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Computational simulation for the analytical and numericaltreatment related to SIRs models

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***Abstract**: This paper addresses the solutions of SI, SIR, SEIR and MSEIR (SIRs) models. Those epidemic models are simplified means of describing the transmission of infectious diseases through individuals. Infectious diseases are a tool which has been used to explore the mechanisms by which diseases dispersal, to prophesy the future course of an outbreak and to esteem strategies to control an epidemic. To find the solution of our models we use the multi-steps differential transform method (Ms-DTM), a reliable and powerful technique that ameliorate reliability and overcome drawbacks advanced in using the standard differential transform method (DTM).Finally, results have been compared with a software package Mathematica using the Parametric ND Solve code and very good agreement is obtained.*

***Keywords:**Nonlinear $SI, SIR, SEIR$ and $MSEIR$ models; differential transform method and multi-step differential transform method; MATHEMATICA10.*

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I. Introduction

Biological Mathematics models begins with Hammer in 1906, who applied the simple mass action principle one for a deterministic epidemic model in discrete time and Ross sitting a simpleEpidemic model in 1911 and propagate epidemic model produced by Kermack and McKendrickin 1927. In the today's world, biologists and doctors still have to deal with plagues and diseases. Millions of people die annually from measles, tuberculosis, malaria, AIDS . . . and billions of others are infected. There was a belief in 1960s that infectious diseases would be soon eliminated with the improvement in sanitation, antibiotics, vaccinations, medical science and medical care. However, they are still the major causes of mortality in the developing countries. Moreover, infectious disease agents adapt and evolve, therefore we can observe new infectious diseases emerging and some already existing diseases re-emerged, sometimes after hundreds of years and/or even mutated. At present we know bacteria which are able to swim in pure bleach or survive in a dose of penicillin. Together with the threat of biological weapons, whose research is lately concerned about microorganisms and lethal infectious diseases, we have great motivation to understand the spread and control of infectious diseases and their transmission characteristics. Mathematical epidemiology contributed to the understanding of the behavior of infectious diseases, its impacts and possible future predictions about its spreading. Mathematical models are used in comparing, planning, implementing, evaluating and optimizing various detection, prevention, therapy and control programs**[1]**. So it is important to validate models by checkingwhether they fit the observed data, and we can clarify the epidemic models as the following:

- $\boldsymbol{SI}$Model is the simplest one among the epidemic models. It's divided the population sizeto the susceptible compartment $s(t)$ and the infectious compartment$i(t)$, and assumesthedisease to be highly infectious but not serious, which means that the infective remain incontact with susceptible for all time$t \geq 0$. It also assumes that the infective continueto spread the disease till the end of the epidemic. The population size to be constant$(s(t) + i(t) = N)$and homogeneous mixing of population.

$$\frac{ds(t)}{dt} = -r\lambda s(t)i(t) \quad (1)$$
$$\frac{di(t)}{dt} = r\lambda s(t)i(t) \quad (2)$$

With initial conditions $s(0) = 1$ and $i(0) = 1$where r the recovery rate is, λ number of infective.

- $\boldsymbol{SIR}$Model, that's assume the population size is large and constant, that can be dividepopulation size into three compartments $s(t)$ - susceptible, $i(t)$ - infective, $r(t)$ - recovered.

$$\frac{ds(t)}{dt} = (1-p)\pi - \beta s(t)i(t) - \mu s(t) \quad (3)$$

$\frac{di(t)}{dt} = \beta s(t)i(t) - (\gamma + \pi)i(t)$(4)
$\frac{dr(t)}{dt} = p\pi + \gamma i(t) - \pi r(t)$ (5)
With initial conditions $s(0) = 0$, $i(0) = 1$and$r(0) = 0$; where p is the fraction of the susceptible individuals,βdisease transmission rate, γrecovery rate and the all positive parameters is constant with $0 < p, \beta, \gamma < 1$.

- ***SEIR***Model, a population supposed constant is divided into different classes, disjoint and based on their disease status. At time t, $s(t)$is the susceptible group, $e(t)$ is the exposedgroup, $i(t)$ is the infective group, and $r(t)$ is the recovered group

$\frac{ds(t)}{dt} = b - \beta s(t)i(t) - \mu s(t)$ (6)
$\frac{de(t)}{dt} = \beta s(t)i(t) - (\sigma + \pi)e(t)$ (7)
$\frac{di(t)}{dt} = \sigma e(t) - (\xi + \mu)i(t)$ (8)
$\frac{dr(t)}{dt} = \xi i(t) - \mu r(t)$ (9)

With initial conditions $s(0) = 0$, $e(0) = 1$, $i(0) = 1$ and$r(0) = 0$; where the all recruitment is done by birth into the class of susceptible and occurs at constant birth rate$b$. The rate constant for non-disease related death is μ; thus $\frac{1}{\mu}$is the average life time, and β is the disease transmission rate, σ is the exposed people becomes infectious with a constant rate, so that $\frac{1}{\sigma}$is the average incubation period. Some infectious individuals will recover after a treatment or a certain period of time at a rate constantξ, making $\frac{1}{\xi}$ the average infectious period **[2]**.

- ***MSEIR*** Model, can divided into five classes the passively-immune group$m(t)$, the susceptible group $s(t)$, the exposed group $e(t)$, the infective group $i(t)$, and the recoveredgroup $r(t)$.

$\frac{dm(t)}{dt} = B - \delta m(t)s(t) - \mu m(t)$(10)
$\frac{ds(t)}{dt} = \delta m(t)s(t) - \beta s(t)i(t) - \mu s(t)$(11)
$\frac{de(t)}{dt} = \beta s(t)i(t) - (\varepsilon + \mu)e(t)$ (12)
$\frac{di(t)}{dt} = \varepsilon e(t) - (\gamma + \mu)i(t)$(13)
$\frac{dr(t)}{dt} = \gamma i(t) - \mu r(t)$ (14)

With the initial conditions $m(0) = 1$, $s(0) = 1$, $e(0) = 0$, $i(0) = 1$ and $r(0) = 0$; where thesymbols stands for is the Recovery rateγ, $\beta$is the Transmission rate, ε is the progression rate,$\delta$is the Immunity rate, $\mu$is the death rate and B is the infectious hepatitis **[3]**.

Some recent studies scrutinize the SIRs models such as Shah et al **[4]** addressed the SEIRmodel and simulation for vector borne diseases. A note on solutions of the SIR models ofepidemics using method of Homotopy analysis is explored by M. Sajid et al **[5]** Ibrahimet al **[6]** construct the nature of MSEIR epidemic model using Homotopy analysis method. Asfour et al **[7]** study the differential fractional transform method to solve the epidemic MSEIR model. Lisi et al **[8]** addressed the analysis of an age-structured MSEIR model. Badshah et al **[9]** indicate the role of dynamics in epidemiology by mathematical models such asSI, SIR, SEIR and MSEIR. The numerical simulation for SI model with variable-order fractional is constructed by Asfour et al **[10]**.In this study, the solutions of those models obtained by using DTM which is one of a semi-numerical method for solving a large diversity of differential equations. The DTM gives exact values of the nth derivative of an analytic function at a pointin terms of unknown and known boundary conditions, so that the solutions are shown in terms ofconvergent series with easily computable components. DTM has some disadvantages, the maindrawbacks is that the obtained solutions usually converges in a very small region and it has slowconvergent rate or completely divergent in the wider region and to overcome the shortcoming,we apply the multi-step differential transform method, that provides the solution in terms ofconvergent series over a sequence of subintervals **[11-14]**.

This attempt is prepared as the following. Section 2, 3study the mathematical analysis of DTM and Ms-DTM. In Section 4. Applications of DTM and Ms-DTM on SI, SIR, SEIRand MSEIR models are progressed. Section 5. Results of Ms-DTM application and discussion are presented.

II. Differential Transformation Method

Consider a general equation of n[th] order ordinary differential equation **[15-17]**
$y(t, f, f', \dots, f^{(n)}) = 0.$
Subject to the initial equations
$f^{(k)}(0) = d_k,\ k = 0, \dots, n-1.$

To demonstrate the differential transformation method (DTM) for solving differential equations, the basic definitions of differential transformation are introduced as follows. Let $f(t)$ be analytic in a domain D and let $t = t_0$ represent any point in$D$. The function $f(t)$ is then represented by one power series whose centre is located at t_0. The differential transformation of the $k-th$ derivative of a function $f(t)$ is defined as the following:

$$F(k) = \left(\frac{1}{k!}\right)\left[\left(\frac{d^{(k)}f(t)}{dt^{(k)}}\right)\right]_{(t=t^0)},\ \forall t \in D. \qquad (15)$$

And the inverse transformation of $F(k)$ can take the form

$$f(t) = \sum_{k=0}^{\infty} F(k)(t-t_0)^{(k)},\ \forall t \in D. \qquad (16)$$

In fact, from Eq. (25) and (26), we obtain

$$f(t) = \sum_{k=0}^{\infty} \frac{(t-t_0)^{(k)}}{k!}\left(\frac{d^{(k)}y(t)}{dt^{(k)}}\right)_{t=t_0}, \forall t \in D. \qquad (17)$$

Eq. (16) implies that the concept of differential transformation is derived from the Taylor series expansion. Form the definitions of (15) and (16); it is easy to prove that the functions comply with the following basic mathematics operations (see Table 1). In real applications, the function $f(t)$ is expressed by a finite series and (27) can be written as:

$$f(t) = \sum_{k=0}^{N} F(k)(t-t_0)^{(k)},\ \forall t \in D. \qquad (18)$$

Eq. (18) implies that $\sum_{k=N+1}^{\infty} F(k)(t-t_0)^{(k)}$ is negligibly small.The following table show that the transformation for some functions and relation by using differential transformation method.

Table 1
Operations of the one dimensional differential transform.

Original function	*Transformed function*
$f(t) = g(t) \mp h(t)$	$F(k) = G(k) \mp H(k)$
$f(t) = \alpha g(t)$	$F(k) = \alpha G(k)$
$f(t) = g(t)h(t)$	$F(k) = \sum_{l=0}^{k} G(l)H(k-l)$
$f(t) = \frac{dg(t)}{dt}$	$F(k) = (k+l)G(k+l)$
$f(t) = \frac{d^n g(t)}{dt^n}$	$F(k) = \frac{(k+l)!}{k!}G(k+n)$
$f(t) = u(t)v(t)w(t)$	$F(k) = \sum_{l=0}^{k}\sum_{r=0}^{k-l} U(l)V(l)w(k-r-l)$

III. Multi-steps Differential Transformation Method [18]

The multi-step DTM is treated as an algorithm in a sequence of intervals for finding accurate approximate solutions for systems of differential equations.Suppose $[0, T]$is the interval over which we want to find the solution for a system of equations$(15-17)$. In actual applications of the DTM, the approximate solution for a system of equations can be expressed by the finite series
$f(t) = \sum_{k=0}^{N} a_{(k)} t^{(k)},\ t \in [0, T]$.(19)
The multi-steps approach introduces a new idea for constructing the approximate solution. Assume that the interval $[0, T]$ is divided into M sub intervals $[t_{m-1}, t_m],\ m = 1,2, \dots, M$ of equal step size $h = \frac{T}{M}$ by using the nodes$t_m = mh$. The main ideas of the Multi-step DTM are as follows. First, we apply the DTM to a system of equations $(15-17)$ over the interval $[0, T]$ we will obtain the following approximate solution

$$f_1(t) = \sum_{k=0}^{N} a_{1n} t^k, t \in [0, t_1], \qquad (20)$$

Using the initial conditions $f^{(k)}(0) = C_k$Form$m \geq 2$ and at each sub interval $[t_{m-1}, t_m]$ we will use the initial conditions $f_m^{(k)}(t_{m=1}) = f_{m=1}^{(k)}(t_{m=1})$ and apply the DTM to Eqs. $(15-17)$over the interval$[t_{m=1}, t_m]$, where t_0 in Eq. (16) is replaced by t_{m-1} the process is repeated and generates a sequence of approximate solution sum. $f_m(t), m = 1,2, \dots, M$for the solution$f(t)$.

$f_m(t) = \sum_{k=0}^{N} a_{mk} \left(t - t_{\{m-1\}}\right)^2, t \in [t_m, t_{m-1}]$ (21)

The new algorithm, multi-step DTM, is simple for computational performance for all values of$h$. It is easily observed that if the step size$h = T$, then the multi-step DTM reduces to the classical DTM.

IV. Analytical solutions by means of the Multi-step DTM [18-21]

This section simulates the solution under the application of DTM to equations ((1)-(14)).

Model 1

$[k+1]S[k+1] = -r\lambda \sum_{l=0}^{k} S[I]I[k-l]$(22)

$[k+1]I[k+1] = r\lambda \sum_{l=0}^{k} S[I]I[k-l]$ (23)

With transformed initial conditions$S(0) = 1$ and$I(0) = 1$.

Model 2

$[k+1]S[k+1] = \delta(k,0)(1-p)\pi - \beta \sum_{l=0}^{k} S[I]I[k-l] - \mu S[k]$ (24)

$[k+1]I[k+1] = \beta \sum_{l=0}^{k} S[I]I[k-l] - (\gamma + \pi)I[k]$ (25)

$[k+1]R[k+1] = p\pi\delta(k,0) + \gamma I[k] - \pi R[k]$ (26)

With transformed initial conditions $S(0) = 0, I(0) = 1$ and$R(0) = 0$

Model 3

$[k+1]S[k+1] = b\delta(k,0) - \beta \sum_{l=0}^{k} S[I]I[k-l] - \mu S[k]$ (27)

$[k+1]E[k+1] = \beta \sum_{l=0}^{k} S[I]I[k-l] - (\sigma + \mu)E[k]$ (28)

$[k+1]I[k+1] = \sigma E(t) - (\xi + \mu)I[k]$ (29)

$[k+1]R[k+1] = \xi I(t) - \mu R[k]$ (30)

With transformed initial conditions $S(0) = 0, E(0) = 1, I(0) = 1$ and$R(0) = 0$

Model 4

$[k+1]M[k+1] = B\delta(k,0) - \delta \sum_{l=0}^{k} M[I]S[k-l] - \mu M[k]$ (31)

$[k+1]S[k+1] = \delta \sum_{l=0}^{k} M[I]S[k-l] - \beta \sum_{l=0}^{k} S[I]I[k-l] - \mu S[k+1]$ (32)

$[k+1]S[k+1] = \beta \sum_{l=0}^{k} S[I]I[k-l] - (\varepsilon + \mu)E[k]$ (33)

$[k+1]I[k+1] = \varepsilon E[k] - (\gamma + \mu)I[k]$ (34)

$[k+1]R[k+1] = \gamma I[k] - \mu R[k]$ (35)

With transformed initial conditions$M(0) = 1$; $S(0) = 1, E(0) = 0, I(0) = 1$ and$R(0) = 0$, where$\delta$is Kronecker's delta.

V. Results of Ms-DTM application and discussion

The solutions obtained by the Ms-DTM are displayed by the aid of graphical illustrations.

1. SI Model

- **Figs.1** (I, II) are prepared to study the variation of susceptible and infectious groups with thetime, it seems that from **Fig.1 (I)** a susceptible group increases with an increase in time butinfectious group decreases with an increase in time with aid of DTM and ParametricNDSolvemethods.

2. SIR Model

- **Figs.2 (a1, a2)** display the influence of increasing in time on the susceptible group, it's observed from **Fig.2 (a1)** susceptible group increases with an increase in time (using DTM), until point (0.56613, 0.4) then it decreases. So we use Ms-DTM, whichprovides the solution in terms of convergent series over a sequence of subintervals. For this from **Fig.2 (a2)** a susceptible group increases with an increase in time till a certain value (19.8223, 0.49), thensusceptible group constant with the increases in time which is agree with ParametricNDSolve.
- **Figs.2 (b1, b2)** show that the impact of increasing in time on the infectious group, it's observed from **Fig.2 (b1)** infectious group decreases with an increase in time (using DTM), until point (0.5831, 0.1.6798) then it increases. Then by using Ms-DTM, **Fig.2 (b2)** show that infectious group decreases with an increase in time till a certain value (18.6324, 0.112), then Infectious group constant with the increases in time.
- **Figs.2 (c1, c2)** depicts that the effects of increasing in time on the recovered group, it's observed from **Fig.2 (c1)** a recovered group increases with an increase in time (using DTM), until point (0.72614, 0.44) then it decreases. Therefore we use Ms-DTM, **Fig.2 (c2)** a recovered group increases with an increase in time till a certain value (9.6213, 0.499), then recovered group constant with the increases in time.
- That's means the Ms-DTM have accuracy as ParametricNDSolve.

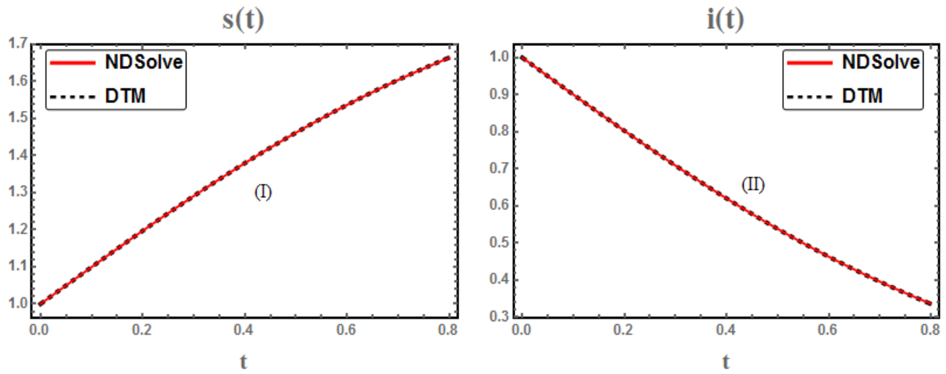

Fig. 1: The comparisons of the results of DTM and ParametricNDSolve package for SI Model with $r = 0.5$ and $\lambda = 0.8$

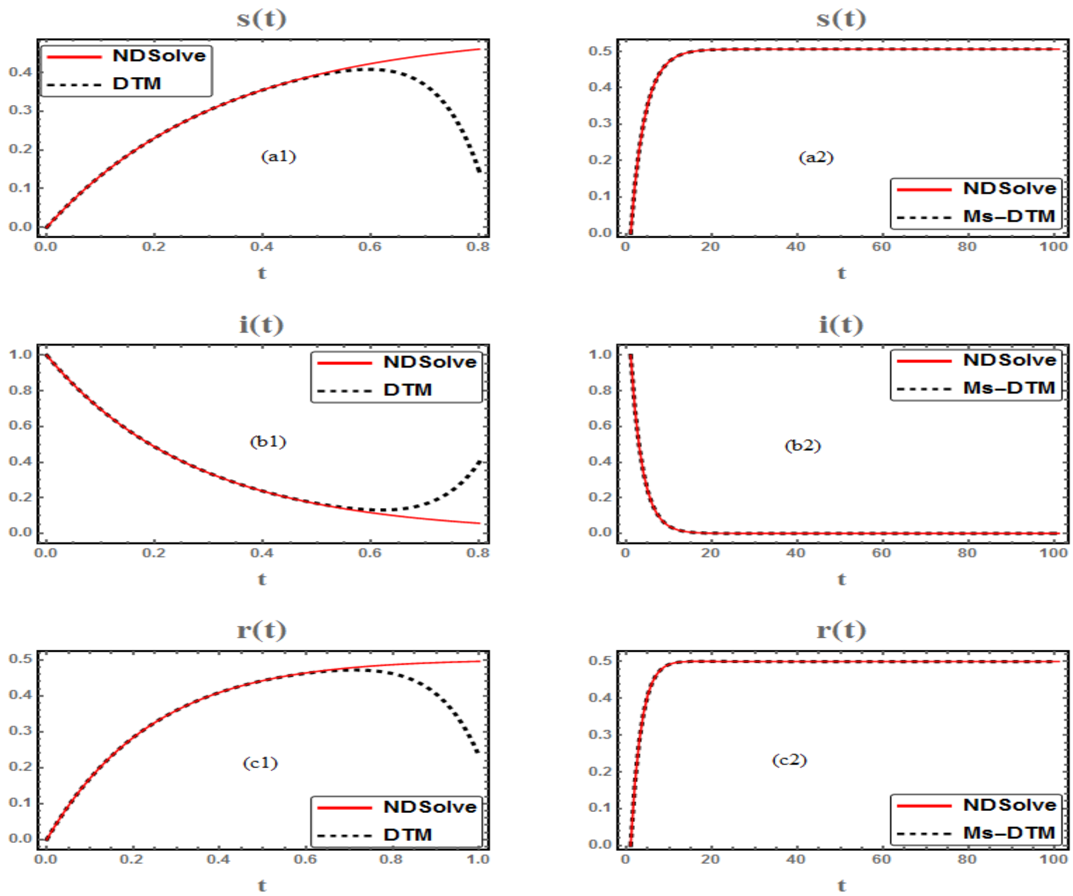

Fig. 2: The comparisons of the results of Ms-DTM and ParametricNDSolve package for sir Model at $p = 0.9, \beta = 0.2, \gamma = 0.5$and $\mu = 0.7$.

3 SEIR Model

- **Figs.3 (d1, d2)** are prepared to study the influence of increasing in time on the susceptible group, that show it's almost like **Figs.3 (a1, a2)**
- **Figs.3 (f1, f2)** show that the impact of increasing in time on the exposed group, it's start from point (0,1) and decreases to reach a zeroth constant point.
- **Figs.3 (g1, g2)** depicts that the effects of increasing in time on the Infectious group, it's seems that infectious group decreases with an increase in time.
- Recovered group increases with an increase in time at point (2.21,0.2765), then start to decrease to reach a fixed value in **Figs.3 (h1, h2)**

4 MSEIR Model

- **Figs.4 (j1, j2)** display the influence of increasing in time on the passively-immune group,that find the passively-immune group start from a super point (0,1) this means that atthe beginning of the treatment, the response is quick and strong at first, but over time theresponse rate is decreases to a fixed point.
- **Figs.4 (k1, k2)** show that the impact of increasing in time on the susceptible group, thetreatment speed is initially at the point (0, 1) and these numbers are maximized and thencontinuously reduced to a fixed value.
- **Figs.4 (L1, L2)** depicts that the effects of increasing in time on the exposed group, exposedgroup start from point (0,0) and increases to reach a maximum point (1.8,0.13), thendecreases to a fixed value.
- **Figs.4 (O1, O2)** and **Figs.4 (Q1, Q2)** show that the infectious and recovered group increaseswith an increase in time

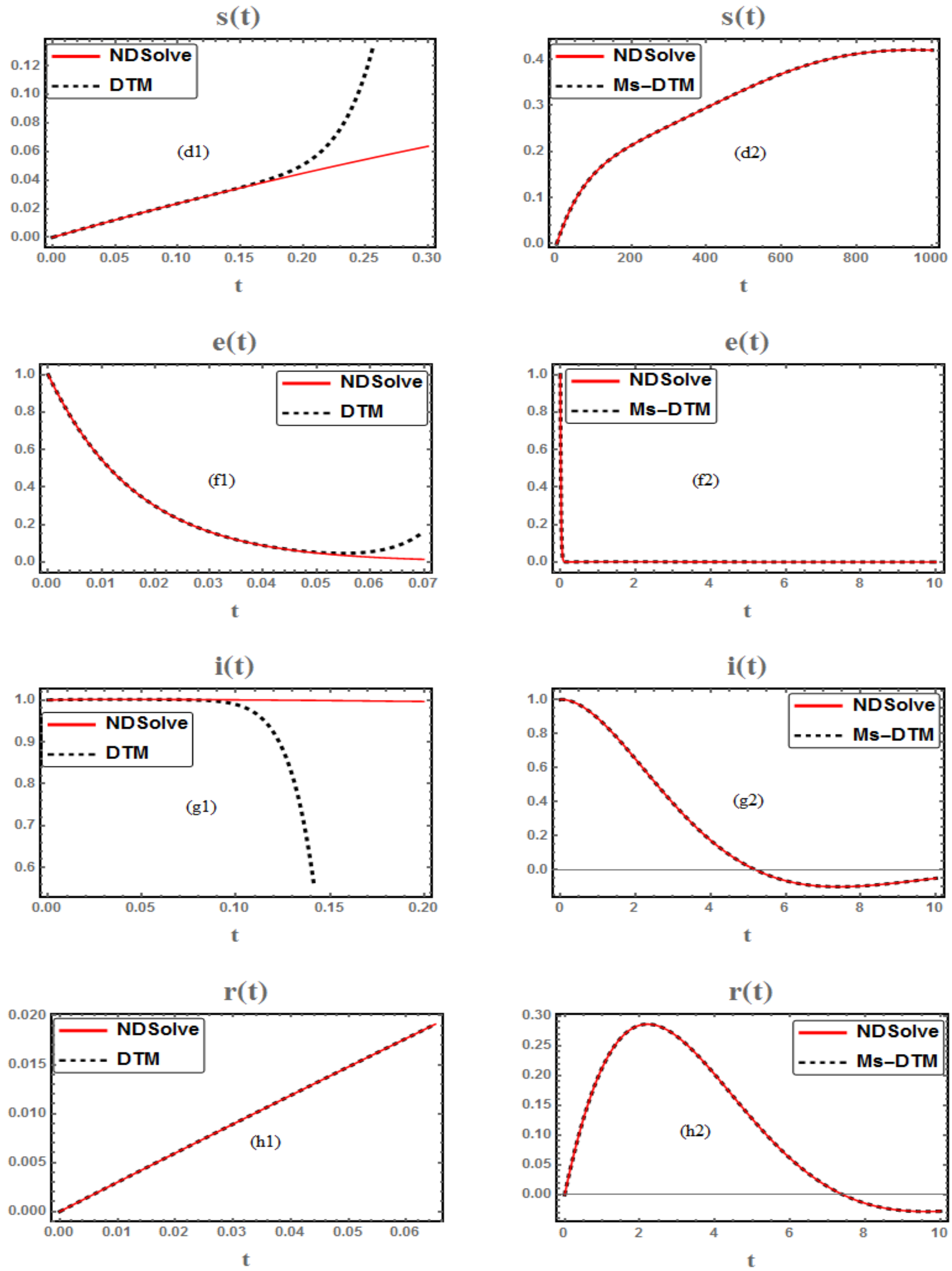

Fig. 3 The comparisons of the results of Ms-DTM and ParametricNDSolve package for *seir* Model at$b = 0.25$, $\beta = 0.5, \mu = 0.625, \xi = 0.3, \gamma = \frac{\pi}{3}and\sigma = 0.1$.

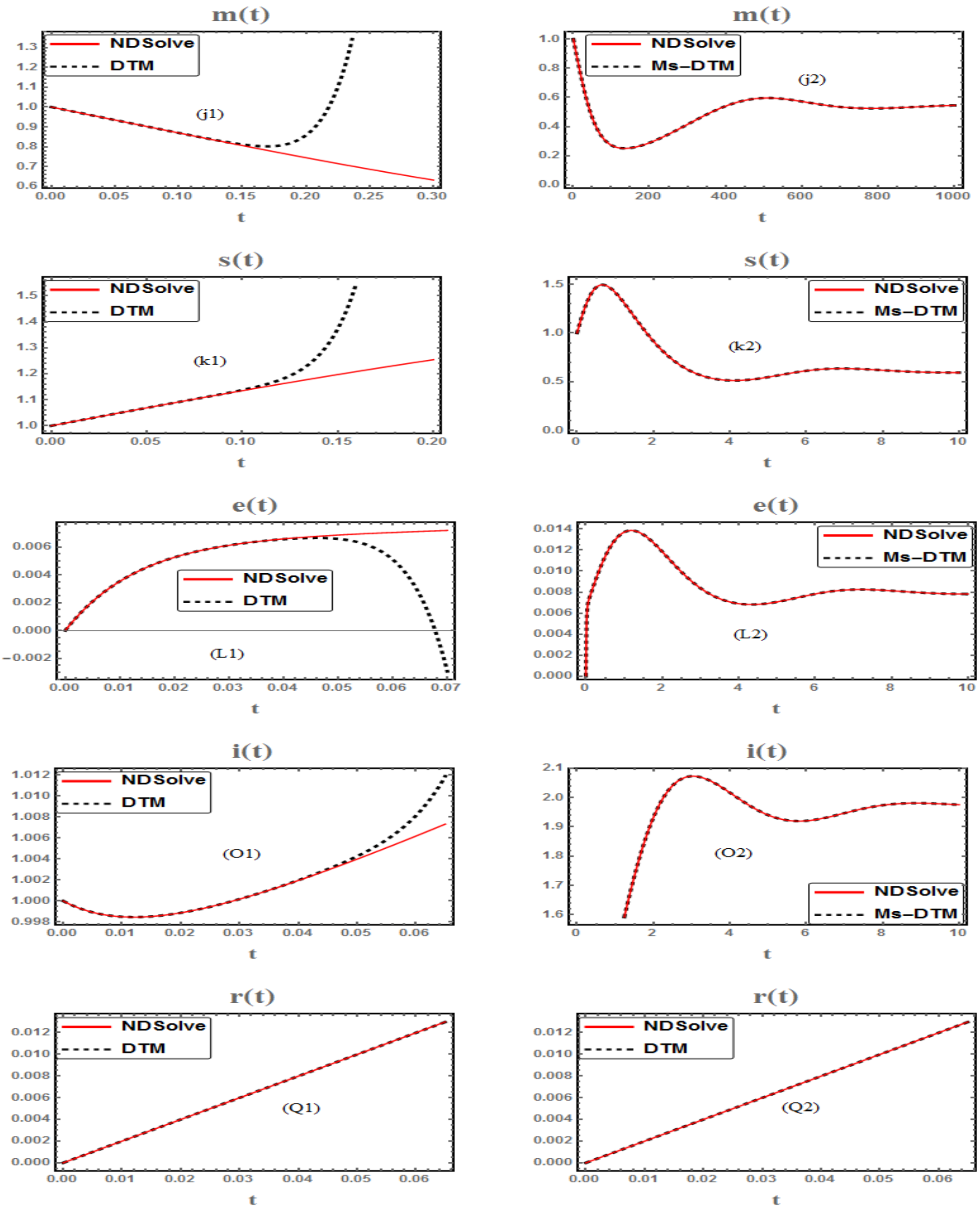

Fig. 4: The comparisons of the results of Ms-DTM and ParametricNDSolve package for $mseir$ Model at$B = 0.6, \delta = 0.9, \mu = 0.1, \varepsilon = 0.3, \beta = 0.5$ and$\gamma = 0.2$.

Table 2: Comparison between ND Solve Method and DTM for SIModel at $r = 0.5$ and $\lambda = 0.8$

t	S(t) NDSolve	S(t) Dtm	Estimate Errors of S (t)	I(t) NDSolve	I(t) Dtm	Estimate Errors of I (t)
0.	1.	1.	0.	1.	1.	0.
1.	1.76159	1.76159	4.19973×10^{-7}	0.238406	0.238405	4.19973×10^{-7}
2.	1.96403	1.96403	1.41303×10^{-7}	0.0359724	0.0359722	1.41303×10^{-7}
3.	1.99505	1.99505	2.95957×10^{-8}	0.00494524	0.00494521	2.95957×10^{-8}
4.	1.99933	1.99933	5.3635×10^{-9}	0.000670698	0.000670692	5.3635×10^{-9}
5.	1.99991	1.99991	9.0771×10^{-10}	0.0000907938	0.0000907929	9.0771×10^{-10}
6.	1.99999	1.99999	1.47513×10^{-10}	0.0000122887	0.0000122885	1.47513×10^{-10}
7.	2.	2.	2.32597×10^{-11}	1.66303×10^{-6}	1.66301×10^{-6}	2.32598×10^{-11}
8.	2.	2.	3.59521×10^{-12}	2.25305×10^{-7}	2.25302×10^{-7}	3.59512×10^{-12}
9.	2.	2.	5.54383×10^{-13}	3.01435×10^{-8}	3.0143×10^{-8}	5.54445×10^{-13}

Table 3: Comparison between ND Solve Method and MS-DTM for SIR Model at$p = 0.9, \beta = 0.2, \gamma = 0.5$ and$\mu = 0.7$.

t	s(t) NDSolve	s(t) Ms–Dtm	Estimate Errors of $s(t)$	i(t) NDSolve	i(t) Ms–Dtm	Estimate Errors of $i(t)$	r(t) NDSolve	r(t) Ms–Dtm	Estimate Errors of r(t)
0.	0.	0	0.	1.	1	0.	0.	0	0.
0.1	0.231553	0.231553	3.01296×10^{-8}	0.485198	0.485198	-9.53175×10^{-8}	0.284113	0.284113	6.42381×10^{-8}
0.2	0.355716	0.355716	6.03135×10^{-8}	0.237041	0.237041	-1.75307×10^{-7}	0.409595	0.409595	1.13598×10^{-7}
0.3	0.423571	0.423571	1.19126×10^{-8}	0.116238	0.116238	-2.87675×10^{-8}	0.463895	0.463895	1.6641×10^{-8}
0.4	0.460924	0.460924	2.47507×10^{-8}	0.0571158	0.0571158	-6.16332×10^{-8}	0.486691	0.486691	3.67058×10^{-8}
0.5	0.481524	0.481524	1.70365×10^{-10}	0.0280967	0.0280967	6.80341×10^{-10}	0.495826	0.495826	-8.28863×10^{-10}
0.6	0.492879	0.492879	2.41667×10^{-8}	0.01383	0.0138301	-4.80767×10^{-8}	0.499213	0.499213	2.41571×10^{-8}
0.7	0.499127	0.499127	5.66023×10^{-9}	0.0068099	0.00680991	-9.52733×10^{-9}	0.500289	0.500289	3.98862×10^{-9}
0.8	0.50256	0.50256	8.11896×10^{-9}	0.00335381	0.00335382	-1.23109×10^{-8}	0.500504	0.500504	4.41334×10^{-9}
0.9	0.504442	0.504442	5.41165×10^{-10}	0.0016519	0.0016519	-1.10088×10^{-9}	0.500441	0.500441	5.68017×10^{-10}
1.	0.505472	0.505472	2.13836×10^{-9}	0.000813674	0.000813677	-2.95146×10^{-9}	0.50032	0.50032	8.83071×10^{-10}
1.1	0.506035	0.506035	8.04212×10^{-10}	0.000400804	0.000400805	-1.1729×10^{-9}	0.500212	0.500212	3.85987×10^{-10}
1.2	0.506342	0.506342	1.03227×10^{-9}	0.000197433	0.000197434	-1.17921×10^{-9}	0.500134	0.500134	1.86534×10^{-10}
1.3	0.506509	0.506509	2.4955×10^{-9}	0.0000972534	0.0000972557	-2.37093×10^{-9}	0.500082	0.500082	1.17164×10^{-11}
1.4	0.5066	0.5066	-7.25574×10^{-10}	0.0000479091	0.0000479083	8.09094×10^{-10}	0.500048	0.500048	-1.16639×10^{-10}
1.5	0.50665	0.50665	-3.29546×10^{-10}	0.0000235999	0.0000235997	2.23065×10^{-10}	0.500028	0.500028	8.80356×10^{-11}
1.6	0.506677	0.506677	-5.73106×10^{-10}	0.0000116253	0.0000116253	4.61066×10^{-11}	0.500016	0.500016	4.70755×10^{-10}
1.7	0.506691	0.506691	-3.74504×10^{-10}	5.72648×10^{-6}	5.72665×10^{-6}	-1.74027×10^{-10}	0.500009	0.500009	4.96191×10^{-10}
1.8	0.506699	0.506699	-6.20771×10^{-11}	2.82069×10^{-6}	2.82097×10^{-6}	-2.84408×10^{-10}	0.500005	0.500005	3.22197×10^{-10}
1.9	0.506703	0.506703	1.5854×10^{-9}	1.38829×10^{-6}	1.38962×10^{-6}	-1.33083×10^{-9}	0.500003	0.500003	-1.45702×10^{-10}
2.	0.506706	0.506706	9.25799×10^{-10}	6.84096×10^{-7}	6.84532×10^{-7}	-4.35349×10^{-10}	0.500002	0.500002	-3.89275×10^{-10}
2.1	0.506707	0.506707	3.77164×10^{-11}	3.37561×10^{-7}	3.37203×10^{-7}	3.58082×10^{-10}	0.500001	0.500001	-3.4527×10^{-10}
2.2	0.506708	0.506708	-1.73429×10^{-9}	1.67106×10^{-7}	1.66107×10^{-7}	9.98323×10^{-10}	0.500001	0.500001	5.80867×10^{-10}
2.3	0.506708	0.506708	-1.53358×10^{-9}	8.26635×10^{-8}	8.18251×10^{-8}	8.38371×10^{-10}	0.5	0.5	5.31104×10^{-10}
2.4	0.506708	0.506708	6.47943×10^{-10}	3.97651×10^{-8}	4.03074×10^{-8}	-5.42218×10^{-10}	0.5	0.5	-9.15078×10^{-11}
2.5	0.506708	0.506708	2.44318×10^{-9}	1.89026×10^{-8}	1.98556×10^{-8}	-9.5295×10^{-10}	0.5	0.5	-1.20771×10^{-9}
2.6	0.506708	0.506708	1.44659×10^{-9}	9.45812×10^{-9}	9.78092×10^{-9}	-3.22803×10^{-10}	0.5	0.5	-9.11988×10^{-10}
2.7	0.506708	0.506708	-1.20123×10^{-10}	5.04105×10^{-9}	4.81812×10^{-9}	2.22929×10^{-10}	0.5	0.5	-7.71816×10^{-11}
2.8	0.506708	0.506708	-5.34434×10^{-10}	2.64035×10^{-9}	2.37342×10^{-9}	2.66927×10^{-10}	0.5	0.5	2.27369×10^{-10}
2.9	0.506708	0.506708	-5.07746×10^{-10}	1.34855×10^{-9}	1.16916×10^{-9}	1.7939×10^{-10}	0.5	0.5	2.75353×10^{-10}
3.	0.506708	0.506708	-3.94293×10^{-10}	6.91917×10^{-10}	5.75931×10^{-10}	1.15985×10^{-10}	0.5	0.5	2.3122×10^{-10}
3.1	0.506708	0.506708	-2.85069×10^{-10}	3.61638×10^{-10}	2.83706×10^{-10}	7.79322×10^{-11}	0.5	0.5	1.70602×10^{-10}
3.2	0.506708	0.506708	-1.96007×10^{-10}	1.89686×10^{-10}	1.39755×10^{-10}	4.99313×10^{-11}	0.5	0.5	1.19532×10^{-10}
3.3	0.506708	0.506708	-1.28435×10^{-10}	9.8355×10^{-11}	6.88436×10^{-11}	2.95114×10^{-11}	0.5	0.5	8.05985×10^{-11}
3.4	0.506708	0.506708	-4.51474×10^{-11}	4.64336×10^{-11}	3.39126×10^{-11}	1.2521×10^{-11}	0.5	0.5	2.77793×10^{-11}
3.5	0.506708	0.506708	2.00217×10^{-10}	-3.10634×10^{-12}	1.67055×10^{-11}	-1.98118×10^{-11}	0.5	0.5	-1.37433×10^{-10}
3.6	0.506708	0.506708	4.85118×10^{-10}	-4.6122×10^{-11}	8.22919×10^{-12}	-5.43511×10^{-11}	0.5	0.5	-3.30776×10^{-10}
3.7	0.506708	0.506708	6.8065×10^{-10}	-7.28474×10^{-11}	4.05373×10^{-12}	-7.69012×10^{-11}	0.5	0.5	-4.63991×10^{-10}
3.8	0.506708	0.506708	6.79771×10^{-10}	-7.3517×10^{-11}	1.99688×10^{-12}	-7.55139×10^{-11}	0.5	0.5	-4.63928×10^{-10}
3.9	0.506708	0.506708	4.31215×10^{-10}	-4.35977×10^{-11}	9.83671×10^{-13}	-4.45813×10^{-11}	0.5	0.5	-2.95522×10^{-10}
4.	0.506708	0.506708	3.01476×10^{-10}	-2.75219×10^{-11}	4.8456×10^{-13}	-2.80064×10^{-11}	0.5	0.5	-2.07746×10^{-10}
4.1	0.506708	0.506708	3.15181×10^{-10}	-2.87145×10^{-11}	2.38696×10^{-13}	-2.89532×10^{-11}	0.5	0.5	-2.17267×10^{-10}
4.2	0.506708	0.506708	3.75764×10^{-10}	-3.59726×10^{-11}	1.17583×10^{-13}	-3.60902×10^{-11}	0.5	0.5	-2.58411×10^{-10}
4.3	0.506708	0.506708	3.87651×10^{-10}	-3.80931×10^{-11}	5.79216×10^{-14}	-3.8151×10^{-11}	0.5	0.5	-2.66195×10^{-10}
4.4	0.506708	0.506708	2.63545×10^{-10}	-2.47526×10^{-11}	2.85324×10^{-14}	-2.47812×10^{-11}	0.5	0.5	-1.81287×10^{-10}
4.5	0.506708	0.506708	1.47039×10^{-10}	-1.1999×10^{-11}	1.40552×10^{-14}	-1.20131×10^{-11}	0.5	0.5	-1.01668×10^{-10}
4.6	0.506708	0.506708	1.01889×10^{-10}	-7.20069×10^{-12}	6.92363×10^{-15}	-7.20761×10^{-12}	0.5	0.5	-7.07542×10^{-11}
4.7	0.506708	0.506708	9.57202×10^{-11}	-6.85425×10^{-12}	3.41061×10^{-15}	-6.85766×10^{-12}	0.5	0.5	-6.64174×10^{-11}
4.8	0.506708	0.506708	9.62049×10^{-11}	-7.45631×10^{-12}	1.68008×10^{-15}	-7.45799×10^{-12}	0.5	0.5	-6.65615×10^{-11}
4.9	0.506708	0.506708	7.11515×10^{-11}	-5.51499×10^{-12}	8.27613×10^{-16}	-5.51581×10^{-12}	0.5	0.5	-4.91851×10^{-11}
5.	0.506708	0.506708	3.38513×10^{-11}	-2.20067×10^{-12}	4.07685×10^{-16}	-2.20108×10^{-12}	0.5	0.5	-2.34579×10^{-11}

Table 4: Comparison between ND Solve Method and MS-DTM for $SEIR$ Model at$b = 0.25,\ \beta = 0.5, \mu = 0.625, \xi = 0.3, \gamma = \frac{\pi}{3}and\sigma = 0.1$.

t	s(t) NDSolve	s(t) Ms–Dtm	Estimate Errors of s (t)	i(t) NDSolve	i(t) Ms–Dtm	Estimate Errors of i (t)	e(t) MS–Dtm	e(t) NDSolve	Estimate Errors of e (t)	r(t) NDSolve	r(t) Ms–Dtm	Estimate Errors of r (t)
0.	0.	0	0.	1.	1	0.	1.	1	0.	0.	0	0.
0.1	0.0447765	0.0447765	2.86342×10^{-12}	0.996324	0.996324	3.0157×10^{-12}	0.000346029	0.000346029	4.62444×10^{-10}	0.0563829	0.0563829	2.95145×10^{-12}
0.2	0.0805946	0.0805946	3.47997×10^{-9}	0.981246	0.981246	6.42469×10^{-10}	0.000631432	0.000631432	3.26757×10^{-10}	0.105561	0.105561	-3.15418×10^{-10}
0.3	0.109396	0.109396	5.93099×10^{-9}	0.957716	0.957716	1.69368×10^{-9}	0.00084886	0.00084886	4.83694×10^{-11}	0.14786	0.14786	-4.32861×10^{-10}
0.4	0.132764	0.132764	1.28926×10^{-8}	0.926973	0.926973	4.87117×10^{-9}	0.00100441	0.00100441	-1.1429×10^{-9}	0.183649	0.183649	-6.95861×10^{-10}
0.5	0.151967	0.151967	1.25049×10^{-8}	0.890183	0.890183	5.94873×10^{-9}	0.00110891	0.00110891	2.0755×10^{-9}	0.213319	0.213319	-4.0838×10^{-10}
0.6	0.168012	0.168012	2.23569×10^{-8}	0.848441	0.848441	1.59424×10^{-8}	0.00117193	0.00117193	-8.84451×10^{-13}	0.23728	0.23728	2.54816×10^{-9}
0.7	0.181685	0.181685	8.85592×10^{-9}	0.802762	0.802762	-2.84742×10^{-10}	0.00120162	0.00120162	5.00592×10^{-11}	0.255953	0.255953	-4.73497×10^{-9}
0.8	0.193599	0.193599	2.94275×10^{-8}	0.754086	0.754086	3.5101×10^{-8}	0.00120472	0.00120472	2.23002×10^{-9}	0.269766	0.269766	1.09983×10^{-8}
0.9	0.204224	0.204224	3.68491×10^{-8}	0.703271	0.703271	5.56553×10^{-8}	0.00118676	0.00118676	1.23794×10^{-10}	0.279144	0.279144	2.19706×10^{-8}
1.	0.213922	0.213922	3.10428×10^{-8}	0.651097	0.651097	5.47709×10^{-8}	0.00115217	0.00115217	2.1213×10^{-11}	0.284512	0.284512	2.3158×10^{-8}
1.1	0.222966	0.222966	3.7969×10^{-8}	0.598269	0.598269	7.93483×10^{-8}	0.00110454	0.00110454	-2.42058×10^{-9}	0.286284	0.286285	3.77685×10^{-8}
1.2	0.231561	0.231561	2.64351×10^{-8}	0.545413	0.545413	6.59574×10^{-8}	0.00104674	0.00104674	1.06352×10^{-10}	0.284866	0.284866	3.4904×10^{-8}
1.3	0.239859	0.239859	3.22042×10^{-8}	0.493083	0.493083	9.61274×10^{-8}	0.000981109	0.000981109	-1.40485×10^{-10}	0.280645	0.280645	5.70261×10^{-8}
1.4	0.247973	0.247973	2.95055×10^{-8}	0.441763	0.441763	9.67054×10^{-8}	0.000909569	0.000909564	-4.88574×10^{-9}	0.273996	0.273996	5.88879×10^{-8}
1.5	0.255979	0.255979	2.3936×10^{-8}	0.391869	0.391869	8.96985×10^{-8}	0.000833702	0.000833697	-5.45584×10^{-9}	0.265272	0.265273	5.88453×10^{-8}
1.6	0.26393	0.26393	2.0205×10^{-8}	0.343753	0.343753	8.51373×10^{-8}	0.00075486	0.000754853	-6.45331×10^{-9}	0.25481	0.25481	5.80853×10^{-8}
1.7	0.271859	0.271859	1.83606×10^{-8}	0.297707	0.297707	9.81166×10^{-8}	0.000674195	0.000674197	2.3189×10^{-9}	0.242921	0.242921	7.80845×10^{-8}
1.8	0.279781	0.279781	1.53994×10^{-8}	0.253969	0.253969	9.98118×10^{-8}	0.000592751	0.000592751	3.11658×10^{-11}	0.229898	0.229898	8.86222×10^{-8}
1.9	0.287702	0.287702	1.14977×10^{-8}	0.212722	0.212722	8.61445×10^{-8}	0.000511427	0.000511425	-1.88209×10^{-9}	0.216009	0.216009	8.29851×10^{-8}
2.	0.295612	0.295612	7.94059×10^{-9}	0.174104	0.174104	7.66648×10^{-8}	0.000431042	0.000431042	5.42985×10^{-10}	0.201501	0.201501	8.51416×10^{-8}
2.1	0.303499	0.303499	4.29347×10^{-9}	0.138208	0.138208	6.92313×10^{-8}	0.000352342	0.000352349	6.41974×10^{-9}	0.186597	0.186597	9.53202×10^{-8}
2.2	0.31134	0.31134	1.50443×10^{-9}	0.105089	0.105089	5.56164×10^{-8}	0.000276023	0.000276023	9.10024×10^{-11}	0.1715	0.1715	9.54292×10^{-8}
2.3	0.319111	0.319111	-1.80969×10^{-9}	0.0747653	0.0747653	4.17982×10^{-8}	0.000202682	0.000202681	-1.45911×10^{-9}	0.156389	0.156389	1.02943×10^{-7}
2.4	0.326781	0.326781	-2.0422×10^{-9}	0.0472242	0.0472243	2.82783×10^{-8}	0.000132866	0.000132872	5.69314×10^{-9}	0.141424	0.141424	9.36848×10^{-8}
2.5	0.334318	0.334318	4.07832×10^{-9}	0.0224261	0.0224261	3.13639×10^{-8}	0.0000670849	0.000067083	-1.96206×10^{-9}	0.126743	0.126743	6.35974×10^{-8}
2.6	0.34169	0.34169	-2.62654×10^{-10}	0.000307041	0.000307053	1.23164×10^{-8}	5.73015×10^{-6}	5.73209×10^{-6}	1.93505×10^{-9}	0.112466	0.112466	7.03401×10^{-8}
2.7	0.348864	0.348864	1.32198×10^{-9}	−0.0192173	−0.0192173	7.17327×10^{-9}	−0.0000508326	−0.0000508327	-9.95728×10^{-11}	0.0986943	0.0986943	5.95323×10^{-8}
2.8	0.355805	0.355805	-2.74338×10^{-9}	−0.0362482	−0.0362482	-1.66105×10^{-8}	−0.000102337	−0.000102336	9.63459×10^{-10}	0.0855118	0.0855118	6.39042×10^{-8}
2.9	0.362482	0.362482	-1.6655×10^{-9}	−0.0509013	−0.0509013	-2.57658×10^{-8}	−0.000148581	−0.000148578	2.47817×10^{-9}	0.0729867	0.0729868	5.59278×10^{-8}
3.	0.368865	0.368865	2.05605×10^{-9}	−0.0633031	−0.0633031	-2.05992×10^{-8}	−0.000189434	−0.000189434	-2.34467×10^{-10}	0.0611723	0.0611724	4.47327×10^{-8}
3.1	0.374927	0.374927	-1.32565×10^{-9}	−0.0735888	−0.0735889	-5.07566×10^{-8}	−0.000224855	−0.000224853	1.56883×10^{-9}	0.0501083	0.0501083	4.50028×10^{-8}
3.2	0.380644	0.380644	3.10602×10^{-10}	−0.0819	−0.0819001	-5.93607×10^{-8}	−0.000254859	−0.000254858	6.43665×10^{-10}	0.0398217	0.0398217	3.75745×10^{-8}
3.3	0.385995	0.385995	1.43652×10^{-9}	−0.088382	−0.088382	-6.95468×10^{-8}	−0.000279539	−0.000279541	-1.30645×10^{-9}	0.0303284	0.0303284	3.12965×10^{-8}
3.4	0.390963	0.390963	-8.9512×10^{-10}	−0.093182	−0.093182	-8.10962×10^{-8}	−0.000299058	−0.000299058	2.65668×10^{-10}	0.0216341	0.0216341	3.21132×10^{-8}
3.5	0.395537	0.395537	-1.51146×10^{-8}	−0.0964476	−0.0964477	-9.692×10^{-8}	−0.000313623	−0.000313623	1.18849×10^{-10}	0.0137355	0.0137356	5.41342×10^{-8}
3.6	0.399707	0.399707	-9.7998×10^{-11}	−0.098325	−0.0983251	-1.07307×10^{-7}	−0.000323503	−0.000323504	-3.85318×10^{-10}	0.00662154	0.00662156	1.99894×10^{-8}
3.7	0.403469	0.40347	5.98691×10^{-9}	−0.0989577	−0.0989578	-1.15469×10^{-7}	−0.000329007	−0.000329008	-3.38014×10^{-10}	0.000274092	0.000274095	3.35755×10^{-9}
3.8	0.406825	0.406825	1.01534×10^{-8}	−0.0984853	−0.0984855	-1.24349×10^{-7}	−0.000330479	−0.000330479	-4.30645×10^{-10}	−0.00533074	−0.00533075	-9.55117×10^{-9}
3.9	0.409779	0.409779	1.6344×10^{-8}	−0.0970426	−0.0970427	-1.28482×10^{-7}	−0.000328288	−0.000328288	-4.87143×10^{-10}	−0.0102218	−0.0102218	-2.33616×10^{-8}
4.	0.412337	0.412337	2.31878×10^{-8}	−0.0947584	−0.0947585	-1.24748×10^{-7}	−0.000322822	−0.000322822	-4.16022×10^{-10}	−0.0144319	−0.014432	-3.37335×10^{-8}
4.1	0.414513	0.414513	2.34027×10^{-8}	−0.0917554	−0.0917555	-1.12559×10^{-7}	−0.000314477	−0.000314477	-4.08007×10^{-10}	−0.0179973	−0.0179973	-2.85354×10^{-8}
4.2	0.416321	0.416321	2.44945×10^{-8}	−0.0881493	−0.0881494	-1.00986×10^{-7}	−0.000303652	−0.000303652	-3.71013×10^{-10}	−0.0209565	−0.0209565	-2.59292×10^{-8}
4.3	0.417779	0.417779	2.7882×10^{-8}	−0.0840485	−0.0840486	-9.06823×10^{-8}	−0.000290738	−0.000290738	-3.37409×10^{-10}	−0.0233498	−0.0233498	-2.96628×10^{-8}
4.4	0.418906	0.418906	3.03767×10^{-8}	−0.0795539	−0.0795539	-7.58884×10^{-8}	−0.000276117	−0.000276117	-2.91054×10^{-10}	−0.0252188	−0.0252189	-2.91349×10^{-8}
4.5	0.419725	0.419725	3.01027×10^{-8}	−0.0747584	−0.0747584	-5.95181×10^{-8}	−0.00026015	−0.00026015	-2.2266×10^{-10}	−0.0266057	−0.0266057	-2.29543×10^{-8}
4.6	0.420258	0.420258	2.78761×10^{-8}	−0.0697473	−0.0697474	-4.95081×10^{-8}	−0.00024318	−0.00024318	-1.76703×10^{-10}	−0.0275525	−0.0275525	-1.74382×10^{-8}
4.7	0.420531	0.420531	2.50629×10^{-8}	−0.0645982	−0.0645983	-4.57998×10^{-8}	−0.000225522	−0.000225523	-1.63317×10^{-10}	−0.0281011	−0.0281011	-1.48349×10^{-8}
4.8	0.420567	0.420567	2.22097×10^{-8}	−0.059381	−0.059381	-4.14134×10^{-8}	−0.000207467	−0.000207467	-1.5023×10^{-10}	−0.0282925	−0.0282925	-1.26852×10^{-8}
4.9	0.420392	0.420392	1.86663×10^{-8}	−0.0541579	−0.0541579	-3.15705×10^{-8}	−0.000189273	−0.000189273	-1.13814×10^{-10}	−0.0281668	−0.0281668	-8.69348×10^{-9}
5.	0.420031	0.420031	1.41418×10^{-8}	−0.048984	−0.0489841	-2.11257×10^{-8}	−0.000171172	−0.000171173	-6.95833×10^{-11}	−0.0277624	−0.0277624	-3.83469×10^{-9}

Table 5: Comparison between ND Solve Method and MS-DTM for $MSEIR$ Model at$B = 0.6, \delta = 0.9, \mu = 0.1, \varepsilon = 0.3, \beta = 0.5$ and$\gamma = 0.2$.

t	m(t) NDSolve	m(t) Ms-Dtm	Estimate Errors of m (t)	s(t) NDSolve	s(t) Ms-Dtm	Estimate Errors of s (t)	e(t) NDSolve	e(t) Ms-Dtm	Estimate Errors of e (t)	i(t) NDSolve	i(t) Ms-Dtm	Estimate Errors of i (t)	r(t) NDSolve	r(t) Ms-Dtm	Estimate Errors of r (t)
0.	1.	1	0.	1.	1	0.	0.	0	0.	1.	1	0.	0.	0	0.
0.1	0.869865	0.869865	9.10826×10^{-11}	1.13509	1.13509	7.72993×10^{-10}	0.00753246	0.00753246	-2.63211×10^{-10}	1.01596	1.01596	-3.23237×10^{-10}	0.0200001	0.0200001	2.62832×10^{-11}
0.2	0.745158	0.745158	3.75964×10^{-8}	1.25427	1.25427	-4.16337×10^{-8}	0.00859256	0.00859256	-7.12631×10^{-10}	1.04555	1.04555	3.40345×10^{-9}	0.0403023	0.0403023	-1.31142×10^{-10}
0.3	0.632758	0.632758	3.52375×10^{-8}	1.35113	1.35113	-3.47385×10^{-8}	0.00960271	0.00960271	-5.2924×10^{-10}	1.08193	1.08193	2.12379×10^{-10}	0.0610598	0.0610598	-2.19931×10^{-10}
0.4	0.536673	0.536673	-5.26448×10^{-9}	1.42256	1.42256	1.20878×10^{-8}	0.0105322	0.0105322	-1.73473×10^{-8}	1.12441	1.12441	1.2774×10^{-8}	0.0823969	0.0823969	-9.07671×10^{-11}
0.5	0.458108	0.458108	6.46965×10^{-9}	1.46851	1.46851	-5.03153×10^{-12}	0.0113577	0.0113577	-9.16791×10^{-11}	1.17213	1.17213	-3.90528×10^{-9}	0.104421	0.104421	-2.756×10^{-10}
0.6	0.396202	0.396202	-3.01855×10^{-8}	1.49112	1.49112	3.68981×10^{-8}	0.0120662	0.0120662	-1.13539×10^{-9}	1.22411	1.22411	-3.35142×10^{-9}	0.12722	0.12722	3.08417×10^{-10}
0.7	0.348936	0.348936	-2.56828×10^{-8}	1.49368	1.49368	3.14346×10^{-8}	0.0126526	0.0126526	2.1959×10^{-10}	1.27934	1.27934	-3.59265×10^{-9}	0.15086	0.15086	4.25804×10^{-12}
0.8	0.313879	0.313879	-2.65653×10^{-8}	1.47986	1.47986	3.02216×10^{-8}	0.0131172	0.0131172	2.70135×10^{-9}	1.33681	1.33681	-4.62405×10^{-9}	0.175388	0.175388	1.18238×10^{-10}
0.9	0.288667	0.288667	-2.62844×10^{-8}	1.45317	1.45317	2.72956×10^{-8}	0.0134636	0.0134636	1.24072×10^{-8}	1.39558	1.39558	-1.28409×10^{-8}	0.20083	0.20083	3.22438×10^{-10}
1.	0.271232	0.271232	-2.5384×10^{-8}	1.41675	1.41675	2.53317×10^{-8}	0.0136978	0.0136978	2.36217×10^{-10}	1.45475	1.45475	2.12776×10^{-11}	0.227194	0.227194	1.99084×10^{-10}
1.1	0.259883	0.259883	-9.46372×10^{-9}	1.37325	1.37325	5.65277×10^{-9}	0.013827	0.013827	1.9685×10^{-9}	1.5135	1.5135	8.58371×10^{-10}	0.25447	0.25447	6.1783×10^{-10}
1.2	0.253288	0.253288	-1.17678×10^{-8}	1.32486	1.32486	9.42491×10^{-9}	0.0138598	0.0138598	-1.92264×10^{-10}	1.57111	1.57111	1.37355×10^{-9}	0.282634	0.282634	3.44741×10^{-10}
1.3	0.250425	0.250425	-2.80401×10^{-8}	1.27336	1.27336	3.31924×10^{-8}	0.0138054	0.0138054	6.93614×10^{-9}	1.62693	1.62693	-1.3196×10^{-8}	0.311648	0.311648	-6.30763×10^{-10}
1.4	0.250521	0.250521	-2.33856×10^{-8}	1.2202	1.2202	2.59821×10^{-8}	0.0136736	0.0136736	3.45684×10^{-10}	1.68041	1.68041	-4.42684×10^{-9}	0.34146	0.34146	-4.98688×10^{-10}
1.5	0.252994	0.252994	-1.37292×10^{-8}	1.16653	1.16653	1.37219×10^{-8}	0.0134749	0.0134749	-1.13322×10^{-8}	1.73107	1.73107	9.46262×10^{-9}	0.372013	0.372013	-8.98257×10^{-11}
1.6	0.257405	0.257405	-1.18695×10^{-8}	1.11326	1.11326	1.20159×10^{-8}	0.0132196	0.0132196	-5.79773×10^{-11}	1.77856	1.77856	-2.15811×10^{-9}	0.40324	0.40324	-8.99703×10^{-11}
1.7	0.263423	0.263423	-6.26448×10^{-9}	1.0611	1.0611	2.62096×10^{-9}	0.0129184	0.0129184	2.56201×10^{-9}	1.82258	1.82258	-1.15891×10^{-9}	0.435066	0.435066	2.17492×10^{-10}
1.8	0.270792	0.270792	-4.34074×10^{-9}	1.01061	1.01061	1.15368×10^{-9}	0.0125814	0.0125814	-9.26423×10^{-10}	1.86293	1.86293	1.87191×10^{-9}	0.467415	0.467415	3.2918×10^{-10}
1.9	0.279314	0.279314	-1.89318×10^{-8}	0.962201	0.962201	4.04233×10^{-8}	0.0122181	0.0122181	-4.22978×10^{-9}	1.89949	1.89949	-1.44439×10^{-8}	0.500208	0.500208	3.64719×10^{-9}
2.	0.288828	0.288828	-1.53102×10^{-8}	0.91618	0.91618	4.54845×10^{-8}	0.0118375	0.0118375	9.04446×10^{-10}	1.93222	1.93222	-3.46825×10^{-8}	0.533364	0.533364	-1.72252×10^{-9}
2.1	0.299203	0.299203	-3.98066×10^{-8}	0.872753	0.872753	9.67927×10^{-8}	0.0114476	0.0114476	-1.57088×10^{-8}	1.96114	1.96114	-2.87812×10^{-8}	0.566803	0.566803	1.12896×10^{-8}
2.2	0.310325	0.310325	-3.91408×10^{-8}	0.83205	0.83205	9.39641×10^{-8}	0.0110554	0.0110554	-3.15214×10^{-9}	1.9863	1.9863	-4.00118×10^{-8}	0.600448	0.600448	1.11875×10^{-8}
2.3	0.322095	0.322095	-4.19786×10^{-8}	0.794137	0.794137	9.5669×10^{-8}	0.010667	0.0106671	1.51563×10^{-8}	2.00784	2.00784	-5.48685×10^{-8}	0.634222	0.634222	1.3661×10^{-8}
2.4	0.334423	0.334423	-5.52784×10^{-8}	0.759031	0.759031	9.52499×10^{-8}	0.0102875	0.0102875	3.57493×10^{-9}	2.02589	2.02589	-1.68997×10^{-8}	0.668053	0.668053	2.50895×10^{-8}
2.5	0.347221	0.347221	-5.74619×10^{-8}	0.726706	0.726706	9.37919×10^{-8}	0.00992102	0.00992103	1.86547×10^{-9}	2.04066	2.04066	-8.1472×10^{-9}	0.701874	0.701874	2.86629×10^{-8}
2.6	0.360409	0.360409	-6.37033×10^{-8}	0.697109	0.697109	8.69521×10^{-8}	0.00957079	0.00957078	-7.95768×10^{-9}	2.05234	2.05234	2.36034×10^{-8}	0.735622	0.735622	3.69859×10^{-8}
2.7	0.373905	0.373905	-6.6155×10^{-8}	0.670162	0.670162	7.2482×10^{-8}	0.00923928	0.00923928	-5.30812×10^{-9}	2.06116	2.06116	4.05929×10^{-8}	0.769237	0.769237	4.02859×10^{-8}
2.8	0.38763	0.387629	-7.28454×10^{-8}	0.645772	0.645773	5.23691×10^{-8}	0.00892832	0.00892833	7.64664×10^{-9}	2.06734	2.06734	6.0765×10^{-8}	0.802667	0.802666	4.67599×10^{-8}
2.9	0.401503	0.401503	-7.97013×10^{-8}	0.623836	0.623836	2.48105×10^{-8}	0.00863915	0.00863915	-1.60511×10^{-9}	2.07112	2.07112	1.1121×10^{-7}	0.835862	0.835862	5.382×10^{-8}
3.	0.415444	0.415444	-8.76108×10^{-8}	0.604243	0.604243	-9.68803×10^{-9}	0.00837246	0.00837246	3.17506×10^{-9}	2.07274	2.07274	1.54861×10^{-7}	0.868781	0.868781	6.0522×10^{-8}
3.1	0.429373	0.429373	-6.99424×10^{-8}	0.586882	0.586881	-1.4311×10^{-8}	0.00812859	0.00812859	9.05189×10^{-10}	2.07242	2.07242	1.4042×10^{-7}	0.901384	0.901384	5.73003×10^{-8}
3.2	0.443208	0.443208	-7.43957×10^{-8}	0.571638	0.571638	-5.08568×10^{-8}	0.00790753	0.00790753	2.47215×10^{-9}	2.07039	2.07039	1.83606×10^{-7}	0.93364	0.933639	6.19868×10^{-8}
3.3	0.456868	0.456868	-7.11981×10^{-8}	0.558402	0.558402	-7.71585×10^{-8}	0.00770905	0.00770904	-9.53232×10^{-9}	2.06688	2.06688	2.18257×10^{-7}	0.965518	0.965518	6.26001×10^{-8}
3.4	0.470272	0.470272	-4.6051×10^{-8}	0.547066	0.547066	-7.2215×10^{-8}	0.00753267	0.00753267	-4.25643×10^{-10}	2.06209	2.06209	1.73763×10^{-7}	0.996997	0.996997	5.6859×10^{-8}
3.5	0.483339	0.483339	-4.27115×10^{-8}	0.537525	0.537525	-9.86756×10^{-8}	0.00737783	0.00737783	-9.24463×10^{-10}	2.05621	2.05621	1.97002×10^{-7}	1.02806	1.02806	5.72829×10^{-8}
3.6	0.49599	0.49599	-4.65041×10^{-8}	0.529679	0.529679	-1.35608×10^{-7}	0.00724382	0.00724383	7.42697×10^{-9}	2.04944	2.04944	2.28611×10^{-7}	1.05868	1.05868	5.82923×10^{-8}
3.7	0.508147	0.508147	-3.87883×10^{-8}	0.523432	0.523432	-1.56884×10^{-7}	0.0071299	0.00712989	-9.09339×10^{-9}	2.04196	2.04196	2.54545×10^{-7}	1.08886	1.08886	5.55201×10^{-8}
3.8	0.519736	0.519736	-3.51978×10^{-8}	0.51869	0.51869	-1.84209×10^{-7}	0.00703517	0.00703518	5.58755×10^{-9}	2.03393	2.03393	2.59294×10^{-7}	1.11858	1.11858	5.31051×10^{-8}
3.9	0.530686	0.530686	-2.28335×10^{-8}	0.515363	0.515363	-2.03267×10^{-7}	0.00695884	0.00695884	-3.07079×10^{-9}	2.0255	2.0255	2.68601×10^{-7}	1.14784	1.14784	4.86255×10^{-8}
4.	0.54093	0.54093	1.31141×10^{-9}	0.513365	0.513365	-2.04778×10^{-7}	0.00689998	0.00689997	-3.84568×10^{-9}	2.01682	2.01682	2.44317×10^{-7}	1.17664	1.17664	4.46796×10^{-8}
4.1	0.550408	0.550408	1.9325×10^{-8}	0.512609	0.512609	-2.12939×10^{-7}	0.00685767	0.00685767	-4.07646×10^{-10}	2.00803	2.00803	2.26104×10^{-7}	1.20498	1.20498	3.98999×10^{-8}
4.2	0.559064	0.559064	3.90407×10^{-8}	0.513008	0.513008	-2.14134×10^{-7}	0.00683099	0.00683098	-3.68089×10^{-9}	1.99925	1.99925	2.07848×10^{-7}	1.23287	1.23287	3.58017×10^{-8}
4.3	0.566853	0.566853	5.58073×10^{-8}	0.514479	0.514478	-2.16355×10^{-7}	0.00681897	0.00681897	1.35718×10^{-9}	1.99058	1.99058	1.85007×10^{-7}	1.2603	1.2603	3.17638×10^{-8}
4.4	0.573737	0.573737	7.8979×10^{-8}	0.516932	0.516932	-1.87375×10^{-7}	0.00682065	0.00682065	-2.57591×10^{-9}	1.98213	1.98213	1.43285×10^{-7}	1.28729	1.28729	3.15618×10^{-8}
4.5	0.579687	0.579687	7.71092×10^{-8}	0.520281	0.52028	-2.1138×10^{-7}	0.00683502	0.00683502	3.83258×10^{-9}	1.97399	1.97399	1.53563×10^{-7}	1.31384	1.31384	2.62066×10^{-8}
4.6	0.584686	0.584686	8.90422×10^{-8}	0.524433	0.524433	-2.14145×10^{-7}	0.00686106	0.00686105	-5.49149×10^{-9}	1.96624	1.96624	1.50375×10^{-7}	1.33997	1.33997	2.24755×10^{-8}
4.7	0.588729	0.588729	9.8831×10^{-8}	0.529296	0.529296	-2.11067×10^{-7}	0.0068977	0.00689769	-1.02833×10^{-8}	1.95895	1.95895	1.40816×10^{-7}	1.36569	1.36569	1.96961×10^{-8}
4.8	0.59182	0.59182	1.09473×10^{-7}	0.534773	0.534773	-2.20316×10^{-7}	0.00694384	0.00694384	2.88493×10^{-9}	1.95219	1.95219	1.20598×10^{-7}	1.39102	1.39102	1.50157×10^{-8}
4.9	0.593978	0.593978	1.23854×10^{-7}	0.540765	0.540765	-2.28125×10^{-7}	0.00699839	0.00699839	-3.86555×10^{-9}	1.94599	1.94599	1.15743×10^{-7}	1.41597	1.41597	1.04282×10^{-8}
5.	0.595231	0.595231	1.28344×10^{-7}	0.547172	0.547172	-2.09897×10^{-7}	0.00706019	0.00706018	-2.72328×10^{-9}	1.94041	1.94041	9.48832×10^{-8}	1.44055	1.44055	9.94084×10^{-9}

VI. Conclusion

In this paper, semi analytical solutions have been evaluated using the Ms-DTM method and compared with ParametricNDSolve solutions. The results are displayed through figures andtables.
The main findings are as follows:

- Ms-DTM provides the solution in terms of convergent series over a sequence of subintervals.
- SIRs models study the influence of treatment on the HIV virus.
- Ms-DTM more reliable and accuracy than others methods.
- Our tables show a good agreement between Ms-DTM and the Mathematica software package (ParametricNDSolve) and compare the errors for the given solutions by the two indicated methods.
- The displayed data show that errors in our solutions are of magnitude less than 4.07685×10^{-16}.

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