^2 = -dt^2 + \alpha_1^2 \, dx_1^2 + \alpha_2^2 \, e^{-2\delta x} dx_2^2 + \alpha_3^2 dx_3^2 \qquad \text{\_\_\_\_\_\_\_\_(1)}$$

Where α_1, α_2 and α_3 are function of t only. The metric (1) has the following non zero christoffel symbols.

$$\Gamma^1_{14} = \frac{\dot{\alpha_1}}{\alpha_1}$$

$$\Gamma^1_{22} = \int \frac{\alpha_2^2}{\alpha_1^2}\, e^{-2\delta x}$$
$$\Gamma^2_{12} = -\delta$$
$$\Gamma^2_{24} = \frac{\dot{\alpha_2}}{\alpha_2}$$
$$\Gamma^3_{34} = \frac{\dot{\alpha_3}}{\alpha_3}$$
$$\Gamma^4_{11} = \alpha_1\, \dot{\alpha_1}$$
$$\Gamma^4_{22} = \alpha_2\, \dot{\alpha_2}\, e^{-2\delta x}$$
$$\Gamma^4_{33} = \alpha_3\, \dot{\alpha_3}$$

Where '.' dots on α_1 , α_2 represents the ordinary differentiation with respect to time t.

The surviving components of the mixed Ricci tensor R^s_r are as following –

$$R^1_1 = \frac{\delta^2}{\alpha_1^2} - \frac{\ddot{\alpha_1}}{\alpha_1} - \frac{\dot{\alpha_1}\dot{\alpha_2}}{\alpha_1\alpha_2} - \frac{\dot{\alpha_1}\dot{\alpha_3}}{\alpha_1\alpha_3} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (2)}$$
$$R^2_2 = \frac{\delta^2}{\alpha_1^2} - \frac{\ddot{\alpha_2}}{\alpha_2} - \frac{\dot{\alpha_1}\dot{\alpha_2}}{\alpha_1\alpha_2} - \frac{\dot{\alpha_2}\dot{\alpha_3}}{\alpha_2\alpha_3} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (3)}$$
$$R^3_3 = -\frac{\ddot{\alpha_3}}{\alpha_3} - \frac{\dot{\alpha_1}\dot{\alpha_3}}{\alpha_1\alpha_3} - \frac{\dot{\alpha_2}\dot{\alpha_3}}{\alpha_2\alpha_3} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (4)}$$
$$R^4_4 = -\frac{\ddot{\alpha_1}}{\alpha_1} - \frac{\ddot{\alpha_2}}{\alpha_2} - \frac{\ddot{\alpha_3}}{\alpha_3} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (5)}$$
$$R^4_1 = \delta\left(\frac{\dot{\alpha_2}}{\alpha_2} - \frac{\dot{\alpha_1}}{\alpha_1}\right) \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (6)}$$

From eqn (2) one finds the following scalar for the Bianchi type III universe

$$R = -2\left[\frac{\ddot{\alpha_1}}{\alpha_1} + \frac{\ddot{\alpha_2}}{\alpha_2} + \frac{\ddot{\alpha_3}}{\alpha_3} + \frac{\dot{\alpha_1}\dot{\alpha_2}}{\alpha_1\alpha_2} + \frac{\dot{\alpha_2}\dot{\alpha_3}}{\alpha_2\alpha_3} + \frac{\dot{\alpha_1}\dot{\alpha_3}}{\alpha_1\alpha_3} - \frac{\delta^2}{\alpha_1^2}\right] \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (7)}$$

The non varnishing component of Einstein tensor

$$G^s_r = R^s_r - \frac{1}{2}R$$
$$G^1_1 = R^1_1 - \frac{1}{2}R$$
$$G^1_1 = \left(\frac{\delta^2}{\alpha_1^2} - \frac{\ddot{\alpha_1}}{\alpha_1} - \frac{\dot{\alpha_1}\dot{\alpha_2}}{\alpha_1\alpha_2} - \frac{\dot{\alpha_1}\dot{\alpha_3}}{\alpha_1\alpha_3}\right)$$
$$-\frac{1}{2}(-2)\left(\frac{\ddot{\alpha_1}}{\alpha_1} + \frac{\ddot{\alpha_2}}{\alpha_2} + \frac{\ddot{\alpha_3}}{\alpha_3} + \frac{\dot{\alpha_1}\dot{\alpha_2}}{\alpha_1\alpha_2} + \frac{\dot{\alpha_2}\dot{\alpha_3}}{\alpha_2\alpha_3} + \frac{\dot{\alpha_1}\dot{\alpha_3}}{\alpha_1\alpha_3} - \frac{\delta^2}{\alpha_1^2}\right) \quad \text{\_\_\_\_\_\_\_\_\_ (8)}$$
$$G^1_1 = \frac{\ddot{\alpha_2}}{\alpha_2} + \frac{\ddot{\alpha_3}}{\alpha_3} + \frac{\dot{\alpha_2}\dot{\alpha_3}}{\alpha_2\alpha_3} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (9)}$$

Similarly

$$G_2^2 = \frac{\ddot{\alpha_1}}{\alpha_1} + \frac{\ddot{\alpha_3}}{\alpha_3} + \frac{\dot{\alpha_1}\dot{\alpha_3}}{\alpha_1\alpha_3} \qquad \text{———— (10)}$$

$$G_3^3 = \frac{\ddot{\alpha_1}}{\alpha_1} + \frac{\ddot{\alpha_2}}{\alpha_2} + \frac{\dot{\alpha_1}\dot{\alpha_2}}{\alpha_1\alpha_2} - \frac{\delta^2}{\alpha_1^2} \qquad \text{———— (11)}$$

$$G_4^4 = \frac{\dot{\alpha_1}\dot{\alpha_2}}{\alpha_1\alpha_2} + \frac{\dot{\alpha_2}\dot{\alpha_3}}{\alpha_2\alpha_3} + \frac{\dot{\alpha_1}\dot{\alpha_3}}{\alpha_1\alpha_3} - \frac{\delta^2}{\alpha_1^2} \qquad \text{———— (12)}$$

$$G_1^4 = \delta\left(\frac{\dot{\alpha_2}}{\alpha_2} - \frac{\dot{\alpha_1}}{\alpha_1}\right) \qquad \text{———— (13)}$$

The energy momentum tensor for cloud of string dust with bulk viscosity.

$$T_r^s = \rho v_r v^s - \mu x_r x^s - \eta v_{;l}^l (g_r^s + v_r v^s) \qquad \text{———— (14)}$$

$$\rho = \rho_p + \mu \qquad \text{———— (15)}$$

Where ρ is the proper energy density for a cloud string with barticle attached them η is the coefficient of bulk viscosity, μ is the string tension density of particle, $\theta = v_{;l}^l$, is the scalar of expansion, ρ_p is the particle energy density, v^r is the four velocity vector of particles and x^r is the unit space like vector representing the direction of string satisfying

$$v_r v^r = -x^r x_r = -1, \quad v^r x_r = 0 \qquad \text{———— (16)}$$

In a co-moving system we get –

$$v^r = (0,0,0,1) \ , \qquad x^r = (\alpha_1^{-1},0,0,0)$$

$$T_1^1 = \eta\theta \qquad \text{———— (17)}$$

$$T_2^2 = \eta\theta \qquad \text{———— (18)}$$

$$T_3^3 = \eta\theta \qquad \text{———— (19)}$$

$$T_4^4 = \rho \qquad \text{———— (20)}$$

$$T_1^4 = 0 \qquad \text{———— (21)}$$

Field equation :-

The Einstein field equation in general relativity with suitable Λ in units are

$$R_r^s - \frac{1}{2} R\, g_r^s = T_r^s - \Lambda\, g_r^s \qquad \text{———— (22)}$$

The field equation (22) and (17) – (21) we get

$$\frac{\dot{\alpha}_2}{\alpha_2} + \frac{\dot{\alpha}_3}{\alpha_3} + \frac{\dot{\alpha}_2\dot{\alpha}_3}{\alpha_2\alpha_3} = \eta\theta - \Lambda \qquad \text{———— (23)}$$

$$\frac{\dot{\alpha}_1}{\alpha_1} + \frac{\dot{\alpha}_3}{\alpha_3} + \frac{\dot{\alpha}_1\dot{\alpha}_3}{\alpha_1\alpha_3} = \eta\theta - \Lambda \qquad \text{———— (24)}$$

$$\frac{\dot{\alpha}_1}{\alpha_1} + \frac{\dot{\alpha}_2}{\alpha_2} + \frac{\dot{\alpha}_1\dot{\alpha}_2}{\alpha_1\alpha_2} - \frac{1}{\alpha_1^2} = \mu + \eta\theta - \Lambda \qquad \text{———— (25)}$$

$$\frac{\dot{\alpha}_1\dot{\alpha}_2}{\alpha_1\alpha_2} + \frac{\dot{\alpha}_2\dot{\alpha}_3}{\alpha_2\alpha_3} + \frac{\dot{\alpha}_1\dot{\alpha}_3}{\alpha_2\alpha_3} - \frac{1}{\alpha_1^2} = \rho + \Lambda \qquad \text{———— (26)}$$

$$\frac{\dot{\alpha}_1}{\alpha_1} - \frac{\dot{\alpha}_2}{\alpha_2} = \theta \qquad \text{———— (27)}$$

The expression for scalar expansion θ and shear scalar σ are

$$\theta = \left(\frac{\dot{\alpha}_1}{\alpha_1} + \frac{\dot{\alpha}_2}{\alpha_2} + \frac{\dot{\alpha}_3}{\alpha_3}\right) \qquad \text{———— (28)}$$

Therefore –

$$\sigma^2 = \frac{1}{2}\sigma_{rs}\sigma^{rs} = \frac{1}{3}\left[\frac{(\dot\alpha_1)^2}{\alpha_1} + \frac{(\dot\alpha_2)^2}{\alpha_2} + \frac{(\dot\alpha_3)^2}{\alpha_3} - \frac{\dot\alpha_1\dot\alpha_2}{\alpha_1\alpha_2} - \frac{\dot\alpha_2\dot\alpha_3}{\alpha_2\alpha_3} - \frac{\dot\alpha_3\dot\alpha_1}{\alpha_3\alpha_1}\right] \qquad (29)$$

Solutions of the field equation :-

The field equ. (23) – (27) are five equations in seven unknown parameter $\alpha_1, \alpha_2, \alpha_3, \rho, \mu, \eta$ and Λ. Therefore we need extra condition to solve the system completely we assume that expansion scalar is condition leads to

$$\alpha_2 = \alpha_3^m \qquad (30)$$

Where m is a constant and the second condition is

$$\Lambda \propto \mathrm{H}$$

$$\Lambda = h_1\mathrm{H} \qquad (31)$$

Where H is Hubble parameter and h_1 is a positive constant

From equ. (27) we get

$$\alpha_1 = h \boxtimes \alpha_2 \qquad (32)$$

Where $h \boxtimes$ is an integrating constant

From eqn. (32) without any loss of generality we can take $h \boxtimes = 1$

$$\alpha_1 = (1)\,\alpha_2$$

$$\alpha_1 = \alpha_2 \qquad (33)$$

In order to obtain the more general solution, we assume that the coefficient of bulk viscosity η is inversely proportional to the expansion θ therefore

$$\eta\ \theta = \mathrm{k}_1 \qquad (34)$$

Substituting eqn. (30) into equation (28) we get –

$$\theta = (2m+1)\frac{\dot\alpha_3}{\alpha_3} \qquad (35)$$

By the use of the equation (30), (32), (33) and (34) , the field equation (23) reduces to

$$\frac{\dot\alpha_3}{\alpha_3} + m\,(m+1)\frac{\dot\alpha_3^2}{\alpha_3^2} = \mathrm{k}_1 - \frac{h_1}{3}(2\mathrm{m}+1)\frac{\dot\alpha_3}{\alpha_3} \qquad (36)$$

Equation (35) can be written us

$$\frac{\ddot\alpha_3/\alpha_3}{\dot\alpha_3/\alpha_3} - \left(\frac{k_1}{m+1}\right)\frac{\alpha_3}{\dot\alpha_3} + \left(\frac{m^2}{m+1}\right)\frac{\dot\alpha_3}{\alpha_3} = -\frac{h_1}{3}\left(\frac{(2m+1)}{(m+1)}\right) \qquad (37)$$

On integrating eqn. (36) we get

$$\dot\alpha_3 = h_3\alpha_3^{-\frac{k_1+m^2}{m+1}} \qquad e^{-\frac{h_2}{3}\left(\frac{2m+1}{m+1}\right)t} \qquad (38)$$

Where h_3 is constant on integration.

Again integrating eqn. (36) we get

$$\alpha_3 = \left[\frac{k_1+m^2+m+1}{m+1}\right]^{\frac{m+1}{k_1+m^2+m+1}}.$$

$$\boxtimes\left[-\frac{3}{h_1}\left(\frac{m+1}{2m+1}\right)\, e^{\frac{h}{3}\left(\frac{2m+1}{m+1}\right)}+h_4\right]^{\frac{m+1}{k_1+m^2+m+1}} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (39)}$$

Where h_4 is constant of integration

$$\alpha_2=\left[\frac{k_1+m^2+m+1}{m+1}\right]^{m\left(\frac{m+1}{k_1+m^2+m+1}\right)}.$$

$$\boxtimes\left[-\frac{3}{h_1}\left(\frac{m+1}{2m+1}\right)\, e^{\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t}+h_4\right]^{m\left(\frac{m+1}{k_1+m^2+m+1}\right)} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (40)}$$

$$\alpha_1=h\left[\frac{k_1+m^2+m+1}{m+1}\right]^{m\left(\frac{m+1}{k_1+m^2+m+1}\right)}.$$

$$\boxtimes\left[-\frac{3}{h_1}\left(\frac{m+1}{2m+1}\right)\, e^{\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t}+h_4\right]^{m\left(\frac{m+1}{k_1+m^2+m+1}\right)} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (41)}$$

$$ds^2=-dt^2+\left[\frac{k_1+m^2+m+1}{m+1}\right]^{\frac{2m(m+1)}{k_1+m^2+m+1}}.$$

$$\boxtimes\left[-\frac{3}{h_1}\left(\frac{m+1}{2m+1}\right)h_3\, e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t}+h_4\right]^{\frac{2m(m+1)}{k_1+m^2+m+1}}\boxtimes\left(h^2dx^2+e^{-2\delta x}dy^2\right)$$

$$+\left[\frac{k_1+m^2+m+1}{m+1}\right]^{2\left(\frac{m+1}{k_1+m^2+m+1}\right)}\boxtimes\left[-\frac{3}{h_1}\left(\frac{m+1}{2m+1}\right)h_3 e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t}+\right.$$

$$\left.h_4\right]^{\frac{2(m+1)}{k_1+m^2+m+1}}dz^2$$

$$\text{\_\_\_\_\_\_\_\_\_\_\_ (42)}$$

For the model (41), the expressions for the energy density δ, the string tension density, μ the particle density ρ_p, the expansion scalar θ, the shear scalar σ and the cosmological term are respectively given by

$$\rho=(m^2+2m)h_3^2\left[\frac{k_1+m^2+m+1}{m+1}\right]^{-2}$$

$$\boxtimes\left[-\frac{3}{h_1}\left(\frac{m+1}{2m+1}\right)h_3 e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t}+h_4\right]e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t}$$

$$-\delta^2\left[\frac{k_1+m^2+m+1}{m+1}\right]^{-2m}$$

$$\boxtimes\left[-\frac{3}{h_1}\left(\frac{m+1}{2m+1}\right)h_3 e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t}+h_4\right]^{-2m\left(\frac{m+1}{k_1+m^2+m+1}\right)}$$

$$-\frac{h_1(2m+1)}{3}h_3\left[\frac{k_1+m^2+m+1}{m+1}\right]^{-1}$$

$$\boxtimes\left[-\frac{3}{h_1}\left(\frac{m+1}{2m+1}\right)h_3e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t}+h_4\right]^{-1}e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t} \quad \_\_(43)$$

$$\mu=\left[\frac{-2m^2h_1+5m\,h_1+h_1}{3\,(m+1)}\right]h_3\left[\frac{k_1+m^2+m+1}{m+1}\right]^{-1}$$

$$\boxtimes\left[-\frac{3}{h_1}\left(\frac{m+1}{2m+1}\right)h_3e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t}+h_4\right]^{-1}$$

$$-\left(\frac{m^3+m^2+2m}{m+1}\right)h_3^2\left[\frac{k_1+m^2+m+1}{m+1}\right]^{-2}$$

$$\boxtimes\left[-\frac{3}{h_1}\left(\frac{m+1}{2m+1}\right)h_3e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t}+h_4\right]^{-2}$$

$$-\delta^2\left[\frac{k_1+m^2+m+1}{m+1}\right]^{-2m\left(\frac{m+1}{k_1+m^2+m+1}\right)}$$

$$\boxtimes\left[-\frac{3}{h_1}\left(\frac{m+1}{2m+1}\right)h_3e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t}+h_4\right]^{-2m\left(\frac{m+1}{k_1+m^2+m+1}\right)}$$

$$-\frac{m-1}{m+1}k \qquad \_\_\_\_\_\_\_\_\_\_\_\_\_\_(44)$$

$$\rho_p=\left(\frac{2m^3+4m^2+4m}{m+1}\right)h_3^2\left[\frac{k_1+m^2+m+1}{m+1}\right]^{-2}$$

$$\boxtimes\left[-\frac{3}{h_1}\left(\frac{m+1}{2m+1}\right)h_3e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t}+h_4\right]^{-2}-e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t}$$

$$-\left[8\frac{mh_1+2h_1}{3(m+1)}\right]h_3\left[\frac{k_1+m^2+m+1}{m+1}\right]^{-1}$$

$$\boxtimes\left[-\frac{3}{h_1}\left(\frac{m+1}{2m+1}\right)h_3e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t}+h_4\right]^{-1}-\left(\frac{m-1}{m+1}\right)k \quad \_(45)$$

$$\theta=(2m+1)\left(\frac{k_1+m^2+m+1}{m+1}\right)$$

$$\boxtimes\left[-\frac{3}{h_1}\left(\frac{m+1}{2m+1}\right)h_3e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t}+h_4\right]^{-1}e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t} \quad \_(46)$$

$$\Lambda=\frac{h_1(2m+1)}{3}h_3\left(\frac{k_1+m^2+m+1}{m+1}\right)^{-1}$$

$$\boxtimes\left[-\frac{3}{h_1}\left(\frac{m+1}{2m+1}\right)h_3e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t}+h_4\right]e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t} \quad \_(47)$$

$$\sigma = \frac{1}{\sqrt{3}}(m-1)h_3\left[\frac{k_1+m^2+m+1}{m+1}\right]^{-1}$$
$$\boxtimes\left[-\frac{3}{h_1}\left(\frac{m+1}{2m+1}\right)h_3 e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t}+h_4\right]e^{-\frac{h_1}{3}\left(\frac{2m+1}{m+1}\right)t} \quad \_\_\_(48)$$

II. Discussion

In this paper we have studied Bianchi type III string cosmological models with the help of cosmological constant and Bulk viscosity. We adapt the condition $\eta \propto \theta$, $\theta \propto \sigma$ and $\Lambda \propto \mathrm{H}$ then the cosmological model for a string cosmology with bulk viscosity and cosmological term is obtained. The energy density $\rho \to \infty$ when $t \to 0$ and $\rho \to 0$ when $t \to \infty$ therefore the model describes a shearing, non rotating continuously expanding inverse with a big bang start. The physical and geometrical aspects of the model are also discussed.

Reference

[1] Nightingale J.P. (1973); J. Astrophys 185, 105.
[2] Pavon D; Bafaluy J. Nad Jou D. (1991); class quantum gravity, 8, 357.
[3] Maartens R. (1995); class quantum Gravity, 12, 455.
[4] Kalyani D. and Singh G.P. (1997); In new direction in relativity and cosmology.
[5] Singh T.; Beesham A. and Mbokazi W.S. (1998); Gen. relativity, Granit 30, 537.
[6] Ng, Y. J. (1992); Int. J. Mod Phys. D1, 145.
[7] We inberg, S. (1989); Rev. Mod. Phys. 611.
[8] R. Bali and S. Dave, “Bianchi type III string cosmological models with bulk viscous fluid in general relativity”. Astrophysics and space science 2002, 282, 1, 461-467.
[9] X. X. wang chin Phys. Lett. 2004, 21, 1205.
[10] Misner C. W. (1967); Nature, 214, 40.
[11] Misner C.W. (1968); J. Astrophys, 151, 43.
[12] Heller M. and Klimek M. (1975); Astrophys, space sci. 33, 261.
[13] Zeldovich, Ya B. (1968); Sor Phys. Vps 11, 381.
[14] Linde, A. D. (1974); Zetp Lett. 19, 183.
[15] Krauss, L. M. and Turner M. S. (1995) Gen. Rol. Grav. 27, 1137.
[16] P. S. Letelier Phys. Rev. D. 1983, 28, 2414.
[17] Singh, J. P. Tiwari, R. K. and Pratibha Shukla (2007) Chin. Phys. Lett. (12) 24.
[18] A. G. Reiss et. al. (2004); Astrono. J. 607, 665.

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