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Generalized Trigonometric Functions and Their Applications

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Annotation: *We consider a class of generalized trigonometric functions given by the divergent trigonometric series; coefficients of these series have a certain order of growth. These functions have no values in usual sense and manifest themselves only as convolution with the test functions. Classes of test functions are formed by even periodical functions that are either defined by chosen Fourier coefficients or have certain differential properties. The operations of differentiation and integration of these generalized trigonometric functions are considered. The class of generalized trigonometric functions of zero order with coefficients forming* N*-periodical sequences is investigated in details. Such functions under certain conditions that are received in this paper have interpolating properties. The results of calculations for the test example are given; these results are well correlated to those predicted by theory.*

Keywords: *generalized trigonometric functions, test functions, divergent trigonometric series, convolution, have interpolation*

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I. Introduction

As you know, the trigonometric series is called the expression

$$\frac{a_0}{2}+\sum_{k=1}^{\infty}a_k\cos kt+b_k\sin kt\ , \tag{1}$$

where a_0, a_k, b_k - real numbers ($k=1,2,\ ...$).

Trigonometric series can be convergent and divergent. In the case where these rows are convergent, their sum are periodic functions with period 2π .

In many cases, we have to deal with divergent trigonometric series, which have no sum in the usual sense. Such series often happen in theoretical studies, during the formal differentiation of convergent trigonometric series etc. For example, in theoretical studies of Fourier series, there is a need for research of series like

$$\frac{1}{2}+\sum_{k=1}^{\infty}\cos kt\ ,$$

which is divergent at each point of the interval $[0,2\pi]$.

One approach to the study of trigonometric divergence series is the approach in which such rows are considered as generalized periodic functions of slow growth; these functions are given in the classes of the main functions K. In the role of the main functions of the class K perform pereodic, infinitely differentiated functions. In this case, the functional, which is the result of the action of a generalized periodic function on the basic functions of the class K, is presented in the form of a convergent numerical series. The expediency of such an approach is explained by the fact that the numerical series through which the functionsionals are presented - are the generalized periodic functions of slow growth in the class K, always convergent in the usual sense. However, this convenience, in our opinion, significantly restricts the use of divergent rows, because the requirement of infinite differentiation of the main functions is rather burdensome.

We propose a different approach to the study of trigonometric divergence series, in which, as before, these series are considered as generalized periodic functions of slow growth; however, these functions are given in the classes of the main functions $K(p)$; in role of functions of class $K(p)$ stand pairs of periodic functions, differentiated p ($p\geq 0$) times. In contrast to the classical approach, the generalized periodic functions that are given in the class of fundamental functions $K(p)$, we will call generalized trigonometric functions. As before, the functional, which is the result of action the generalized trigonometric function on the main functions of the class $K(p)$, given in the form of a numerical series; the value of the same parameter p are selected in such way, to to ensure the convergence of these numerical rows. This approach was quite resultive; so, he leads to the construction of classes of interpolation trigonometric splines. Note that the class of trigonometric splines is quite broad; In such way, in particular, it also includes the class of periodic simple polynomial splines.

II. Analysis of researches and publications

Divergent series in different years were considered by well-known mathematicians Euler, Leibniz, d'Alembert, Largange, Bernoulli, Abel, Frobenius, Gelder, Poisson, Voroniy, S.Bernshtein, and others [1]. In considering such series, methods of generalized summation were used. This approach was considered in [2].

Another approach to the study of divergent trigonometric series was proposed by Schwartz [3]. In this approach, divergent trigonometric series are regarded as objects of a new type, which are called distributions or generalized periodic functions; such distributions are given as functionals in classes of infinitely differentiated main functions. This approach was considered in the works of Mikunskiy, Edwards, Nikolsiy etc.

In this paper we consider another approach to the construction of generalized trigonometric functions, in which the classes of functions differentiated by finite number of times are selected as the classes of basic functions.

III. The purpose of the work.

Development of the basic of the theory of generalized trigonometric functions, which are given in classes of basic functions, differentiated finite number of times; obtaining conditions for the existence of generalized trigonometric functions; Investigation of combinations of generalized trigonometric functions with basic functions; obtaining conditions while such convolutions have interpolation properties. Illustration of theoretical positions by test case.

IV. The main part.

Let's consider class $G(\rho)$ divergent trigonometric series of order ρ ($0 \le \rho < \infty$),

$$\frac{a_0}{2} + \sum_{k=1}^{\infty} a_k(\rho)\cos kt + b_k(\rho)\sin kt \tag{1}$$

Where coefficients $a_k(\rho)$, $b_k(\rho)$, ($k = 1, 2, ...$) are satisfied the conditions

$$|a_k(\rho)|,\ |b_k(\rho)| \le Ck^{\rho}, \quad 0 < C < \infty. \tag{2}$$

Of course, coefficients $a_k(0)$, $b_k(0)$ of functions of class $G(0)$ are limited numerical sequences.

For such classes of trigonometric series was preposed by Schwarts [3 the approach, at which this divergent series defines a generalized periodic function (distribution) $\varphi(\rho, t)$, so

$$\varphi(\rho, t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k(\rho)\cos kt + b_k(\rho)\sin kt\ . \tag{3}$$

Let the class of infinitely differentiated principal pairwise periodic functions, which can be represented by Fourier series

$$\lambda(t) = \frac{1}{2} + \sum_{k=1}^{\infty} \nu_k \cos kt\ , \tag{4}$$

Then, according to [3], the result of the generalized periodic function (distribution) (φ, λ) on the main function $\lambda(t) \in K$ can be consoderes like

$$(\varphi, \lambda) = \int_{-\pi}^{\pi} \varphi(\rho, t)\lambda(\mathrm{t})dt\,(\varphi, \mu) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \nu_k\left[a_k(\rho) + b_k(\rho)\right], \tag{5}$$

and convolution $\varphi(\rho, t) * \lambda(t)$ - like

$$\varphi(\rho, t) * \lambda(t) = \int_{-\pi}^{\pi} \varphi(\rho, t - \tau)\lambda(\tau)d\tau = \frac{a_0}{2} + \sum_{k=1}^{\infty} \nu_k\left[a_k(\rho)\cos kt + b_k(\rho)\sin kt\right]. \tag{6}$$

As we have said earlier, the requirement of infinite differentiation of the main functions $\lambda(t) \in K$ limits the use of the apparatus of generalized periodic functions. We have proposed another approach to constructing generalized periodic functions, in which the class of functions having a finite number of derivatives

is selected as the basic functions. This approach, which receives generalized trigonometric functions, is a certain analogue of the approach of Schwarts. Let's consider this approach in more detail.
As before, we are considering functions $\varphi(\rho,t)$ of class $G(\rho)$. However, now as functions of the main functions we will consider functions $\mu(\alpha,r,t)$, which can be presented as Fourier series

$$\mu(\alpha,r,t)=\frac{1}{2}+\sum_{k=1}^{\infty}\nu_k(\alpha,r)\cos kt\,,\qquad (r>1),\tag{7}$$

Where Furier $\nu_k(\alpha,r)$, ($k=1,2,\,...$), have a decreasing order $O(k^{-(r+2)})$, ($0\le r<\infty$), and depends the parameter α. It is clear that such basic functions are pairwise periodic functions, which have a continuous derivative r of order ($r=0,1,\,...$); the class of such functions is denoted $P(\alpha,r)$.
Every function $\varphi(\rho,t)\in G(\rho)$ we put the generalized trigonometric function of the form into conformity

$$\left(\varphi,\mu\right)=\int_{-\pi}^{\pi}\varphi(\rho,t)\mu(\alpha,r,t)dt\tag{8}$$

assuming, of course, that this functional exists.

If function $\varphi(\rho,t)\in G(\rho)$ looks like (3), and function $\mu(\alpha,r,t)\in P(\alpha,r)$ looks like (7), so functional (8) takes shape

$$\left(\varphi,\mu\right)=\int\varphi(\rho,t)\mu(\alpha,r,t)dt=\frac{a_0}{2}+\sum_{k=1}^{\infty}\nu_k(\alpha,r)\left[a_k(\rho)+b_k(\rho)\right]\tag{9}$$

It is easy to obtain conditions under which generalized trigonometric functions exist (9). Because the coefficients $a_k\left(\rho\right),\ b_k\left(\rho\right)$, ($k=1,2,\,...$) have a growth order ρ ($\rho=0,1,\,...$), and coefficients $\nu_k(\alpha,r)$, ($k=1,2,\,...$), have decreasing order $O(k^{-(r+2)})$, ($r=0,1,\,...$), then it is clear that for the convergence of the series on the right side (6) the condition must be satisfied $r-\rho>-1$;
taking into account that ρ i r whole, this condition takes on shape

$$r\ge\rho\,.\tag{10}$$

As before, the convolution of a generalized trigonometric function $\varphi(\rho,t)$ and basic function $\mu(\alpha,r,t)$ given like expression

$$\varphi(\rho,t)*\mu(\alpha,r,t)=\left\{\frac{a_0}{2}+\sum_{-N}^{N}\nu_k(\alpha,r)\left[a_k(\rho)\cos kt+b_k(\rho)\sin kt\right]\right\}.\tag{11}$$

In this case, depending on the parameters ρ i r, we can obtain both a normal function and a generalized trigonometric function. So, when the condition is satisfied $r\ge\rho$ we get an ordinary continuous function; at $r-\rho\le-2$ we obtain a generalized trigonometric function; at other values ρ i r series (11) requires additional research.

In the future, we will consider convolutions of generalized and basic functions in assuming the satisfying of the condition $r\ge\rho$; In this case, we obtain ordinary periodic (trigonometric) functions $S(\alpha,p,t)$, which depends from parameter α, differentiated $p=r-\rho$ times, have an argument t and given nearby (11).

An important role in the theory of generalized trigonometric functions is played by the methods of constructing the basic functions of the class $P(\alpha,r)$. In [2] Some variants of the choice of basic functions were considered, the Fourier coefficients of which satisfy this condition. Thus, in the role of Fourier coefficients of the main functions, members of the sequence of type $\frac{\psi(\alpha,k)}{k^{r+2}}$, $\sin\frac{\psi(\alpha,k)}{k^{r+2}}$, etc, where $\psi(\alpha,k)$ - some limited features of the number k and parameter α, not equal to 0. In this case, the main functions are given evenly in convergent series

$$\mu(r,t)=\frac{1}{2}+\sum_{k=1}^{\infty}\frac{\psi(k)}{k^{r+2}}\cos kt\,,\quad \mu(r,t)=\frac{1}{2}+\sum_{k=1}^{\infty}\sin\frac{\psi(k)}{k^{r+2}}\cos kt\,.$$

Such approach provides the ability to construct the necessary basic functions. In the role of basic functions one can also use periodic, continuous continuous functions with continuous derivatives of order r inclusive, and derivatives of order $r+1$ have a limited variation. Under these conditions, the Fourier coefficients of these functions have a decreasing order $O(\mathrm{r}+2)$. In this approach, the functions of controlled smoothness can act as functions of the main functions, in particular, the classes of polynomial B - splines, classes of trigonometric and polynomial fundamental functions of corresponding orders, considered by us in [2], etc.

V. Remark.

Generally speaking, for a convolution of generalized and basic functions the condition of existence has the form $r-\rho>-2$. This is explained by the fact that for trigonometric series there are signs of convergence, which do not demand the convergence of a series made up of coefficients of these series (for example, the sign of Dirichlet). But such rows do not coincide evenly, and therefore, their amounts are not continuous functions. However, sometimes and such convolutions are of interest.

VI. Differentiation and integration.

Generalized trigonometric functions can be integrated and differentiated. Yes, formally differentiating (1) p times, ($p=1,2,\ldots$), and marked $\frac{d^p}{dt^p}\varphi(\rho,t)=D_t^p\varphi(\rho,t)$, we will get

$$D_t^p\varphi(\rho,t)=\sum_{k=1}^{\infty}k^p\left[a_k(\rho)\cos(kt+\frac{\pi}{2}\text{p})+b_k(\rho)\sin(kt+\frac{\pi}{2}\text{p})\right].$$

So, as a result of the differentiation of a generalized trigonometric function of order ρ we obtain a new generalized trigonometric order function $\rho+p$; In other words, differentiation increases the order of the generalized trigonometric function. Otherwise, the case seems to be with the integration of generalized trigonometric functions. So, formally integrating (1), (note that in integrating we always consider functions with zero mean), we obtain

$$I_t^p\varphi(\rho,t)==\sum_{k=1}^{\infty}\frac{1}{k^p}\left[a_k(\rho)\cos(kt-\frac{\pi}{2}\text{p})+b_k(\rho)\sin(kt-\frac{\pi}{2}\text{p})\right].$$

As a result of the integration of a generalized trigonometric function of order, we obtain a new generalized trigonometric function of order $\rho-p$; In other words, integration minimizes the order of the generalized trigonometric function, and with values $p>\rho$ we get (in case of convergence of a row) the usual functions.

Considering the convolution of generalized trigonometric functions $D_t^p\varphi(\rho,t)$ та $I_t^p\varphi(\rho,t)$ with the main functions, it is easy to see that changes in the order of generalized trigonometric functions that occur when they are integrated and differentiated, can be taken into account by changing the order of the main functions. Particular attention, however, should be given to the change of the arguments of the trigonometric functions of the members of a series, which, in each integration (differentiation), are interchanged in a cofunction with the corresponding signs.

VII. Interpolation convolutions.

An important subclass of generalized trigonometric functions is the set of functions 0 - n order, the sequence of coefficients of which $a_k(0)$, $b_k(0)$, ($k=1,2,\ldots$) are N - periodic sequences ($N=2n+1$, $n=1,2,\ldots$). Denote the functions of this subclass through $G(0,N)$; it is seen that $G(0,N)\subset G(\rho)$. Note that the operations of differentiating and integrating the class function are locked, that is, they again lead to class functions $G(0,N)$.

The expediency of allocating a subclass $G(\rho,N)$ generalized trigonometric functions due to the fact that the convolution of the functions of this subclass with the main functions have certain interpolation properties. Let's consider this question in more detail.

Let it be on the line $[0,2\pi]$ given a continuous function Let's also have a uniform grid on this segment $\Delta_N=\{t_i\}_{i=1}^N$, $t_i=\frac{2\pi}{N}(i-1)$, $N=2n+1$, $n=1,2,\ldots$. Note through $\{f(t_i)\}_{i=1}^N=\{f_i\}_{i=1}^N$ set of function values $f(t)$ in the nodes of the grid Δ_N. Consider a trigonometric polynomial

$$T_n^*(t)=\frac{a_0^*}{2}+\sum_{k=1}^{n}a_k^*\cos kt+b_k^*\sin kt\,, \tag{12}$$

which interpolates the function $f(t)$ on the grid Δ_N. Then the coefficients of this polynomial are determined by the formulas

$$a_k^* = \frac{2}{N}\sum_{j=1}^{N} f_j \cos kt_j, \qquad b_k^* = \frac{2}{N}\sum_{j=1}^{N} f_j \sin kt_j, \tag{13}$$
$$k = 0,1,...,n; \qquad k = 1,2,...,n.$$

Brought in line in a trigonometric polynomial with a divergent trigonometric series

$$\frac{a_0^*}{2} + \sum_{k=1}^{\infty} a_k^* \cos kt + b_k^* \sin kt, \tag{14}$$

coefficients a_k^*, b_k^* which are calculated by the formulas (13) for any values k ($k = 1, 2, ...$).

It's easy to make sure that the sequences of the coefficients a_k^*, b_k^* are periodic with a period N and series (14) is divergent; although its coefficients are limited, they do not go up to 0 with growth k. So, this series represents a generalized trigonometric function 0-th order. Taking to attention, that coeddicient a_0^* in the vast majority of cases is not of interest, we will submit (14) in a different form and formally denote

$$\varphi(0,N,t) = \frac{a_0^*}{2} + \sum_{k=1}^{n}\left[a_k^* C_k(N,t) + b_k^* S_k(N,t)\right], \tag{15}$$

where

$$C_k(N,t) = \cos kt + \sum_{m=1}^{\infty} \cos(mN+k)t + \cos(mN-k)t;$$
$$S_k(N,t) = sinkt + \sum_{m=1}^{\infty} \sin(mN+k)t - \sin(mN-k)t$$

Then, as we already said, the convolution $F(N,\alpha,r,t) = \varphi(0,N,t) * \mu(\alpha,r,t)$, $\mu(\alpha,r,t) \in P(\alpha,r)$, $r \geq 2$, can be filed in the form

$$F(N,\alpha,r,t) = \frac{a_0^*}{2} + \sum_{k=1}^{n}\left[a_k^* C_k(\alpha,\nu,r,N,t) + b_k^* S_k(\alpha,\nu,r,N,t)\right], \tag{16}$$

where

$$C_k(\alpha,\nu,r,N,t) = \nu_k(\alpha,r)\cos kt + \sum_{m=1}^{\infty} \nu_{mN+k}(\alpha,r)\cos(mN+k)t + \nu_{mN-k}(\alpha,r)\cos(mN-k)t,$$
$$S_k(\alpha,\nu,r,N,t) = \nu_k(\alpha,r)\sin kt + \sum_{m=1}^{\infty} \nu_{mN+k}(\alpha,r)\sin(mN+k)t - \nu_{mN-k}(\alpha,r)\sin(mN-k)t.$$

It is clear that the series (16) coincides evenly. We will now require a convulation $F(N,\alpha,r,t)$ interpolated trigonometric polynomial (12) (and the function $f(t)$) in the nodes of the grid Δ_N. To do this, we introduce the expression (16) multiplier $H_k(\alpha,r,N)^{-1}$ which one will choose from the conditions of interpolation.

Calculating the value of the convolution in the nodes of the grid Δ_N, we will get

$$F(N,\alpha,r,t_i) = \frac{a_0^*}{2} + \sum_{k=1}^{n} \frac{1}{H_k(\alpha,r,N)}\left[a_k^* C_k(\alpha,\nu,r,N,t_i) + b_k^* S_k(\alpha,\nu,r,N,t_i)\right]$$

Taking to attention that,

$$cos(mN \pm k)t_i = coskt_i, \quad \sin(mN+k)t_i = \sin kt_i, \quad \sin(mN-k)t_i = -\sin kt_i,$$

So equality,

$$C_k(\alpha,\nu,r,N,t_i) = \cos kt_i\left[\nu_k(\alpha,r) + \sum_{m=1}^{\infty} \nu_{mN+k}(\alpha,r) + \nu_{mN-k}(\alpha,r)\right],$$
$$S_k(\alpha,\nu,r,N,t_i) = \sin kt_i\left[\nu_k(\alpha,r) + \sum_{m=1}^{\infty} \nu_{mN+k}(\alpha,r) + \nu_{mN-k}(\alpha,r)\right],$$

In the end we obtain,

$$F(N,\alpha,r,t_i) = \frac{a_0^*}{2} + \sum_{k=1}^{n} \frac{1}{H_k(\alpha,r,N)}\left[\nu_k(\alpha,r) + \sum_{m=1}^{\infty} \nu_{mN+k}(\alpha,r) + \nu_{mN-k}(\alpha,r)\right] \times \left[a_k^* \cos kt_i + b_k^* \sin kt_i\right]. \tag{17}$$

Comparing the obtained expression with (12), it follows that the convolution interpolates a trigonometric polynomial $T_n^*(t)$ (and accordingly the function $f(t)$) in the nodes of the grid Δ_N if, and only if equality is fulfilled

$$\frac{1}{H_k(\alpha,r,N)}\left[\nu_k(\alpha,r)+\sum_{m=1}^{\infty}\nu_{mN+k}(\alpha,r)+\nu_{mN-k}(\alpha,r)\right]=1,$$

$$(k=1,2,\ldots,n).$$

Consequently, there is a **theorem**. If multiplier $H_k(\alpha,r,N)$ looks like

$$H_k(\alpha,r,N)=\left[\nu_k(\alpha,r)+\sum_{m=1}^{\infty}\nu_{mN+k}(\alpha,r)+\nu_{mN-k}(\alpha,r)\right], \quad (k=1,2,\ldots,n). \tag{18}$$

So convulation $F(N,\alpha,r,t)$ interpolates a trigonometric polynomial $T_n^*(t)$ in the nodes of the grid Δ_N. Considering (18), interpolation convolution $F(\varphi,0,N,\mu,r,t)$ can be written as

$$F(N,\alpha,r,t_i)=\frac{a_0^*}{2}+\sum_{k=1}^{n}\frac{1}{H_k(\alpha,r,N)}\left[a_k^*C_k(\alpha,\nu,r,N,t_i)+b_k^*S_k(\alpha,\nu,r,N,t_i)\right]. \tag{19}$$

Let's turn to the consideration of specific classes of basic functions $\mu(\alpha,r,t)\in P(\alpha,r)$, Fourier coefficients of which have certain decreasing orders. As we have already said, there are several possible approaches here.

1. In the role of the main functions, select functions with known analytical representation, calculate their Fourier coefficients and apply them to calculate the convolution by the formula (17).
2. Specify sequences with a certain decreasing order; members of this sequence are further regarded as Fourier coefficients and calculate a convolution of formula (17). If necessary, the main function is constructed through a Fourier series.
3. Combined approach, in which the Fourier coefficients of the main function are multiplied by the corresponding sequence members with a certain decreasing order. Consider these approaches in more detail. In the first approach, without losing generality, we restrict ourselves to considering the case when in the role of basic functions of the classes of periodic polynomial B- splines, which were considered in [2]. Fourier coefficients $\sigma_k(r)$ B- splines of order r, built on a grid Δ_N, looks like

$$\sigma_k(\alpha,r)=\left[\operatorname{sinc}(\alpha k)\right]^{r+1}, \text{ where } \operatorname{sinc}x=\frac{\sin x}{x}, \quad (r=0,1,\ldots) \tag{20}$$

and, respectively, have a descending order $O\left(k^{-(r+1)}\right)$; Note that in this case $\alpha=\dfrac{\pi}{N}$.

In the other approach, in the role of Fourier coefficients, members of the sequence of type can be selected $\eta(k)k^{-(r+2)}$, $\eta(k)\sin\left(k^{-(r+2)}\right)$, $\eta(k)\sin^{r+2}\left(k^{-1}\right)$ etc, where $\eta(k)$ - some limited function is not identically identical to zero. Of course, the main functions corresponding to these Fourier coefficients are constructed as sum of uniformly convergent (when $r\geq 0$) Fourier series. Note that this approach, in which the Fourier coefficients were used like $k^{-(r+2)}$, is dealt with details [2].

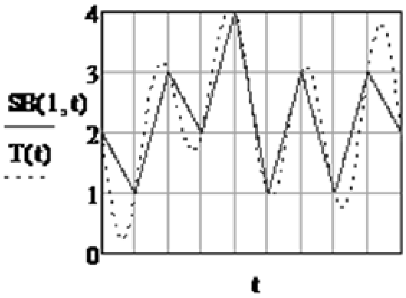
Finally, the third approach combines the first and second approaches, for example, coefficients B-spline of order m multiplied by a member of the sequence $\eta(k)k^{-q}$ ($m,q=1,2,\ldots$); received expressions

$$\frac{\eta(k)}{k^{-q}}\left[\operatorname{sinc}\left(\frac{\pi k}{N}\right)\right]^{m+1}, \quad (k=1,2,\ldots);$$

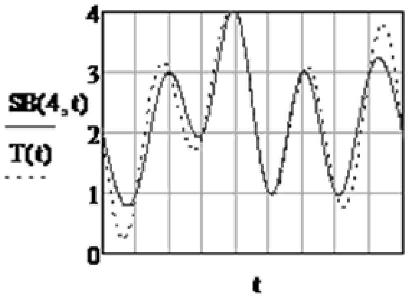
considered in the future as Fourier coefficients in decreasing order $O\left(k^{-(m+q+1)}\right)$.

Consider an example that illustrates the behavior of interpolation convulation. Let it be on the line $[0,2\pi)$ given a continuous test function $f(t)$. Let's also $N=9$, and a sequence of function values $\{f_i\}_{i=1}^{9}$ on the grid Δ_9 looks like $\{2,1,3,2,4,1,3,1,3\}$. Calculating the coefficients a_k^*, b_k^*, $(k=1,2,\ldots,n)$ interpolation trigonometric polynomial $T_n^*(t)$ with formulas (13), we create a generalized trigonometric function 0- th order according to (16).

In the role of the main functions we choose B - splines, for example, of the first and fourth orders, the coefficients of which have the form (20). Calculate convolutions $FI(\varphi, N, \nu, r, t)$ ($N=9$, $r=1,4$) by formulas (19). The graphs of these convulations and the polynomial graph $T_n^*(t)$ is seen on graph 1, 2.



Graph1.



Graph.2.

Based on the results of numerical experiments, we can conclude that these results are in good agreement with the theoretical positions.

Of course, the illustrative material given does not in any way exhaust the variability of interpolation cones constructed by the methods above.

VIII. Conclusions

1. A new approach to the construction of generalized trigonometric functions is considered, which differs from those known by the fact that these functions are given in the classes of finitely differentiated basic functions.
2. Considered the construction of classes of basic functions; It is shown that these classes can be specified in at least three ways.
3. The operations of differentiation and integration of generalized trigonometric functions are introduced.
4. Issues of interpolation of trigonometric polynomials in knots of even-dimensional grids by convolutions of generalized trigonometric functions with principal functions are considered. The obtained results extend to the problem of interpolation of continuous functions in the same nodes.
5. Developed methods for constructing interpolation functions can be considered as methods for constructing functions with given differential properties; In particular, these methods can be considered as methods of constructing interpolation trigonometric splines arbitrary in-line.
6. The class of interpolation trigonometric splines is quite broad and, in particular, includes a well-known class of simple polynomial periodic splines.
7. The results of the numerical calculations are in good agreement with the theoretical provisions.
8. Developed methods of interpolation using classes of polynomial and trigonometric fundamental functions require further research.

References in English

[1]. *Hardy G. H.* Divergent Series. – Oxford, 1949.
[2]. *Denysiuk V. P.* Trigonometric series and splines. – K.: "NAU", 2017. – 212 p. (Ukrainian).
[3]. Scwartz L. Méthodes mathématiques pour les sciences physiques. – Hermann, 115, Boulevard Saint-Germain, Paris VI, 1961. – 412p.

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