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Some allied regular spaces via gsp-open sets in topology

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Abstract:*In this paper we define and study some allied regular spaces using gsp-open sets and gsp-closed sets , namely (sp,gsp)-regular spaces, gsp-regular spaces,(gsp,gs) -regular spaces, weakly g^*regular spaces, (gsp,sp)-regular spaces,(p,gsp)-regular spaces and strongly gsp-regular spaces. also, we defined some basic characterization of above mentioned regular spaces.*

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***Key words:** semipreopen sets, gsp-closed sets .g^*-closed sets preopen sets ,gs-closed sets, gsp-irresoluteness, and strongly g^*-continuums functions*

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I. Introduction

In 1982 A S Mashhour et al[10] have defined and studied the concept of pre-open sets and Spre-continuous functions of topology. In 1983 S.N.Deeb et al [7] have defined and studied the concept of pre-closed sets ,precloseropearater,p-regular spaces and pre-closed functions in topology. In 1986, D. Andrijivic [1] introduced and studied the notion of semipre open sets, semipreclosed sets ,semipreinterior operator and semipre-closed operator in topological spaces. Later, many topologists have been studied these above mentioned sets in the literature. For the first time , N.Levine [9] has introduced the notion g-closed sets and g-open sets in topology. S P Arya et.al[2] have defined and studied the nontion of gs-closed sets and gs-open sets in 1990. In 1995 , J.Dontchev[6] has defined and studied of concept of gsp-closed sets, gsp-open sets , gsp-continuous function and gsp-irresoluteness in topology. In 2000 M.K.R.S . Veera kumar[11] has defined and studied of properties of g*-closed sets in topological spaces. In this paper , using pre-closed sets ,semipre-open sets ,gsp-closed sets ,gs-open sets , g*-closed sets. We define and study the concepts of (sp,gsp)-regular spaces,gsp-regular spaces,(gsp,gs) -regular spaces, weakly g^*regular spaces, (gsp,sp)-regularspaces,(p,gsp)-regular spaces and strongly gsp-regular spaces

II. Preliminaries

Throughout this paper (X , τ) and (Y, σ) (or simply X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated . If A be a subset of X, the Closure of A and Interior of A denoted by Cl(A) and Int(A)respectivly.

We give the following define are useful in the sequel :

DEFINITION 2.1 : A subset A of space (X , τ) is said to becalled

(i) semi-open set [8] if $A \subset$ Cl (Int (A))

(ii) pre-open set [10] if A $\subset$IntCl(A)

(iii) semi-pre open set [1] if $A \subset$ Cl (Int (Cl(A)))

The complement of a semiopen (resp. preopen , semipreopen) set of a space X is called semiclosed [3] (resp. preclosed [7] ,semipreclosed [1]) set in X.

The family of all semi open (resp. preopen ,semi-pre open) sets of X will be denoted by SO(X) (resp. PO(X) , SPO(X)).

Definition 2.2[4] : The intersection of all semi-closed sets of X containing subset A is called the semi-closure of A and is denoted by sCl(A).

Definition 2.3[1] : The intersection of all semipre-closed sets of X containing subset A is called the semipre-closure of A and is denoted by spCl(A).

Definition 2.4[5]: The union of all semi-open sets of X contained in A is called the semi-interior of A and is denoted by sInt (A).

Definition 2.5[1]: The union of all semipre-open sets of X contained in A is called the semipre-interior of A and is denoted by spInt(A).

Definition 2.6 : A sub set A of a space X is said to be :
(i) a generalized closed (briefly, g- closed) **[9]** set if Cl(A)$\subseteq U$ whenever A$\subseteq U$ and U is open set in (X, τ)
(ii) a generalized semi-closed (briefly, gs- closed) [**2**] set if sCl(A)$\subseteq U$ whenever A$\subseteq U$ and U is open set in (X, τ)
(iii) a generalized semi-preclosed (briefly, gsp- closed) [**6**] set if spCl(A)$\subseteq U$ whenever A$\subseteq U$ and U is open in (X, τ)
(iv) a g^*-closed set[7] if Cl(A)$\subseteq U$ whenever A$\subseteq U$ and U is g-open set in (X, τ)

Definition 2.7 [11] : A topological space X is said to be g-regular if for each g–closed set F of X and each point x$\notin$ F there exist disjoint open sets U and V of X such that x$\in$U and F$\subset$V

III. Properties of (gsp,sp)-regular spaces

We ,define the following

Definition 3.1: A topological space X is said to be gsp- regular if for each gsp-closed set F of X and each point x$\in X-F$, there exist disjoint open sets U and V of X such that x$\in U$ and $F\subset V$
Since every g-closed set is gsp-closed set so every gsp-regular space is g-regular space.

Theorem 3.2 A topological space X is gsp-regular if and only if for each gsp-closed set F of X and each point x$\in X-F$, there exist open sets U and V of X such that x$\in U$: $F\subset V$ and Cl(U)$\bigcap Cl(V)=\emptyset$

Proof: Necessity: Let F be a gsp-closed set of X and x$\in X-F$. There exist open sets U_0 and V of X such that x$\in$ U_0 , $F\subset V$ and, $U_0\cap V=\emptyset$, hence $U_0\cap Cl(V)=\emptyset$. Since X is gsp-regular, there exist open sets G and H of X such that x$\in$G Cl(V)$\subset H$ and , G$\cap$H= $\emptyset$, hence Cl(G)$\cap$H= $\emptyset$ .Now put U= $U_0\cap G$, then U and V are open sets of X such that x$\in$ U , $F\subset V$ and Cl(U)$\bigcap Cl(V)=\emptyset$.

Sufficiency: This is obvious.
Theorem 3.3: Let X be a topological space then the following statements are equivalent:
(i) X is gsp-regular space
(ii) For each point x$\in$X and for each gsp-open neighbourhood W of x, there exists a open set of x , such that Cl(V)$\subseteq W$
(iii) For each point of x$\in$X and for each gsp-closed not containing x, then there exists a open set V of X such that Cl(V)$\bigcap F=\emptyset$.

Proof: (i)$\Longrightarrow$**(ii)**: Let W be a gsp-open neighbourhood of x. Then there exists a gsp-open set G such that x$\in$X$\subseteq W$. Since (X-G) is gsp-closed set and $x\notin(X-G)$, by hypothesis there exist open sets U and V such that (X-G) $\subseteq U$, $x \in$ V and U∩V=∅ and so $V \subset (X - U)$. Now $Cl(V) \subseteq Cl(X - U) = (X - U)$ and (X-G) $\subseteq U$ implies $(X - U) \subseteq G \subseteq W$. There fore $Cl(V) \subseteq W$

(ii)⟹**(i)**: Let F be any gsp-closed set of $x \notin F$. Then $x \in X - F$ and (X-F) is gsp-open and so (X−F) is an gsp-open neighbourhood of x. By hypothesis there exists a open V of x such that $x \in V$ and $Cl(V) \subseteq (X - F)$ which implies $F \subseteq (X - Cl(V))$. Then $(X - Cl(V))$. is open set containing F and $V \cap (X - Cl(V)) = \emptyset$. Therefore X is gsp-regular space .

(ii)⟹(iii): Let $x \in X$ and F be an gsp-closed set such that $x \notin F$. Then (X-F) is an gsp-open neighbourhood of x and by hypothesis there exists a open set V of x such that $Cl(V) \subseteq (X - F)$ and there fore $Cl(V) \cap F = \emptyset$

(iii)⟹**(ii)**: Let $x \in X$ and W be an gsp-open neighbourhood of x then there exists an gsp-open set G such that $x \in G \subseteq W$. Since (X-G) is gsp-closed and $x \notin (X - G)$ by hypothesis there exists a open set V of x such that $Cl(V) \cap (X - G) = \emptyset$ There fore $Cl(V) \subseteq G \subseteq W$.

Theorem 3.4: A topological space X is an gsp-regular space if and only if given any x∈U and any open set U of X there is gsp-open set V such that x∈ V ⊂gspCl(V)⊆U.

Proof: Let U be an open set, x∈U. So X−U is closed set such that x∉U. Since X is a gsp-regular space then there exist gsp-open sets V_1 and V_2 such that $V_1 \cap V_2 = \emptyset$, $X - U \subset V_2$, $x \in V_1$. Since $V_1 \cap V_2 = \emptyset$, we have $gspCl(V_1) \subset gspCl(X - V_2) = X - V_2$. Since $X - U \subset V_2$, we have $X - V_2 \subset U$. Hence we have $x \in V_1 \subset gspCl(V_1) \subset (X - V_2) \subseteq U$.

Conversely, let F be a closed set in X and x∈X−F. So X−F is an open set such that x∈X−F. Hence there exists a gsp-open set U such that x∈U⊂gspCl(U) ⊂(X−F). Let V= X−gspCl(V). So V is a gsp-open set which contains F and U∩V=∅. Hence X is an gsp-regular space.

Theorem 3.5: Let X and Y be topological space and Y is a regular. If f:X→Y is closed gsp-irresolute and one to one then X is an gsp-regular space.

Proof: Let F be closed set in X, x∉F. Since f is closed mapping, then f(F) is closed set in Y, f(x)=y∉f(F). But Y is gsp-regular space then there are two open sets U and V in Y such that f(F) ⊆V, y∈U, U∩V=∅. Since f is gsp-irrersolute mapping and one to one so $f^{-1}(U)$, $f^{-1}(V)$ are two open sets X and x∈ $f^{-1}(U)$, F∈ $f^{-1}(V)$, $f^{-1}(U)$ ∈ $f^{-1}(V)$= ∅. Hence X is gsp-regular space.

We define the following.

Definition:3.6: A topological space X is said to be (gsp, gs)-regular if for each gsp-closed set of X and each point x∈X−F there exist disjoint gs-open sets U and V of X such that x∈U and F⊂V

Theorem 3.7: A topological space X is (gsp,gs)-regular if and only if for each gsp-closed set F of X and each point x∈X−F, there exist disjoint gs-open sets U and V of X such that x∈U,F⊂V and sCl(U) ∩sCl(V)= ∅.

Proof is similar to Theorem 3.6 above.

Theorem 3.8: Let X be a topological space then the following statements are equivalent:

(i) X is (gsp,gs)-regular space

(ii) For each point $x \in$ X and for each gsp-open neighbourhood W of x, there exists a open set of x , such that sCl(V) $\subseteq W$

(iii) For each point of $x \in$ X and for each gsp-closed not containing x, then there exists a gs-open set x such that sCl(V)∩ F = ∅.

Proof is similar to Theorem 3.3 above.

Theorem 3.9: A topological space X is an(gsp,gs)-regular space if and only if given gsp-open set U with x∈U, there exists gs-open set V such that x∈ *V* ⊂sCl(V)⊆U.

Proof: Let U be a gsp-open set, x∈U. So X−U isa gsp-closed set such that x∉X-U. Since X is a (gsp,gs)-regular space then there exist gs-open sets V_1 and V_2 such that $V_1 \cap V_2=\emptyset$, X−U⊂V_2, x∈V_1. Since $V_1 \cap V_2=\emptyset$, we have sCl(V_1) ⊂sCl(X−V_2)= X−V_2. Since X−U⊂V_2, we have X− V_2 ⊂ U. Hence we have x∈V_1⊂sCl(V_1) ⊂(X−V_2))⊆U.

Conversely, let F be a gsp- closed set in X and x∈X−F. So X−F is an gs- open set such that x∈X−F. Hence there exists a gs-open set U such that x∈U⊂sCl(V) ⊂(X−F). Let V= X−gspCl(V). So V is a gs-open set which contains F and U∩V=∅. Hence X is an (gsp, gs)-regular space.

In view of the definitions of g^*-closed sets, g-closed sets, αg-closed sets, gs-closed sets, sg-closed sets, gsp-closed sets, pre-closed sets, semipre-closed sets, semi-closed sets. we have the following implications.

Note 3.10 : Clearly, every closed set is a every g^*-closed set

Definition 3.11: A space X is said to be g^*-regular space if for each g^*-closed set F and for each x∈X−F there exist two disjoint open sets U and V such that x∈U and F⊂ V

Definition 3.12: A space X is said to be weakly g^*-regular space if for each closed set F and for each x∈X−F there exist two disjoint g^*-open sets U and V such that x∈U and F⊂ **V**

Theorem 3.13: A topological space X is g^*-regular if and only if for each g^*-closed set F of X and each point x∈X−F, there exist open sets U and V of X such that x∈U and F⊂ V and Cl(U) ∩Cl(V)= ∅

Proof: Necessity: Let F be a g^*-closed set of X and x∈X−F, there exists open sets U_0 and V of X such that x∈U_0 F⊂ V and $U_0 \cap V=\emptyset$, hence $U_0 \cap$Cl(V)=∅. Since X is g^*-regular, there exists open sets G and H of X such that x∈G,Cl(V) ⊂H and G∩H=∅, hence Cl(G)∩H=∅.Now put U= $U_0 \cap$G, then U and V are open sets of X such that x∈U, F⊂**V** and Cl(U) ∩Cl(V)= ∅.

Sufficiency: This is obvious.

The routine proof of the following theorem is omitted.

Theorem 3.14: Let X be a topological space, then the following statements are equivalent:

(i) X is g^*-regular space

(ii) For each point x∈X and for each closed neighbourhoods W of x, there exists a g^*-open set V of X such that g^*Cl(V) ⊆W

(iii) For each point x∈X and for each g^*-closed not containing x, then there exists g^*-open set V of X such that g^*Cl(V) ∩F=∅.

Theorem 3.15: A topological space X is an g^*-regular space if and only if given any x∈X and open set U of X there is g^*-open set V such that x∈V⊂ g^*Cl(V) ⊆U.

Proof: Let U be any open set, x∈U. So X−U is closed set such that x∉U. Since X is a g^*-regular space then there exist g^*-open sets V_1 and V_2 such that $V_1 \cap V_2=\emptyset$, X−U⊂ V_2, x∈ V_1. Since $V_1 \cap V_2=\emptyset$, we have X−V_2⊂U. Hence we have x∈V_1⊂ g^*Cl(V_1) ⊂(X−V_2) ⊆U.

Conversely, let F be a closed set in X and x∈ X−F so X−F is an open set such that x∈ X−F.Hence there exists a g-open set U such that x∈ *U* ⊂g^*Cl(V) ⊆X−F. Let V= X−g^*Cl(V). So V is a g^*-open set which contains F and U∩V=∅. Hence X is an g^*-regular space.

Now ,we define the following.

Definition 3.16: A function f:X→Y is called always g^*-closed if the image of each g^*-closed sets of X is g^*-closed in Y

Definition 3.17: A function $f:X\to Y$ is called g^*-closed if the image of each closed set of X is g^*-closed in Y

We prove the following

Theorem 3.18: Let X and Y be topological space and Y is regular space. If $f:X\to Y$ is closed, g^*-continuous and bijective, then X is weakly g^*-regular space.

Proof: Let F be a closed set in X, $x\notin F$. Since f is closed function, then f(F) is closed set in Y, $f(x)=y\notin f(F)$. But Y is regular space then there are two open sets U and V in Y such that $f(F)\subseteq V$. $y\in U$, $U\cap V=\emptyset$. Since $f^{-1}(U)$, $f^{-1}(V)$ are two g^*-open sets in X and $x\in f^{-1}(U)$, $F\subset f^{-1}(V)$, $f^{-1}(U)\cap f^{-1}(V)=\emptyset$. Hence X is weakly g^*-regular space.

We, define the following.

Definition 3.19: A function $f:X\to Y$ is strongly g^*-continuous if the inverse image of each g^*-closed set of Y is closed in X, equivalently, if the inverse image of each g^*-open set of Y is open in X.

Theorem 3.20: Let X and Y be topological space and Y is g^*-regular space. If $f:X\to Y$ is always g^*-closed, strongly g^*-continuous and bijective, then X is g^*-regular.

Proof is similar to **3**.18.

Definition 3.21: A topological space X is said to be (gsp, sp)-regular if for each gsp-closed set F of X and each point $x\in X-F$, there exist disjoint semipre-open sets U and V of X such that $x\in U$ and $F\subset V$.

Lemma 3.22: A subset A of a space X is said to be gs-open if $F\subseteq sInt(A)$ whenever $F\subseteq A$ and F is closed in X.

Theorem 3.23: The following properties are equivalent for a space in X;

(i) X is (gsp,sp) regular

(ii) for each gsp-closed set F and each point $x\in X-F$ there exist $U\in SPO(X)$ and a gsp-open set V such that $x\in U$, $F\subset V$ and $U\cap V=\emptyset$

(iii) for each subset of X and each gsp-closed set F such that $A\cap F=\emptyset$, there exist $U\in SPO(X)$ and a gsp-open set V such that $A\cap U\neq\emptyset$. $F\subset V$ and $U\cap V=\emptyset$

(iv) for each gsp-closed set F of X. $F=\cap\{spCl(V); F\subset V$ and V is gsp-open$\}$

Proof: (i)$\Longrightarrow$(ii): This proof is obvious since every semipre -open set is gsp-open set.

(ii)$\Longrightarrow$(iii): Let A be a subset of X and F a gsp-closed set X such that $A\cap F=\emptyset$ For a point $x\in A$, $x\in X-F$ and hence there exist $U\in SPO(X)$ and a gsp-open set V such that $x\in U$, $F\subset V$ and $A\cap F=\emptyset$

(iii)$\Longrightarrow$(i): Let F be any gsp-closed set of X and $x\in X-F$. Then $\{x\}\cap F=\emptyset$ and there exist $U=SPO(X)$ and a gsp-open set W such that $x\in U$, $F\subset W$ and $U\cap W=\emptyset$. Put $V=spInt(W)$, then by lemma 5.2.22. We have $F\subset V$, $V=SPO(X)$ and $U\cap V=\emptyset$There fore X is (gsp,sp)-regular.

(i)$\Longrightarrow$(iv): For any gsp-closed set F of X. We obtain $F\subset\cap\{spCl(V): F\subset V$ and V is gsp-open$\}$ $\subset\cap\{spCl(V): F\subset V$ and $V\in SPO(X)\}=F$**.** Therefore $F\subset\cap\{spCl(V): F\subset V$ and V is gsp-open$\}$

(iv)$\Longrightarrow$(i): Let F be any gsp-closed set of X and $x\in X-F$.By (iv), there exists a gsp-open set W of X and such that $F\subset W$ and $x\in X-spCl(W)$. Since F is gsp-closed, $F\subset spInt(W)$ by lemma 5.2.22. Put $V=SPO(X)$. Since $x\in X-spCl(W)$, $x\in X-spCl(V)$.Put $U=X-spCl(V)$, then $x\in U$, $U\in SPO(X)$ and $U\cap V=\emptyset$.This show that X is (gsp,sp)-regular.

Theorem 3.24: Let X be a topological space then the following statements are equivalents;

(i) X is (gsp,sp) -regular space

(ii) For each point $x\in X$ and for each gsp neighbourhood W of x, there exist sp-open sets V of x such that $spCl(V)\subseteq W$

(iii) For each point $x\in X$ and for each gsp -closed not containing x, there exists a sp-open set V of x such that $spCl(V)\cap F=\emptyset$

Proof is similar to Theorem 3.3 above.
We, define the following

Definition 3.25: A space X is said be (p,gsp) regular if for each preclosed set F and for each x∈ X−F there exist two disjoint gsp-open sets U and V of X such that x∈U and F⊂V.

Theorem 3.26: A topological space X is (p,gsp)-regular if and only if for each pre-closed set F of X and each point x∈ X−F, there exist gsp-open sets U and V of X. Such that x∈U, F⊂V and gspCl(U) ∩gspCl(V)= ∅.

Proof: Necessity: Let F be a pre-closed set of X and x∈ X−F, there exist gsp-open sets U_0, F⊂V and U_0∩ V=∅, hence U_0∩ gspCl(V)=∅. Since X is (p,gsp)-regular, there exist gsp-open sets G and H of X such that x∈G, gspCl(A) ⊂H and G∩H=∅,hence gspCl(G)∩H=∅. Now put U=U_0∩G, then U and V are gsp-open sets of X such that x∈U, F⊂V and gspCl(U) ∩gspCl(V)= ∅.

Sufficiency: This is obvious.

Theorem 3.27: Let X be a topological space then the following statements are equivalents;

(i) X is (p,gsp) -regular space

(ii) For each point x∈X and for each pre-neighbourhood W of x, there exist gsp-open sets V of x such that gspCl(V) ⊆W

(iii)For each point x∈X and for each pre-closed not containing x, there exists a gsp-open set V of x such that gspCl(V) ∩F=∅.

Proof is similar to Theorem 3.3 above.

4. Properties of strongly gsp regular spaces

Definition 4.1: A topological space X is said to be strongly gsp-regular if for each closed set F of X and each point x∈X−F there exist disjoint gsp-open sets U and V of X such that x∈U and F⊂V.

Theorem 4.2: Let X be a topological space then the following statements are equivalents;

(i) X is strongly -regular space

(ii) For each point x∈X and for each gspopen-neighbourhood W of x, there exist gsp-open sets V of x such that gspCl(V) ⊆W

(iii) For each point x∈X and for each closed not containing x, there exists a gsp-open set V of x such that gspCl(V) ∩F=∅.

Proof: (i)⟹(ii): Let W be a gsp-open neighbourhood W of x. Then there exists a gsp-open set G such that x∈X⊆W. Since (X−G) is closed set and x∉(X−G), by hypothesis there exist gsp-open sets U and V such that (X−G)⊆U, x∈V and U∩V=∅ and so V⊂(X−G). Now gspCl(V) ⊆ gspCl(X−U)=(X−U) and (X−G)⊆U implies (X−U)⊆G⊆W.Therefore gspCl**(V)** ⊆W.

(ii)⟹(i): Let F be any closed set if x∉F.Then x∈ (X−F) and (X−F) is gsp-open and so (X−F) is an gsp-open neighbourhood of x. By hypothesis there exists a gsp-open set V of x such that x∈Vand gspCl(V) ⊆(X−*F*) which implies F⊆ X−gspCl(V). Then X−gspCl(V) is gsp-open set containing F and V∩(X−gspCl(V))= ∅.Therefore X is strongly gsp-regular space.

(ii)⟹(iii): Let x∈X and F be an closed set such that x∉F. Then (X−F) is a gsp-open neighbourhood of x and by hypothesis there exists a gsp-open set V of x such that gspCl(V)⊆(X−F) and therefore gspCl(V) ∩F=∅.

(iii)⟹(ii): Let x∈X and W be a gsp-open neighbourhood of x. Then there exists an gsp-open sets of G such that x∈G⊆W. Since (X−G) is open set and x∉(X−G) by hypothesis there exists a gsp-open set V of x such that gspCl(V) ∩(X−G)= ∅. Therefore gspCl(V) ⊆G⊆W.

We define the following

Definition 4.3: A function f:X→Y is strongly g^*-continuous if the inverse image each g^*-closed set of Y is closed in X, equivalently if the inverse image of each g^*-open set of Y is open in X.

Theorem 4.4: Let X and Y be topological space X is (sp,gsp)-regular space. If f:X→Y is semipre-closed, gsp-irresolute and bijective, then X is strongly gsp-regular space.

Proof: Let F be a closed set in X, $x\notin F$. Since f is semipre-closed function, then f(F) is semipre-closed set in Y, $f(x)=y\notin f(F)$. But Y is (gs,gsp)-regular space, then there are two gsp-open sets U and V in Y such that $f(F)\subseteq V$. $y\in U$, $U\cap V=\emptyset$. Since $f^{-1}(U)$, $f^{-1}(V)$ are two gsp-open sets in X and $x\in f^{-1}(U)$, $F\subset f^{-1}(V)$, $f^{-1}(U)\cap f^{-1}(V)=\emptyset$. Hence X is strongly gsp-regular space.

Define the following

Definition 4.5: A space X is said to be (gs,gsp)-regular if for each gs-closed set F and for each $x\in(X-F)$ there exist two disjoint gsp-open sets U and V such that $x\in X$ and $F\subset V$.

Clearly, every (gs,gsp)-regular space is strongly gsp-regular space.

Definition 4.6: A space X is said to be (g^*,gs)-regular if for each g^*-closed set F and for each $x\in(X-F)$ there exist disjoint gs-open sets U and V such that $x\in U$ and $F\subset V$

Definition 4.7: A space X is said to be (g^*,gsp)-regular space if for each g^*-closed set F and for each $x\in(X-F)$ there exist disjoint gsp-open sets U and V such that $x\in U$ and $F\subset V$.

Since every gs-open set is gsp-open set and hence it is clearly that ,every (g^*,gs)-regular space is (g^*,gsp)-regular space.

The routine proof of the following theorem is omitted.

Theorem 4.8: The following properties are equivalent for a space X;

(i) X is (g^*,gs)-regular space

(ii) For each g^*-closed set F and each point $x\in(X-F)$ there exist $U\in SO(X)$ and gs- open set V such that $x\in U$, $F\subset V$ and $U\cap V=\emptyset$

(iii) For each subset A of X and each g^*-closed set F such that $A\cap F=\emptyset$,there exist $V\in SO(X)$ and a gs-open set V such that $A\cap V\neq\emptyset$ $F\subset V$ and $U\cap V=\emptyset$.

(iv) For each g^*-closed set F of X, $F=\cap\{sCl(V); F\subset V$ and V is gs-open$\}$.

Next, we characterized the (g^*,gsp)-regular space in the following

Theorem 4.9: The following properties are equivalent for a space X;

(i) X is (g^*, gsp)-regular space

(ii) For each g^*-closed set F and each point $x\in(X-F)$ there exist $U\in SPO(X)$ and gsp- open set V such that $x\in U$, $F\subset V$ and $U\cap V=\emptyset$

(iii)For each subset A of X and each g^*-closed set F such that $A\cap F=\emptyset$,there exist $U\in SPO(X)$ and a gsp-open set V such that $A\cap U\neq\emptyset$, $F\subset V$ and $U\cap V=\emptyset$.

(iv) For each g^*-closed set F of X, $F=\cap\{spCl(V); F\subset V$ and V is gsp-open$\}$

Proof is similar to Theorem 3.23 above.

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