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A Fuzzy Vertex Graceful Labeling On Friendship and Double Star Graphs

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Abstract: *A labeling graph G which can be gracefully numbered is said to be graceful. A graph which admits a fuzzy graceful labeling is called a fuzzy graceful graph. In this paper we introduced fuzzy vertex gracefulness and discussed to Friendship graphs and Double star graphs.*
Key words: *Graceful labeling, Fuzzy graceful labeling*

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I. Introduction

Zadeh introduced the fuzzy set as a class of object with a continuum of grades of membership. In contrast to classical crisp sets where a set is defined by either membership or non-membership, the fuzzy approach relates to a grade of membership between [0,1], defined in terms of the membership function of a fuzzy number. Fuzzy relation on a set was first defined by Zadeh in 1965, the first definition of a fuzzy graph was introduced by Kaufmann in 1973 and the structure of fuzzy graphs developed by Azriel Rosenfeld in 1975. Fuzzy graphs have many more applications in modeling real time systems where the level of information inherent in the system varies with different levels of precision. The concept of a graceful labeling has been introduced by Rosa in 1966.This note is a further contribution on fuzzy labeling. Fuzzy labeling for fuzzy Friendship graph and fuzzy Double star graph are called a fuzzy labeling Friendship graph and fuzzy labeling Double star graph respectively.

II. Preliminaries and Main results

Definition 1
Let U and V be two sets. Then ρ is said to be a fuzzy relation from U into V if ρ is a fuzzy set of U x V.

Definition 2
A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \rightarrow [0, 1]$ and $\mu : VxV \rightarrow [0, 1]$, where for all $u, v \in V$, we have $\mu (u, v) \leq \sigma (u) \wedge \sigma (v)$.

Definition 3
A labeling of a graph is an assignment of values to the vertices and edges of a graph.

Definition 4
A graceful labeling of a graph G with q edges is an injection $f : V(G) \rightarrow \{0,1,2,3,\ldots,q\}$ such that when edge xy $\in E(G)$ is assigned the label $|f(x) - f(y)|$, all of edge labels are distinct.

Definition 5
A graph $G = (\sigma, \mu)$ is said to be a fuzzy labeling graph if $\sigma : V \rightarrow [0,1]$ and $\mu : VxV \rightarrow [0,1]$ is bijective such that the membership value of edges and vertices are distinct and $\mu (u, v) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

Definition 6
A Friendship graph F_n is a graph which consist n triangles with a common vertex.

Definition 7
A Friendship graph with fuzzy labeling is called a fuzzy Friendship graph.

Definition 8
In a fuzzy Friendship graph if all vertex values are distinct then it is called fuzzy vertex graceful labeling Friendship graph.
A Friendship in a fuzzy graph consists of two node sets V and U with $|V| = 1$ and $|U| > 1$, such that $\mu(v, u_i) > 0$, where i = 1 to n - 1 and $\mu(u_i, u_{i+1}) > 0$, Where i = 1 to n – 2.

Definition 9
Double star graph is obtained from K_2 by joining in pendent edges to both the end and n pendent edges to the other end of K_2.

Definition 10
A Double star graph with fuzzy labeling is called a fuzzy Double star graph.

Definition 11
In a fuzzy Double star graph if all vertex values are distinct then it is called fuzzy vertex graceful labeling Double star graph.
A fuzzy Double star graph $DS_{1,n}$ consists of two node sets V and W with $|V| = 1$ and $|W| > 1$, such that $\mu(v, w_i) > 0$, where i = 1 to n and $\mu(w_i, w_{i+1}) = 0$, Where i = 1 to n.

Remark
In the fuzzy Friendship graph $F_{1,n}$ the vertex labeling $\sigma : F \to [0,1]$ satisfies the condition that if the values of F starts only from n - 1/10, n/10, n + 1/10, n + 2/10, etc., then the Friendship graph is a fuzzy vertex graceful labeling Friendship graph.

Preposition 1
Every fuzzy fan graph $F_{1,n}$ is a fuzzy vertex graceful fan graph.

Proof
A Friendship graph F_n is a graph with n vertices for $n \geq 2$.
A Friendship in a fuzzy graph consists of two node sets F and F_n with
$|F| = 1$ and $|F_n| > 1$, such that $\mu(F, F_i) > 0$, where i = 1 to n - 1 and
$\mu(F_i, F_{i+1}) = 0$ if i is even, where i = 1 to n - 2.
In the Friendship graph F is the central vertex and F_i denotes the vertices in the outer triangle.
Here when $\sigma : F \to [0,1]$ and $\sigma : F_i \to [0,1]$ defined by
$\sigma(F_i) = \sigma(F) - \mu(F, F_i)$, where $\mu(F, F_i) = 0.01\times2^{i-1}$, i = 1 to n - 1 and
$\mu(F_i, F_{i+1}) = \mu(F, F_i)$ where i = 1 to n - 2.
(or)
$\mu(F_{n-2}, F_{n-1}) = \mu(F, F_{n-2})$.
Examples

Case (1)
When $\sigma(F)$ starts from n - 3/10 to n + 3/10
Here when $\sigma : F \to [0,1]$ and $\mu : F_i \to [0,1]$,
$\sigma(F_1) = \sigma(F) - 0.01$
$\sigma(F_2) = \sigma(F) - 0.02$
$\sigma(F_3) = \sigma(F) - 0.04$
$\sigma(F_4) = \sigma(F) - 0.08$
$\sigma(F_5) = \sigma(F) - 0.16$ etc.,
$\sigma(F_i) = \sigma(F) - (0.01)\times2^{i-1}$, where i = 1 to n - 1.
That is $\sigma(F_i) = \sigma(F) - \mu(F, F_i)$, where $\mu(F, F_i) = (0.01)\times2^{i-1}$, i = 1 to n - 1.
Also
$\mu(F_1,F_2) = \mu(F, F_1)$
$\mu(F_2,F_3) = \mu(F, F_2)$
$\mu(F_3,F_4) = \mu(F, F_3)$
$\mu(F_4,F_5) = \mu(F, F_4)$
$\mu(F_i,F_{i+1}) = \mu(F, F_i)$ where i = 1 to n – 2.

Case (2)

When σ (F) starts from n - 3/100 to n + 3/100

Here when $\sigma : F \to [0,1]$ and $\mu : F_i \to [0,1]$,

$\sigma(F_1) = \sigma(F) - 0.001$

$\sigma(F_2) = \sigma(F) - 0.002$

$\sigma(F_3) = \sigma(F) - 0.004$

$\sigma(F_4) = \sigma(F) - 0.008$

$\sigma(F_5) = \sigma(F) - 0.016$ etc.,

$\sigma(F_i) = \sigma(F) - (0.001)\times 2^{i-1}$, where i = 1 to n - 1.

That is $\sigma(F_i) = \sigma(F) - \mu(F, F_i)$, where $\mu(F, F_i) = (0.001)\times 2^{i-1}$, i = 1 to n - 1.

Also

$\mu(F_1,F_2) = \mu(F, F_1)$

$\mu(F_2,F_3) = \mu(F, F_2)$

$\mu(F_3,F_4) = \mu(F, F_3)$

$\mu(F_4,F_5) = \mu(F, F_4)$

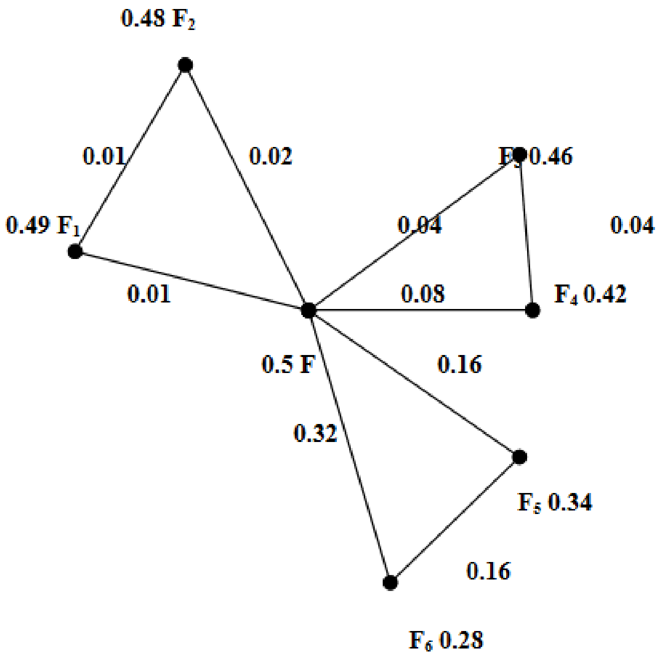
$\mu(F_i,F_{i+1}) = \mu(F, F_i)$ where i = 1 to n – 2

Like this for any values of $\sigma : F \to [0, 1]$, the labeling of all vertices in the outer triangle F_i are distinct where i = 1 to n - 1.

That is $\sigma(F_i) = \sigma(F) - \mu(F, F_i)$, where i = 1 to n - 1.

According to this condition the friendship graph F_n is a fuzzy vertex graceful friendship graph.

Example



Fuzzy vertex graceful labeling Friendship graph F_3

Preposition 2

Every fuzzy Double star graph $DS_{1,n}$ is a fuzzy vertex graceful Double star graph.

Proof

A Double star graph $DS_{1,n}$ is a graph with n vertices only if $n \geq 1$.

A Double star in a fuzzy graph consists of two node sets V and W with

$|V| = 1$ and $|W| > 1$, such that $\mu(v, w_i) > 0$, where i = 1 to n.

In the Double star graph V is the central vertex and W_i denotes the inner and outer vertices.

Here when $\sigma : V \to [0,1]$ and $\sigma : W_i \to [0,1]$ defined by

$\sigma(W_i) = \sigma(V) - \mu(V, W_i)$, where $\mu(V, W_i) = 0.001 \times i$ for inner vertices and

$\sigma(W_{i+n}) = \sigma(w_i) - \mu(W_i, W_{i+n})$, where $\mu(W, W_{i+n}) = 0.001 \times (n + i)$ for i =1 to n.

Examples

Case (1)

When σ (V) starts from n - 1/100

Here when $\sigma : V \rightarrow [0,1]$ and $\mu : W_i \rightarrow [0,1]$

For inner vertices,

$\sigma (W_1) = \sigma (W) - 0.001$

$\sigma (W_2) = \sigma (W) - 0.002$

$\sigma (W_3) = \sigma (W) - 0.003$

$\sigma (W_4) = \sigma (W) - 0.004$ and etc.,

that is $\sigma (W_i) = \sigma (V) – \mu (V, W_i)$, where $\mu(V, W_i) = 0.001 \times i$ for i = 1 to n.

Also

$\mu(V, W_1) = 0.001$

$\mu(V, W_2) = 0.002$

$\mu(V, W_3) = 0.003$

For outer vertices,

$\sigma (W_{i+n}) = \sigma (w_i) – \mu (W_i, W_{i+n})$, where $\mu(W, W_{i+n}) = 0.001 \times (n + i)$ for i =1 to n.

$\sigma (W_5) = \sigma (w_1) – 0.006$

$\sigma (W_6) = \sigma (w_i) – 0.007$

$\sigma (W_7) = \sigma (w_i) – 0.008$

Also

$\mu(W_1, W_5) = 0.006$

$\mu(W_2, W_6) = 0.007$

$\mu(W_3, W_7) = 0.008$

Case (2)

When σ (V) starts from n - 1/1000

Here when $\sigma : V \rightarrow [0,1]$ and $\mu : W_i \rightarrow [0,1]$

For inner vertices,

$\sigma (W_1) = \sigma (W) - 0.0001$

$\sigma (W_2) = \sigma (W) - 0.0002$

$\sigma (W_3) = \sigma (W) - 0.0003$ and etc.,

that is $\sigma (W_i) = \sigma (V) – \mu (V, W_i)$, where $\mu(V, W_i) = 0.0001 \times i$ for i = 1 to n.

Also

$\mu(V, W_1) = 0.0001$

$\mu(V, W_2) = 0.0002$

$\mu(V, W_3) = 0.0003$

For outer vertices,

$\sigma (W_{i+n}) = \sigma (w_i) – \mu (W_i, W_{i+n})$, where $\mu(W, W_{i+n}) = 0.0001 \times (n + i)$ for i =1 to n.

$\sigma (W_5) = \sigma (w_1) – 0.0006$

$\sigma (W_6) = \sigma (w_i) – 0.0007$

$\sigma (W_7) = \sigma (w_i) – 0.0008$

Also

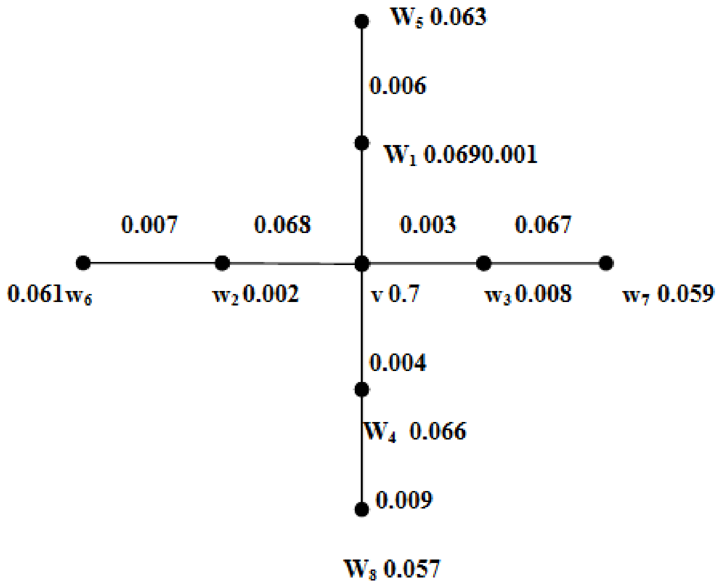
$\mu(W_1, W_5) = 0.0006$

$\mu(W_2, W_6) = 0.0007$

$\mu(W_3, W_7) = 0.0008$

Hence every double star graph is a fuzzy vertex graceful labeling Double star grap

`Example



Fuzzy vertex graceful labeling Double star graph $DS_{1,4}$

III. Conclusion

In this paper, the concepts of fuzzy vertex graceful labeling Double Star graph and fuzzy vertex graceful labeling Friendship graph have been discussed. We further extend this study on some more special classes of graphs.

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