

Analysis on Time Taken For Recruitment in a Single Grade Marketing Organization with Classified Policy Decisions Having Non-Instantaneous Wastages as a Geometric Process with Two Control Limits

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Abstract: In this paper, for a single grade marketing organization with non- instantaneous shortage in manpower (wastage) due to policy decisions, classified according to their intensity of attrition, the moments of time taken for recruitment are obtained using a suitable policy for recruitment when (i) the system has a control limit for alertness (referred as optional threshold), in addition to the breakdown threshold (referred as mandatory threshold) for cumulative shortage in manpower (ii) the shortage process of manpower due to exits form a geometric process (iii) both the thresholds have an additional component for the cumulative shortage in manpower due to frequent breaks (iv) the exits form an ordinary renewal process with exponential inter-exits, according as the inter-policy decision times form a geometric process or an order statistics.

Keywords: Single grade marketing organization; Classified policy decisions; Non-instantaneous exits; Geometric process; optional and mandatory thresholds with two components and Policy of recruitment.

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I. Introduction

The authors in [1, 2, 3] are the pioneers in the study of stochastic manpower models using suitable statistical techniques. In any marketing organization, shortage in manpower occurs due to policy decisions. As recruitment carries cost, frequent recruitment is not a good option. Hence the shortage in manpower is permitted till it reaches a level called threshold. As in reliability theory, for replacement of systems using shock model approach, recruitment is done in the organization when the cumulative shortage in manpower exceeds this threshold. In [4] the author has studied the problem with instantaneous exits by considering an optional control limit as an alertness level prior to mandatory threshold. In [5] the authors have studied the manpower planning problem for a single grade system with non-instantaneous exits by considering single threshold which is a mandatory threshold. In [7, 11, 13] the authors have studied the work in [5] by considering optional and mandatory control limits for cumulative shortage process and associating different stochastic processes for successive decision times and shortage processes respectively. In [8] the authors have studied their work in [7] when the decisions are classified according to attrition rate. In [9] the authors have analyzed the problem in [8] when both the thresholds have an additional component for the cumulative shortage in manpower due to frequent breaks. Recently, in [10, 12] the authors have extended their work in [9] for correlated and order statistics shortage processes. The present paper extends the research work in [13] when both the thresholds have an additional component for the cumulative shortage in manpower system due to frequent breaks.

II. Description Of The Model

A single grade marketing organization is assumed to take policy decisions at random epochs in $(0, \infty)$ and random number of persons quit this organization leading to shortage in manpower. It is assumed that the exits are not instantaneous and shortage in manpower is linear and cumulative. While the shortage in manpower caused at the i^{th} exit point is denoted by X_i , the cumulative shortage in manpower in the first k exit points is denoted by S_k . It is assumed that $\{X_i\}_{i=1}^{\infty}$ form a geometric process with rate d, ($d > 0$). The distribution of X_1 is $M(t) = 1 - e^{-\alpha t}$, $\alpha > 0$. It is assumed that the decisions are classified into two types according to high and low rates of attritions. The proportions of decisions having high attrition rate λ_1 is denoted by p_1 . The proportions of decisions having low attrition rate λ_2 is denoted by $(1 - p_1)$. The sequence of inter-decision times and inter-

exit times are respectively denoted by $\{U_k\}_{k=1}^\infty$ and $\{W_i\}_{i=1}^\infty$. It is assumed that W_i 's are independent random variables with cumulative distribution function $G(x) = 1 - e^{-\alpha x}$ and probability density function $g(\cdot)$. The number of exit points in $(0,t]$ is denoted by $N_e(t)$ and it is assumed to be independent of X_i for each i and for all $t > 0$. The mandatory threshold level for the cumulative shortage in manpower in the organization is denoted by Z and the additional control limit for alertness prior to Z is denoted by Y . It is assumed that Y and Z are independent of shortage process $\{X_i\}_{i=1}^\infty$. While the first component of Y (Z) corresponding to cumulative shortage in manpower due to attrition is denoted by Y_1 (Z_1), the second component of Y (Z) corresponding to frequent breaks taken by the personnel working in the system is represented by Y_2 (Z_2). Clearly $Y = Y_1 + Y_2$ and $Z = Z_1 + Z_2$. The cumulative distributions of Y and Z are denoted by $H_1(\cdot)$ and $H_2(\cdot)$ respectively. It is assumed that (i) Y_1 and Y_2 are independent with respective distributions $H_{11}(y) = 1 - e^{-\theta_1 y}$ and $H_{12}(y) = 1 - e^{-\theta_2 y}$ and (ii) Z_1 and Z_2 are independent with respective distributions $H_{21}(z) = 1 - e^{-\gamma_1 z}$ and $H_{22}(z) = 1 - e^{-\gamma_2 z}$. The organization may or may not go for recruitment when the cumulative shortage in manpower exceeds Y . It is assumed that the organization makes recruitment with probability ' p ' when the cumulative shortage in manpower exceeds Y . It is also assumed that policy decisions produce exit with probability ' q ' ($q \neq 0$). The time taken for recruitment is denoted by T with cumulative distribution function $L(\cdot)$ and probability density function by $l(\cdot)$. The mean and variance of T are denoted by $E(T)$ and variance $V(T)$ respectively. The Laplace transform of the function $a(\cdot)$ is denoted by $\bar{a}(\cdot)$. Regarding the policy of recruitment, it is to be noted that when cumulative shortage in manpower exceeds Y , the organization has the option to go or not to go for recruitment. However, recruitment is to be done when this cumulative shortage exceeds Z .

III. Main Result

From the recruitment policy, the tail distribution of time taken for recruitment is given by

$$P(T > t) = P[S_{N_e(t)} \leq Y] + P[Y < S_{N_e(t)} \leq Z]p \tag{1}$$

Since $N_e(t)$ is independent of X_i and $P[N_e(t) = k] = G_k(t) - G_{k+1}(t)$ with $G_0(t) = 1$, from Renewal theory [6], using law of total probability, we get

$$P(T > t) = \sum_{k=0}^{\infty} [G_k(t) - G_{k+1}(t)]P(S_k \leq Y) + p \sum_{k=0}^{\infty} [G_k(t) - G_{k+1}(t)]P(S_k > Y)P(S_k \leq Z) \tag{2}$$

Since $\{X_i\}$ is a geometric process with rate d , it can be shown that

$$\bar{m}_k(s) = \prod_{i=1}^k \bar{m}\left(\frac{s}{d^{i-1}}\right), k=1,2,3,\dots \tag{3}$$

Since Y is independent of S_k , $Y = Y_1 + Y_2$, Y_1 and Y_2 are independent exponential random variables with means $\frac{1}{\theta_1}$ and $\frac{1}{\theta_2}$ respectively by hypothesis, conditioning upon S_k and using law of total probability and on

simplification, we get

$$P(S_k \leq Y) = (1 + N_1)\bar{m}_k(\theta_2) - N_1\bar{m}_k(\theta_1), \tag{4}$$

Similarly, we get

$$P(S_k \leq Z) = (1 + N_2)\bar{m}_k(\gamma_2) - N_2\bar{m}_k(\gamma_1), \tag{5}$$

where $N_1 = \frac{\theta_2}{(\theta_1 - \theta_2)}$ and $N_2 = \frac{\gamma_2}{(\gamma_1 - \gamma_2)}$. \tag{6}

From (2), (4) and (5), it can be shown that

$$P(T > t) = 1 + N_1 \sum_{k=1}^{\infty} G_k(t) S_1 - (1 + N_1) \sum_{k=1}^{\infty} G_k(t) S_2 + p \left\{ N_2 \sum_{k=1}^{\infty} G_k(t) S_3 - (1 + N_2) \sum_{k=1}^{\infty} G_k(t) S_4 \right\} + p \left\{ -N_1(1 + N_2) \sum_{k=1}^{\infty} G_k(t) S_5 + (1 + N_1)(1 + N_2) \sum_{k=1}^{\infty} G_k(t) S_6 + N_1 N_2 \sum_{k=1}^{\infty} G_k(t) S_7 - (1 + N_1) N_2 \sum_{k=1}^{\infty} G_k(t) S_8 \right\}. \quad (7)$$

Since $\bar{l}(s)$ is same as the Laplace –Stieltjes transform of $\frac{d}{dt}[1 - P(T > t)]$, using convolution theorem for Laplace transform, from (7), we get

$$\bar{l}(s) = -N_1 \sum_{k=1}^{\infty} (\bar{g}(s))^k S_1 + (1 + N_1) \sum_{k=1}^{\infty} (\bar{g}(s))^k S_2 - p \left\{ N_2 \sum_{k=1}^{\infty} (\bar{g}(s))^k S_3 - (1 + N_2) \sum_{k=1}^{\infty} (\bar{g}(s))^k S_4 \right\} + p \left\{ -N_1(1 + N_2) \sum_{k=1}^{\infty} (\bar{g}(s))^k S_5 + (1 + N_1)(1 + N_2) \sum_{k=1}^{\infty} (\bar{g}(s))^k S_6 + N_1 N_2 \sum_{k=1}^{\infty} (\bar{g}(s))^k S_7 - (1 + N_1) N_2 \sum_{k=1}^{\infty} (\bar{g}(s))^k S_8 \right\}. \quad (8)$$

where $S_1 = \overline{m_{k-1}}(\theta_1) - \overline{m_k}(\theta_1)$, $S_2 = \overline{m_{k-1}}(\theta_2) - \overline{m_k}(\theta_2)$, $S_3 = \overline{m_{k-1}}(\gamma_1) - \overline{m_k}(\gamma_1)$, $S_4 = \overline{m_{k-1}}(\gamma_2) - \overline{m_k}(\gamma_2)$
 $S_5 = \overline{m_{k-1}}(\theta_1) \overline{m_{k-1}}(\gamma_2) - \overline{m_k}(\theta_1) \overline{m_k}(\gamma_2)$, $S_6 = \overline{m_{k-1}}(\theta_2) \overline{m_{k-1}}(\gamma_2) - \overline{m_k}(\theta_2) \overline{m_k}(\gamma_2)$,
 $S_7 = \overline{m_{k-1}}(\theta_1) \overline{m_{k-1}}(\gamma_1) - \overline{m_k}(\theta_1) \overline{m_k}(\gamma_1)$, $S_8 = \overline{m_{k-1}}(\theta_2) \overline{m_{k-1}}(\gamma_1) - \overline{m_k}(\theta_2) \overline{m_k}(\gamma_1)$ (9)

The raw moments of T can be computed from (8) using the result

$$E(T^r) = (-1)^r \left[\frac{d^r}{ds^r} \bar{l}(s) \right]_{s=0}, \quad r = 1, 2, 3, \dots \quad (10)$$

Therefore from (8) and (10), we get

$$E(T) = \bar{g}'(0) \left\{ N_1 \sum_{k=1}^{\infty} k S_1 - (1 + N_1) \sum_{k=1}^{\infty} k S_2 + p \left[N_2 \sum_{k=1}^{\infty} k S_3 - (1 + N_2) \sum_{k=1}^{\infty} k S_4 - N_1(1 + N_2) \sum_{k=1}^{\infty} k S_5 \right] + p \left[(1 + N_1)(1 + N_2) \sum_{k=1}^{\infty} k S_6 + N_1 N_2 \sum_{k=1}^{\infty} k S_7 - (1 + N_1) N_2 \sum_{k=1}^{\infty} k S_8 \right] \right\} \quad (11)$$

$$E(T^2) = -N_1 \sum_{k=1}^{\infty} k S_1 \left[\bar{g}''(0) + (k-1)(\bar{g}'(0))^2 \right] + (1 + N_1) \sum_{k=1}^{\infty} k S_2 \left[\bar{g}''(0) + (k-1)(\bar{g}'(0))^2 \right] - p \left\{ N_2 \sum_{k=1}^{\infty} k S_3 \left[\bar{g}''(0) + (k-1)(\bar{g}'(0))^2 \right] - (1 + N_2) \sum_{k=1}^{\infty} k S_4 \left[\bar{g}''(0) + (k-1)(\bar{g}'(0))^2 \right] - N_1(1 + N_2) \sum_{k=1}^{\infty} k S_5 \left[\bar{g}''(0) + (k-1)(\bar{g}'(0))^2 \right] + (1 + N_1)(1 + N_2) \sum_{k=1}^{\infty} k S_6 \left[\bar{g}''(0) + (k-1)(\bar{g}'(0))^2 \right] + N_1 N_2 \sum_{k=1}^{\infty} k S_7 \left[\bar{g}''(0) + (k-1)(\bar{g}'(0))^2 \right] - (1 + N_1) N_2 \sum_{k=1}^{\infty} k S_8 \left[\bar{g}''(0) + (k-1)(\bar{g}'(0))^2 \right] \right\} \quad (12)$$

Equations (11) and (12) give the first two moments of time taken for recruitment for single grade marketing organization.

We now determine moments of time taken for recruitment for two different cases on inter-policy decision times.

Case(i): The process of inter-policy decision times is a geometric process with rate $c, (c > 0)$ and the distribution $F(\cdot)$ of U_1 is $F(t) = 1 - [p_1 e^{-\lambda_1 t} + (1 - p_1) e^{-\lambda_2 t}]$, $\lambda_1, \lambda_2 > 0$.

An explicit result connecting the distributions of inter-exit times and inter-decision times is given by

$$G(x) = q \sum_{n=1}^{\infty} (1 - q)^{n-1} F_n(x). \quad (13)$$

Therefore $\bar{g}(s) = q \sum_{n=1}^{\infty} (1 - q)^{n-1} \bar{f}_n(s)$, where $\bar{f}_n(s) = \prod_{k=1}^n \bar{f}\left(\frac{s}{c^{k-1}}\right)$ (14)

From (14), we get

$$\bar{g}'(0) = \frac{c}{(c-1+q)} \bar{f}'(0), \text{ where } \bar{f}'(0) = -\left(\frac{p_1}{\lambda_1} + \frac{1-p_1}{\lambda_2}\right) \quad (15)$$

From (11) and (15), we get

$$E(T) = \frac{c}{(c-1+q)} \bar{f}'(0) \left\{ N_1 \sum_{k=1}^{\infty} k S_1 - (1+N_1) \sum_{k=1}^{\infty} k S_2 + p \left[N_2 \sum_{k=1}^{\infty} k S_3 - (1+N_2) \sum_{k=1}^{\infty} k S_4 - N_1(1+N_2) \sum_{k=1}^{\infty} k S_5 \right] \right. \\ \left. + p \left[(1+N_1)(1+N_2) \sum_{k=1}^{\infty} k S_6 + N_1 N_2 \sum_{k=1}^{\infty} k S_7 - (1+N_1) N_2 \sum_{k=1}^{\infty} k S_8 \right] \right\} \quad (16)$$

From (14) and on simplification, we get

$$\bar{g}''(0) = \frac{c^2}{(c^2-1+q)} \bar{f}''(0) + \frac{2c^2(1-q)}{(c^2-1+q)(c-1+q)} (\bar{f}'(0))^2, \text{ where } \bar{f}''(0) = 2 \left(\frac{p_1}{\lambda_1^2} + \frac{1-p_1}{\lambda_2^2} \right) \quad (17)$$

Variance of time taken for recruitment for case (i) can be determined from (16) and (12) using (15) and (17).

Case(ii): The process of inter-policy decision times is associated with an order statistics where the sample of size r is selected from a hyper-exponential population of independent and identically distributed inter-policy decision times with a common distribution function $F(\cdot)$ given as in case(i).

From [8], the probability density function $f_{u(j)}(\cdot)$ of the j^{th} order statistics is given by

$$f_{u(j)}(t) = j \binom{r}{j} [F(t)]^{j-1} f(t) [1-F(t)]^{r-j}, \quad j = 1, 2, \dots, r. \quad (18)$$

Suppose $f(t) = f_{u(1)}(t)$.

From (14) and (18), we get

$$\bar{g}'(0) = \frac{-1}{q} \overline{f_{u(1)}}'(0) \quad (19)$$

From (11) and (19), we get

$$E(T) = \frac{-1}{q} \overline{f_{u(1)}}'(0) \left\{ N_1 \sum_{k=1}^{\infty} k S_1 - (1+N_1) \sum_{k=1}^{\infty} k S_2 + p \left[N_2 \sum_{k=1}^{\infty} k S_3 - (1+N_2) \sum_{k=1}^{\infty} k S_4 - N_1(1+N_2) \sum_{k=1}^{\infty} k S_5 \right] \right. \\ \left. + p \left[(1+N_1)(1+N_2) \sum_{k=1}^{\infty} k S_6 + N_1 N_2 \sum_{k=1}^{\infty} k S_7 - (1+N_1) N_2 \sum_{k=1}^{\infty} k S_8 \right] \right\} \quad (20)$$

From (14), (18) and on simplification, we get

$$\bar{g}''(0) = \frac{1}{q^2} \left[q \overline{f_{u(1)}}''(0) + 2(1-q) \left(\overline{f_{u(1)}}'(0) \right)^2 \right] \quad (21)$$

$$\overline{f_{u(1)}}'(0) = -A_r, \text{ where } A_r = \sum_{n=0}^r \frac{{}^r C_n p_1^n (1-p_1)^{r-n}}{[\lambda_1 n + (r-n)\lambda_2]} \quad (22)$$

$$\overline{f_{u(1)}}''(0) = 2B_r, \text{ where } B_r = \sum_{n=0}^r \frac{{}^r C_n p_1^n (1-p_1)^{r-n}}{[\lambda_1 n + (r-n)\lambda_2]^2} \quad (23)$$

Variance of time taken for recruitment for case (ii) when $f(t) = f_{u(1)}(t)$ can be determined from (20) and (12) using (21), (22) and (23).

Suppose $f(t) = f_{u(r)}(t)$.

From (14) and (18), we get

$$\overline{g}'(0) = \frac{1}{q} \overline{f_{u(r)}'}(0) \tag{24}$$

From (11) and (24), we get

$$E(T) = \frac{-1}{q} \overline{f_{u(r)}'}(0) \left\{ N_1 \sum_{k=1}^{\infty} k S_1 - (1 + N_1) \sum_{k=1}^{\infty} k S_2 + p \left[N_2 \sum_{k=1}^{\infty} k S_3 - (1 + N_2) \sum_{k=1}^{\infty} k S_4 - N_1 (1 + N_2) \sum_{k=1}^{\infty} k S_5 \right] + p \left[(1 + N_1)(1 + N_2) \sum_{k=1}^{\infty} k S_6 + N_1 N_2 \sum_{k=1}^{\infty} k S_7 - (1 + N_1) N_2 \sum_{k=1}^{\infty} k S_8 \right] \right\} \tag{25}$$

From (14), (18) and on simplification, we get

$$\overline{g}''(0) = \frac{1}{q^2} \left[q \overline{f_{u(r)}''}(0) + 2(1 - q) \left(\overline{f_{u(r)}'}(0) \right)^2 \right] \tag{26}$$

$$\overline{f_{u(r)}'}(0) = C_r, \text{ where } C_r = \sum_{n=0}^r \sum_{n_1=0}^{r-n} \frac{{}^r C_n (-1)^{r-n} {}^{(r-n)} C_{n_1} p_1^{n_1} (1 - p_1)^{r-n-n_1}}{[\lambda_1 n_1 + (r - n - n_1) \lambda_2]} \tag{27}$$

$$\overline{f_{u(r)}''}(0) = -2D_r, \text{ where } D_r = \sum_{n=0}^r \sum_{n_1=0}^{r-n} \frac{{}^r C_n (-1)^{r-n} {}^{(r-n)} C_{n_1} p_1^{n_1} (1 - p_1)^{r-n-n_1}}{[\lambda_1 n_1 + (r - n - n_1) \lambda_2]^2} \tag{28}$$

Variance of time taken for recruitment for case (ii) when $f(t) = f_{u(r)}(t)$ can be determined from (25) and (12) using (26), (27) and (28).

Note:

1. Analytical result which is not yet attempted on variance of time taken for recruitment when the inter-decision times are independent and identically distributed exponential random variables can be obtained by taking $c=1$ in case (i).
2. Our result on variance of time taken for recruitment in case (i) when $c = 1$ and $p = 0$ subsumes the analogous result of case (i) in [5] when the shortage process is a geometric process.
3. Our result on variance of time taken for recruitment in case (i) when $c = 1$ and $p_1 = 1$ subsumes the analogous result of case (i) in [7] when the shortage process is a geometric process.
4. Depending upon the situation, a marketing organization may either wish to elongate or advance the time taken for recruitment. This can be done by observing the impact on the variation of nodal parameters over the performance measures such as the average time taken for recruitment.

IV. Conclusion

The Stochastic model discussed in this paper is new in the context of bringing another type of dependence among loss of manpower by associating geometric process to the shortage process and associating another geometric process (Case – (i)) and order statistics (Case – (ii)) to inter-decision process in the presence of (i) Non-instantaneous exits (ii) a chance factor for any decision to have exit points (iii) provision of an alertness level as an additional control limit and (iv) classified policy decisions with different attrition rates in the manpower system.

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