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On Decomposition of βg^* Closed Sets in Topological Spaces

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Abstract: *The aim of this paper is to introduced and study the classes of βg^*-locally closed set and different notions of generalization of continuous functions namely βg^*lc-continuity, βg^*lc*-continuity and βg^*lc**-continuity and their corresponding irresoluteness were studied..*

Keywords: *βg^*-separated, βg^*-dense, βg^*-submaximal, βg^*lc-continuity, βg^*lc*-continuity βg^*lc**-continuity.*

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I. Introduction:

The first step of locally closedness was done by Bourbaki [2]. He defined a set A to be locally closed if it is the intersection of an open and a closed set. In literature many general topologists introduced the studies of locally closed sets. Extensive research on locally closedness and generalizing locally closedness were done in recent years. Stone [7] used the term LC for a locally closed set. Ganster and Reilly used locally closed sets in [4] to define LC-continuity and LC-irresoluteness. Balachandran et al [1] introduced the concept of generalized locally closed sets. The aim of this paper is to introduce and study the classes of βg^* locally closed set and different notions of generalization of continuous functions namely βg^*lc-continuity, βg^*lc*-continuity and βg^*lc**-continuity and their corresponding irresoluteness were studied.

II. Preliminary Notes

Throughout this paper (X,τ), (Y,σ) are topological spaces with no separation axioms assumed unless otherwise stated. Let $A\subseteq X$. The closure of A and the interior of A will be denoted by Cl(A) and Int(A) respectively.

Definition 2.1. A Subset S of a space (X,τ) is called
(i) locally closed (briefly lc)[6] if $S=U\cap F$, where U is open and F is closed in (X,τ).
(ii) r-locally closed (briefly rlc) if $S=U\cap F$, where U is r-open and F is r-closed in (X,τ).
(iii) generalized locally closed (briefly glc) [1] if $S=U\cap F$, where U is g-open and F is g-closed in (X,τ).

Definition 2.2. [4] A subset A of a topological space (X,τ) is called βg^*-closed if $gcl(A)\subseteq U$ whenever $A\subseteq U$ and U is β-open subset of X.

Definition 2.3. For a subset A of a space X, $\beta g^*\text{-cl}(A) = \bigcap\{F: A\subseteq F, F \text{ is } \beta g^* \text{closed in } X\}$ is called the βg^*-closure of A.

Remark 2.4. For a topological space (X,τ), the following statements hold:
(1) Every closed set is βg^*-closed but not conversely [4].
(2) Every g-closed set is βg^*-closed but not conversely [4].
(3) Every g*-closed set is βg^*-closed but not conversely [4].
(4) A subset A of X is βg^*-colsed if and only if βg^*-cl(A)=A.
(5) A subset A of X is βg^*-open if and only if βg^*-int(A)=A.

Corollary 2.5. If A is a βg^*-closed set and F is a closed set, then $A\cap F$ is a βg^*-closed set.

Definition 2.6[5]: A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called βg^* continuous if $f^{-1}(V)$ is βg^* closed subset of (X,τ) for every closed subset V of (Y,σ).

Definition 2.7. A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is called
i) LC-continuous [6] if $f^{-1}(V)\in LC(X,\tau)$ for every $V\in\sigma$.
ii) GLC-continuous [1] if $f^{-1}(V)\in GLC(X,\tau)$ for every $V\in\sigma$.

Definition 2.8. A subset S of a space (X,τ) is called
(i) submaximal [3] if every dense subset is open.
(ii) g-submaximal [1] if every dense subset is g-open.

III. βg^* Locally Closed Set

Definition 3.1: A subset A of (X,τ) is said to be βg^*locally closed set (briefly βg^*lc) if A=L∩M where L is βg^*-open and M is βg^*-closed in (X,τ).
Definition 3.2: A subset A of (X,τ) is said to be βg^*lc^*set if there exists a βg^*-open set L and a closed set M of (X,τ) such that A=L∩M.
Definition 3.3: A subset B of (X,τ) is said to be βg^*lc^{**}set if there exists an open set L and a βg^*-closed set M such that A=L∩M.
The class of all βg^*lc (resp. βg^*lc^* & βg^*lc^{**}) sets in X is denoted by βg^*LC(X).(resp. $\beta g^*LC^*(X)$ & $\beta g^*LC^{**}(X)$)
From the above definitions we have the following results.
Proposition :3.4
i) Every locally closed set is βg^*lc.
ii) Every glc-set is βg^*lc.
iii) Every g^*lc-set is βg^*lc.
iv) Every rlc -set is βg^* lc.
v) Every gr lc-set is βg^*lc.
vi) Every βg^*lc^*-set is βg^*lc^{**}.
viii) Every βg^*lc^{**}-set is βg^*lc^{**}.
However the converses of the above are not true as seen by the following examples
Example 3.5. Let X={a,b,c} with τ={ϕ,{a},X}. βg^*lc={{a},{b},{c},{a,b},{a,c},{b,c}, ϕ,X}. Then A={a} is βg^*lc-set but not locally closed.
Example 3.6. Let X={a,b,c} with τ={ϕ,{a},X}. Then A={a} is βg^* lc-set but not glc-set.
Example 3.7. In example 3.5, Let A={b} is βg^*lc-set but not βg^*lc^*-set.
Example 3.8 Let X={a,b,c} with τ={ϕ,{a},X}. Then A={a} is βg^*lc-set but not g^*lc-set.
Example 3.9. In example 3.8, Let A={a} is βg^*lc set but not grlc-set.
Example 3.10 In example 3.8, Let A={b} is βg^*lc-set but not rlc-set.
Example 3.11 In example 3.8. Let A={b} is βg^*lc-set but not βg^*lc^*-set.
Example 3.12. In example 3.8. Let A={b} is βg^*lc-set but not βg^*lc^{**}-set.
Remark 3.13. The concepts of βg^*lc^*set and βg^*lc^{**} sets are independent of each other as seen from the following example.
Example 3.14. In example 3.6, Let A={b,c} is βg^*lc^*-set but not βg^*lc^{**}-set and Let A={a} is βg^*lc^{**}-set but not βg^*lc^*-set.
Remark 3.15. Union of two βg^*lc-sets are βg^*lc-sets.

IV. βg^*-DENSE SETS AND βg^*-SUBMAXIMAL SPACES

Definition 4.1. A subset A of (X,τ) is called βg^*-dense if βg^*-cl(A)=X.
Example 4.2. Let X={a,b,c,d}with τ={ϕ,{b},{a,c},{a,b,c},X}. Then the set A={a,b,c} is βg^*-dense in (X,τ).
Recall that a subset A of a space (X,τ) is called dense if cl(A)=X.
Proposition 4.3. Every βg^*-dense set is dense.
Let A be a βg^*-dense set in (X,τ). Then βg^*-cl(A)=X. Since βg^*-cl(A)⊆gcl(A)⊆cl(A). we have cl(A)=X and so A is dense.
The converse of the above proposition need not be true as seen from the following example.
Example 4.4. Let X={a,b,c,d}with τ={ϕ,{b},{c,d},{b,c,d},X}. Then the set A={b,c} is a dense in (X,τ) but it is not βg^*-dense in (X,τ).
Definition 4.5. A topological space (X,τ) is called βg^*-submaximal if every dense subset in it is βg^*-open in (X,τ).
Proposition 4.6. Every submaximal space is βg^*-submaximal.
Proof. Let (X,τ) be a submaximal space and A be a dense subset of (X,τ). Then A is open. But every open set is βg^*-open and so A is βg^*-open. Therefore (X,τ) is βg^*-submaximal.
The converse of the above proposition need not be true as seen from the following example.
Example 4.7. Let X = {a,b,c}with τ = {ϕ,X}. Then βg^*O(X) = P(X). we have every dense subset is βg^*-open and hence (X,τ) is βg^*-submaximal. However, the set A={c} is dense in (X,τ), but it is not open in (X,τ). Therefore (X,τ) is not submaximal.
Proposition 4.8. Every g-submaximal space is βg^*-submaximal.
Proof. Let (X,τ) be a g-submaximal space and A be a dense subset of (X,τ). Then A is g-open. But every g-open set is βg^*open and A is βg^*-open. Therefore (X,τ) is βg^*-submaximal.
The converse of the above proposition need not be true as seen from the following example.

Example 4.9. Let $X = \{a,b,c,d\}$ with $\tau = \{\phi,\{d\},\{a,b,c\},X\}$. Then $GO(X) = P(X)$ and $GO(X)=\{\phi,\{d\},\{a,b,c\},X\}$. we have every dense subset is βg^*-open and hence (X,τ) is βg^*-submaximal. However, the set $A=\{a\}$ is dense in (X,τ), but it is not g-open in (X,τ). Therefore (X,τ) is not g-submaximal.

Proposition 4.10. Every r-submaximal space is βg^*-submaximal.

The converse of the above proposition need not be true as seen from the following example.

Example 4.11. Let $X=\{a,b,c,d\}$ with $\tau=\{\phi,\{a,d\},\{b,c\},X\}$. Then $RO(X)=P(X)$ and $\beta g^*O(X)=\{\phi,\{b,c\},\{a,d\},X\}$.Every dense subset is r-open and hence (X,τ) is r-submaximal. However the set $A=\{a\}$ is dense in (X,τ), but it is not βg^*-open in (X,τ). Therefore (X,τ) in not βg^*-submaximal.

Theorem 4.12. Assume that $\beta g^*C(X)$ is closed under finite intersections. For a subset A of (X,τ) the following statements are equivalent:

(1) $A\in \beta g^*LC(X)$,

(2) $A=S\cap\beta g^*\text{-cl}(A)$ for some βg^*-open set S,

(3) $\beta g^*\text{-cl}(A)-A$ is βg^*-closed,

(4) $A\cup(\beta g^*\text{-cl}(A))^c$ is βg^*-open,

(5) $A\subseteq \beta g^*\text{-int}(A\cup(\beta g^*\text{-cl}(A))^c)$.

Proof. (1)⇒(2). Let $A\in\beta g^*LC(X)$. Then $A=S\cap G$ where S is βg^*-open and G is βg^*-closed. Since $A\subseteq G$, $\beta g^*\text{-cl}(A)\subseteq G$ and so $S\cap \beta g^*\text{-cl}(A)\subseteq A$. Also $A\subseteq S$ and $A\subseteq \beta g^*\text{-cl}(A)$ implies $A\subseteq S\cap \beta g^*\text{-cl}(A)$ and therefore $A=S\cap \beta g^*\text{-cl}(A)$.

(2)⇒(3). $A=S\cap \beta g^*\text{-cl}(A)$ implies $\beta g^*\text{-cl}(A)-A= \beta g^*\text{-cl}(A)\cap S^c$ which is βg^*-closed since S^c is βg^*-closed and $\beta g^*\text{-cl}(A)$ is βg^*-closed.

(3)⇒(4). $A\cup(\beta g^*\text{-cl}(A))^c=(\beta g^*\text{-cl}(A)-A)^c$ and by assumption, $(\beta g^*\text{-cl}(A)-A)^c$ is βg^*-open and so is $A\cup(\beta g^*\text{-cl}(A))^c$.

(4)⇒(5). By assumption, $A\cup(\beta g^*\text{-cl}(A))^c= \beta g^*\text{-int}(A\cup(\beta g^*\text{-cl}(A))^c)$ and hence $A\subseteq \beta g^*\text{-int}(A\cup(\beta g^*\text{-cl}(A))^c)$.

(5)⇒(1). By assumption and since $A\subseteq \beta g^*\text{-cl}(A)$, $A= \beta g^*\text{-int}(A\cup(\beta g^*\text{-cl}(A))^c)\cap \beta g^*\text{-cl}(A)$. Therefore, $A\in \beta g^*LC(X)$.

Theorem 4.13 For a subset A of (X,τ), the following statements are equivalent:

(1) $A\in \beta g^*LC^*(X)$,

(2) $A=S\cap\text{cl}(A)$ for some βg^*-open set S,

(3) $\text{cl}(A)-A$ is βg^*-closed,

(4) $A\cup(\text{cl}(A))^c$ is βg^*-open.

Proof. (1)⇒(2). Let $A\in \beta g^*LC^*(X)$. There exist an βg^*-open set S and a colsed set G such that $A=S\cap G$. Since $A\subseteq S$ and $A\subseteq\text{cl}(A)$, $A\subseteq S\cap\text{cl}(A)$. Also since $\text{cl}(A)\subseteq G$, $S\cap\text{cl}(A)\subseteq S\cap G=A$. Therefore $A=S\cap\text{cl}(A)$.

(2)⇒(1). Since S is βg^*-open and cl(A) is a closed set, $A=S\cap\text{cl}(A)\in \beta g^*LC^*(X)$.

(2)⇒(3). Since $\text{cl}(A)-A=\text{cl}(A)\cap S^c$, $\text{cl}(A)-A$ is βg^*-closed

(3)⇒(2). Let $S=(\text{cl}(A)-A)^c$. Then by assumption S is βg^*-open in (X,τ) and $A=S\cap\text{cl}(A)$.

(3)⇒(4). Let $G=\text{cl}(A)-A$. Then $G^c=A\cup(\text{cl}(A))^c$ and $A\cup(\text{cl}(A))^c$ is βg^*-open.

(4)⇒(3). Let $S=A\cup(\text{cl}(A))^c$. Then S^c is βg^*-closed and $S^c=\text{cl}(A)-A$ and so $\text{cl}(A)-A$ is βg^*-closed.

Theorem 4.14. A space (X,τ) is βg^*-submaximal if and only if $P(X)= \beta g^*LC^*(X)$.

Proof. Necessity. Let $A\in P(X)$ and let $V=A\cup(\text{cl}(A))^c$. This implies that $\text{cl}(V)=\text{cl}(A)\cup(\text{cl}(A))^c=X$. Hence $\text{cl}(V)=X$. Therefore V is a dense subset of X. Since (X,τ) is βg^*-submaximal, V is βg^*-open. Thus $A\cup(\text{cl}(A))^c$ is βg^*-open and by theorem 6.2.13 we have $A\in \beta g^*LC^*(X)$.

Sufficiency. Let A be a dense subset of (X,τ). This implies $A\cup(\text{cl}(A))^c=A\cup X^c=A\cup\phi=A$. Now $A\in \beta g^*LC^*(X)$ implies that $A=A\cup(\text{cl}(A))^c$ is βg^*-open by Theorem 6.2.13. Hence (X,τ) is βg^*-submaximal.

Theorem 4.15. Let A be a subset of (X,τ). Then $A\in \beta g^*LC^{**}(X)$ if and only if $A=S\cap \beta g^*\text{-cl}(A)$ for some open set S.

Proof. Let $A\in \beta g^*LC^{**}(X)$. Then $A=S\cap G$ where S is open and G is βg^*-closed. Since $A\subseteq G$, $\beta g^*\text{-cl}(A)\subseteq G$. We obtain $A=A\cap \beta g^*\text{-cl}(A)=S\cap G\cap \beta g^*\text{-cl}(A)=S\cap \beta g^*\text{-cl}(A)$.

Converse part is trivial.

Theorem 4.16. Let A be a subset of (X,τ). If $A\in \beta g^*LC^{**}(X)$, then $\beta g^*\text{-cl}(A)-A$ is βg^*-closed and $A\cup(\beta g^*\text{-cl}(A))^c$ is βg^*-open.

Proof. Let $A\in \beta g^*LC^{**}(X)$. Then by theorem 4.15, $A=S\cap \beta g^*\text{-cl}(A)$ for some open set S and $\beta g^*\text{-cl}(A)-A= \beta g^*\text{-cl}(A)\cap S^c$ is βg^*-closed in (X,τ). If $G= \beta g^*\text{-cl}(A)-A$, then $G^c=A\cup(\beta g^*\text{-cl}(A))^c$ and G^c is βg^*-open and so is $A\cup(\beta g^*\text{-cl}(A))^c$.

Proposition 4.17 Assume that $\beta g^*O(X)$ forms a topology. For subsets A and B in (X,τ), the following are true:

(1) If A, $B\in \beta g^*LC(X)$, then $A\cap B\in \beta g^*LC(X)$.

(2) If A, $B\in \beta g^*LC^*(X)$, then $A\cap B\in \beta g^*LC^*(X)$.

(3) If A, $B\in \beta g^*LC^{**}(X)$, then $A\cap B\in \beta g^*LC^{**}(X)$.

(4) If $A\in \beta g^*LC(X)$ and B is βg^*-open (resp. βg^*-closed), then $A\cap B\in \beta g^*LC(X)$.

(5) If A∈ $\beta g^*LC^*(X)$ and B is βg^*-open (resp. closed), then A∩B∈ $\beta g^*LC^*(X)$.
(6) If A∈ $\beta g^*LC^{**}(X)$ and B is βg^*-closed (resp. open), then A∩B∈ $\beta g^*LC^{**}(X)$.
(7) If A∈ $\beta g^*LC^*(X)$ and B is βg^*-closed, then A∩B∈ $\beta g^*LC(X)$.
(8) If A∈ $\beta g^*LC^{**}(X)$ and B is βg^*-open, then A∩B∈ $\beta g^*LC(X)$.
(9) If A∈ $\beta g^*LC^{**}(X)$ and B∈ $\beta g^*LC^*(X)$, then A∩B∈ $\beta g^*LC(X)$.
Proof. By Remark 2.4, (1) to (8) hold.
(9). Let A=S∩G where S is open and G is βg^*-closed and B=P∩Q where P is βg^*-open and Q is closed. Then A∩B=(S∩P)∩(G∩Q) where S∩P is βg^*-open and G∩Q is βg^*-closed, .Therefore A∩B∈ $\beta g^*LC(X)$.
Definition 4.18 . Let A and B be subsets of (X,τ). Then A and B are said to be βg^*-separated if A∩ βg^*-cl(B)=ϕ and βg^*-cl(A)∩B=ϕ.
Example 4.19 Let X={a,b,c}with τ={ϕ,{a},{c},{a,c},{b,c},X}. Let A={a} and B={b}. Then βg^*-cl(A)={a} and βg^*-cl(B)={b} and so the sets A and B are βg^*-separated.
Proposition 4.20. Assume that $\beta g^*O(X)$ forms a topology. For a topological space (X,τ), the following are true:
(1) Let A, B∈ $\beta g^*LC(X)$. If A and B are βg^*-separated then A∪B∈ $\beta g^*LC(X)$.
(2) Let A, B∈ $\beta g^*LC^*(X)$. If A and B are separated (i.e., A∩cl(B)=ϕ and cl(A)∩B=ϕ), then A∪B∈ $\beta g^*LC^*(X)$.
(3) Let A, B∈ $\beta g^*LC^{**}(X)$. If A and B are βg^*-separated then A∪B∈ $\beta g^*LC^{**}(X)$.
Proof. (1) Since A, B∈ $\beta g^*LC(X)$. by theorem 4.13, there exists βg^*-open sets U and V of (X,τ) such that A=U∩ βg^*-cl(A) and B=V∩ βg^*-cl(B). Now G=U∩(X− βg^*-cl(B)) and H=V∩(X− βg^*-cl(A)) are βg^*-open subsets of (X,τ). Since A∩ βg^*-cl(B)=ϕ, A⊆(βg^*-cl(B))^c. Now A=U∩ βg^*-cl(A) becomes A∩(βg^*-cl(B))^c=G∩ βg^*-cl(A). Then A=G∩ βg^*-cl(A). Similarly B=H∩ βg^*-cl(B). Moreover G∩ βg^*-cl(B)=ϕ and H∩ βg^*-cl(A)=ϕ. Since G and H are βg^*-open sets of (X,τ), G∪H is βg^*-open. Therefore A∪B=(G∪H)∩ βg^*-cl(A∪B) and hence A∪B∈ $\beta g^*LC(X)$.
(2) and (3) are similar to (1), using Theorems 4.13 and 4.14.
Lemma 4.21 If A is βg^*-closed in (X,τ) and B is βg^*-closed in (Y,σ), then A×B is βg^*-closed in (X×Y, τ×σ).
Theorem 4.22. Let (X,τ) and (Y,σ) be any two topological spaces. Then
i) If A∈ $\beta g^*LC(X,\tau)$ and B∈ $\beta g^*LC(Y,\sigma)$, then A×B∈ $\beta g^*LC(X\times Y, \tau\times\sigma)$.
ii) If A∈ $\beta g^*LC^*(X,\tau)$ and B∈ $\beta g^*LC^*(Y,\sigma)$, then A×B∈ $\beta g^*LC^*(X\times Y, \tau\times\sigma)$.
iii) If A∈ $\beta g^*LC^{**}(X,\tau)$ and B∈ $\beta g^*LC^{**}(Y,\sigma)$, then A×B∈ $\beta g^*LC^{**}(X\times Y, \tau\times\sigma)$.
Proof. Let A∈ $\beta g^*LC(X,\tau)$ and B∈ $\beta g^*LC(Y,\sigma)$. Then there exists βg^*-open sets V and V' of (X,τ) and (Y,σ) respectively and βg^*-closed sets W and W' of (X,τ) and (Y,σ) respectively such that A=V∩W and B=V'∩W'. Then A×B= (V∩W)×(V'∩W')=(V×V')∩(W×W') holds and hence A×B∈ $\beta g^*LC(X\times Y, \tau\times\sigma)$.
The proofs of (ii) and (iii) are similar to (i).

V. βg^*LC -CONTINUOUS AND βg^*LC-IRRESOLUTE FUNCTIONS

In this section, we define βg^*LC-continuous and βg^*LC-irresolute functions and obtain a pasting lemma for βg^*LC^{**}-continuous functions and irresolute functions.
Definition 5.1. A function f:(X,τ)→(Y,σ) is called
i) βg^*LC-continuous if $f^{-1}(V)$∈ $\beta g^*LC(X,\tau)$ for every V∈σ.
ii) βg^*LC^*-continuous if $f^{-1}(V)$∈ $\beta g^*LC^*(X,\tau)$ for every V∈σ.
iii) βg^*LC^{**}-continuous if $f^{-1}(V)$∈ $\beta g^*LC^{**}(X,\tau)$ for every V∈σ.
iv) βg^*LC-irresolute if $f^{-1}(V)$∈ $\beta g^*LC(X,\tau)$ for every V∈ $\beta g^*LC(Y,\sigma)$.
v) βg^*LC^*-irresolute if $f^{-1}(V)$∈ $\beta g^*LC^*(X,\tau)$ for every V∈ $\beta g^*LC^*(Y,\sigma)$.
vi) βg^*LC^{**}-irresolute if $f^{-1}(V)$∈ $\beta g^*LC^{**}(X,\tau)$ for every V∈ $\beta g^*LC^{**}(Y,\sigma)$.
Proposition 5.2. If f:(X,τ)→(Y,σ) is βg^*LC-irresolute, then it is βg^*LC-continuous.
Proof. Let V be open in Y. Then V∈ $\beta g^*LC(Y,\sigma)$. By assumption, $f^{-1}(V)$∈ $\beta g^*LC(X,\tau)$. Hence f is βg^*LC-continuous.
Proposition 5.3. Let f:(X,τ)→(Y,σ) be a function, then
1. If f is LC-continuous, then f is βg^*LC-continuous.
2. If f is βg^*LC^*-continuous, then f is βg^*LC-continuous.
3. If f is βg^*LC^{**}-continuous, then f is βg^*LC-continuous.
4. If f is glc-continuous, then f is βg^* LC -continuous.
Remark 5.4. The converses of the above are not true may be seen by the following examples.
Example 5.5. 1. Let X=Y={a,b,c,d}, τ={ϕ,{c},{a,b},{a,b,c},X} and σ={ϕ,{a},{a,d},X}. Let f: X→Y be the identity map. Then f is βg^* LC-continuous but not LC-continuous. Since for the open set {a,d}, f^{-1}{a,d}= {a,d} is not locally closed in X.
2. Let X=Y={a,b,c,d}, τ={ϕ,{c},{a,b},{a,b,c},X} and σ={ϕ,{d},{a,d},{a,c,d},X}. Let f: X→Y be the identity map. Then f is βg^* LC-continuous but not βg^* LC^*-continuous. Since for the open set {a,c,d}, f^{-1}{a,c,d} = {a,c,d}is not βg^* LC^*-closed in X.

3. Let X=Y={a,b,c,d}, τ={ϕ,{c},{a,b},{a,b,c},X} and σ={ϕ,{a},{a,c},X}. Let f: X→Y be the identity map. Then f is βg^* LC-continuous but not βg^* LC^{**}-continuous. Since for the open set {a,c}, f^{-1}{a,c} = {a,c} is not βg^* LC^{**}-closed in X.
4. Let X=Y={a,b,c,d}, τ={ϕ,{a,d},{b,c},X} and σ={ϕ,{a},{a,b},{a,b,d},X}. Let f: X→Y be the identity map. Then f is βg^* LC -continuous but not glc-continuous. Since for the open set {a,b,d}, f^{-1}{a,b,d} = {a,b,d} is not βg^* LC -set in X.

We recall the definition of the combination of two funtions: Let X=A∪B and f:A→Y and h:B→Y be two functions. We say that f and h are compatible if f∕A∩B=h∕A∩B. If f:A→Y and h:B→Y are compatible, then the functions (f∆h)(X)=h(X) for every x∈B is called the combination of f and h.
Pasting lemma for βg^* LC^{**}-continuous (resp. βg^* LC^{**}-irresolute) functions.

Theorem 5.6. Let X=A∪B, where A and B are βg^* -closed and regular open subsets of (X,τ) and f:(A, τ∕B)→(Y,σ) and h:(B,τ∕B)→(Y,σ) be compatible functions.
a) If f and h are βg^*LC^{**}-continuous, then (f∆h):X→Y is βg^*LC^{**}-continuous.
b) If f and h are βg^*LC^{**}-irresolute, then (f∆h):X→Y is βg^*LC^{**}-irresolute.

Next we have the theorem concerning the composition of functions.

Theorem 5.7. Let f:(X,τ)→(Y,σ) and g:(Y,σ)→(Z,η) be two functions, then
a) g∘f is βg^*LC-irresolute if f and g are βg^*LC-irresolute.
b) g∘f is βg^*LC^*-irresolute if f and g are βg^*LC^*-irresolute.
c) g∘f is βg^*LC^{**}-irresolute if f and g are βg^*LC^{**}-irresolute.
d) g∘f is βg^*LC-continuous if f is βg^*LC-irresolute and g is βg^*LC-continuous.
e) g∘f is βg^*LC^*-continuous if f is βg^*LC^*-continuous and g is continuous.
f) g∘f is βg^*LC-continuous if f is βg^*LC-continuous and g is continuous.
g) g∘f is βg^*LC^*-continuous if f is βg^*LC^*-irresolute and g is βg^*LC^*-continuous.
h) g∘f is βg^*LC^{**}-continuous if f is βg^*LC^{**}-irresolute and g is βg^*LC^{**}-continuous.

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