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A Study on Circular Motion and Its Applications

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Abstract: *In this paper we have discussed about the circular motion and its applications. The basic concept of dynamics and derivation of circular motion is derived. The application problem such as satellite in a circular orbit, moving car are solved by using derivation of circular motion.*
Keywords: *centripetal force, centrifugal force, clothoid, conical pendulum, uniform magnetic field,*

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I. Introduction

In physicscircularmotion is a movement of an object along the circumference of a circle or rotation along a circular path. It can be uniform, with constant angular rate of rotation and constant speed, or non-uniform with a changing rate of rotation. The rotation around a fixed axis of a three-dimensional body involves circular motion of its parts. The equations of motion describe the movement of the center of mass of a body.

Examples of circular motion include: an artificial satellite orbiting the Earth at a constant height, a stone which is tied to a rope and is being swung in circles, a car turning through a curve in a race track, an electron moving perpendicular to a uniform magnetic field, and a gear turning inside a mechanism.

Since the object's velocity vector is constantly changing direction, the moving object is undergoing acceleration by a centripetal forcein the direction of the center of rotation. Without this acceleration, the object would move in a straight line, according to Newton's laws of motion.

Principle of circular motion

If the direction of the acting force is in the same direction of motion , The velocity of the moving object increases , the direction ofmotion does not change .

Example : when the motorcyclist pumps more fuel , the motorbike is acted upon by a force in the same direction of motion and accelerates or increases velocity .

If the direction of the acting force is opposite to the direction of motion , The velocityof the moving object decreases , The direction of motion does not change .

Example : When the motorcyclist applies the brakes , the motorbike is acted upon by a force opposite to the direction of motion and decelerates or decreases velocity **.**

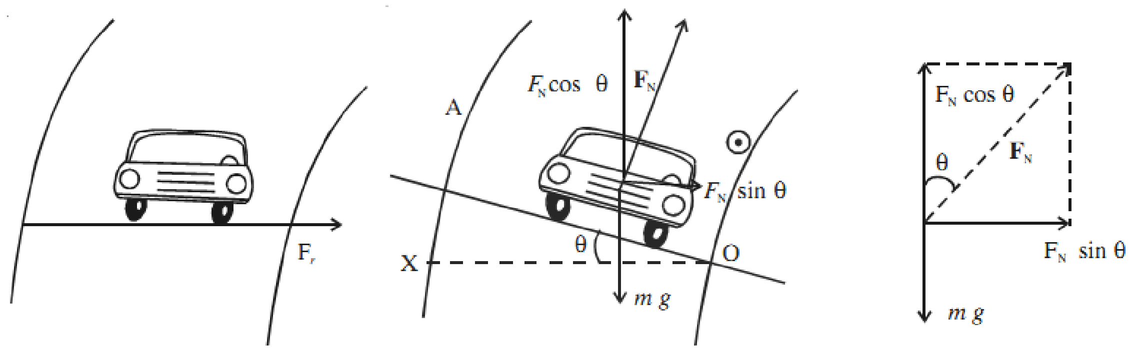
APPLICATIONS OF UNIFORM CIRCULAR MOTION
BANKING OF ROADS

While riding a bicycle and taking a sharp turn, you may have felt that something is trying to throw you away from your path.

You tend to be thrown out because enough centripetal force has not been provided to keep you in the circular path. Some force is provided by the friction between the tyres and the road, but that may not be sufficient. When you slow down, the needed centripetal force decreases and you manage to complete this turn.

Consider a car of mass m, travelling with speed v on a curved section of a highway. To keep the car moving uniformly on the circular path, a force must act on it directed towards the centre of the circle and its magnitude must be equal to mv^2/r.

If the road is levelled, the force of friction between the road and the tyres provides the necessary centripetal force to keep the car in circular path. This causes a lot of wear and tear in the tyre and may not be enough to give it a safe turn. The roads at curves are, therefore, banked, where banking means the raising of the outer edge of the road above the level of the inner edge.



ROLLER COASTERS AND CENTRIPETAL FORCE

People ride roller coasters, of course, for the thrill they experience, but that thrill has more to do with centripetal force than with speed. By the late twentieth century, roller coasters capable of speeds above 90 MPH (144 km/h) began to appear in amusement parks around America; but prior to that time, the actual speeds of a roller coaster were not particularly impressive. Seldom, if ever, did they exceed that of a car moving down the highway. On the other hand, the acceleration and centripetal force generated on a roller coaster are high, conveying a sense of weightlessness (and sometimes the opposite of weightlessness) that is memorable indeed.

Few parts of a roller coaster ride are straight and flat—usually just those segments that mark the end of one ride and the beginning of another. The rest of the track is generally composed of dips and hills, banked turns, and in some cases, clothoid loops. The latter refers to a geometric shape known as a clothoid , rather like a teardrop upside-down.

Because of its shape, the clothoid has a much smaller radius at the top than at the bottom—a key factor in the operation of the roller coaster ride through these loops. In days past, roller-coaster designers used perfectly circular loops, which allowed cars to enter them at speeds that were too high, built too much force and resulted in injuries for riders. Eventually, engineers recognized the clothoid as a means of providing a safe, fun ride.

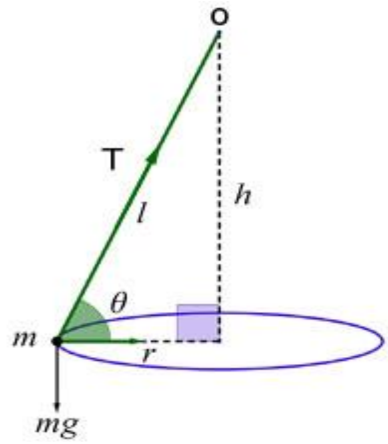
As you move into the clothoid loop, then up, then over, and down, you are constantly changing position. Speed, too, is changing. On the way up the loop, the roller coaster slows due to a decrease in kinetic energy, or the energy that an object possesses by virtue of its movement. At the top of the loop, the roller coaster has gained a great deal of potential energy, or the energy an object possesses by virtue of its position, and its kinetic energy is at zero. But once it starts going down the other side, kinetic energy—and with it speed—increases rapidly once again.

Throughout the ride, you experience two forces, gravity, or weight, and the force (due to motion) of the roller coaster itself, known as normal force. Like kinetic and potential energy—which rise and fall correspondingly with dips and hills—normal force and gravitational force are locked in a sort of "competition" throughout the roller-coaster ride. For the coaster to have its proper effect, normal force must exceed that of gravity in most places.

The increase in normal force on a roller-coaster ride can be attributed to acceleration and centripetal motion, which cause you to experience something other than gravity. Hence, at the top of a loop, you feel lighter than normal, and at the bottom, heavier. In fact, there has been no real change in your weight: it is, like the idea of "centrifugal force" discussed earlier, a matter of perception.

CONICAL PENDULUM

A conical pendulum consists of a weight (or bob) fixed on the end of a string or rod suspended from a pivot. Its construction is similar to an ordinary pendulum; however, instead of swinging back and forth, the bob of a conicalpendulum moves at a constant speed in a circle with the string (or rod) tracing out a cone.



Let OP = L Be the string, o being fixed and the particle being attached at P. Theparticle describes a circle of radius r, h being the depth of the circle below O. Let θ be the inclination of the string with the vertical ON

Then, $\sin\theta = \frac{r}{l}$

$l \sin\theta = r$

$r = l\sin\theta$

$\cos\theta = \frac{h}{l}$

$h = l\cos\theta$

$r = l\sin\theta$

$h = l\cos\theta$

If $w(=\dot{\theta})$ be the angular velocity then the central acceleration w^2r towards N Now the forces acting on P are

Its weight mg downwards,

The tension T in the direction p_0 of the string

The equations of motion for P are

$T\cos\Theta - mg = 0 \quad \to 1$

$T\cos\theta = mg$

$T = \frac{mg}{\cos\theta}$

Since there is no motion for p in the vertical direction

$mrw^2 = T\sin\theta \quad \to 2$

Eliminating T between 1 and 2

$mrw^2 = \frac{mg}{cos}\sin\theta$

$w^2 = \frac{mg}{mr}\frac{\sin\theta}{\cos\theta}$

$w^2 = \frac{g}{r}\frac{\sin\theta}{\cos\theta}$

$w^2 = \frac{g}{r}\frac{r/l}{h/l}$

$w^2 = \frac{g}{r}\frac{r}{l}\frac{l}{h} = \frac{g}{h} \to 3$

$w^2 = \frac{g}{h}$

$w = \sqrt{\frac{g}{h}}$

Thus the angular velocity varies inversely as the square root of the depth of p below O

Time of one complete revolution = $\frac{2\pi}{w}$

$= \frac{2\pi}{\sqrt{\frac{g}{h}}}$

$= 2\pi\sqrt{\frac{h}{g}}$

If the particle makes n revolution per second the angular velocity w=2πh radian/sec

From 3,

$w^2 = \frac{g}{r} = \frac{\sin\theta}{\cos\theta}$ (by 3)

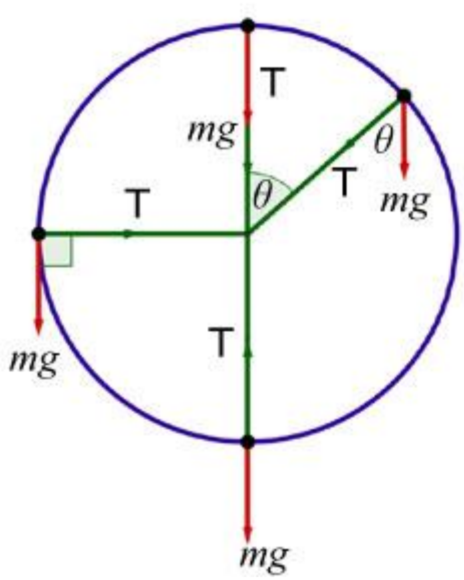
$r\,4\,\pi^2 n^2 = \frac{g}{\cos\theta}\,\sin\theta$

$\theta \neq 0,$

$\cos\theta = \frac{g\sin\theta}{l sin\theta\ 4\pi^2 n}$

cos $\theta = \frac{g}{4\pi^2 n^2 l}$
For θ to be real the conditions is ,
$4\pi^2 n^2 l > g$
Tension T=$\frac{mg}{cos\theta}$
T = $\frac{mg\,4\pi^2 n^2 l}{g}$
T= m$4\pi^2 n^2 l$
Hence Proved

MASS PERFORMING VERTICAL CIRCULAR MOTION UNDERGRAVITY
Consider a mass m performing circular motion under gravity , the circle with radius r



The centripetal force on the mass varies at different position on the circle
Top mg + T = $\frac{mv^2}{r}$
Middle T = $\frac{mv^2}{r}$
Bottom T-mg = $\frac{mv^2}{r}$
String at an angle θ to the vertical
mg cosθ+T = $\frac{mv^2}{r}$
Hence the proof

II. Problems

PROBLEM: 1
In a conical pendulum making n revolution per second, the length l of the string in diminished by a small length x show that the increase in the number of revolution per second is approximately $\frac{nx}{2l}$ the inclination of the string to the vertical and the tension remaining the same.
Solution:
We know that, the tension T In a conical pendulum making n revolution per second is given by T = $4\pi^2 n^2$lm
Assuming T, m, $4\pi^2$ as constant taking logarithms,
logT =log$4\pi^2$+2logn+logl
Taking differentials,
0= 0+2.$\frac{dn}{n}$+$\frac{dl}{l}$
$-\frac{dl}{l}$ =2.$\frac{dn}{n}$
$-\frac{dl}{2l}$ =.$\frac{dn}{n}$
dn = -n $\frac{dl}{2l}$
dl= -x

dn = $-\frac{n(-x)}{2l}$

dn = $\frac{nx}{2l}$

RESULT:

The increase in the number of revolution per second is approximately $\frac{nx}{2l}$

The inclination of the string to vertical and the tension remaining the same

PROBLEM: 2

A 20g mass move as a conical pendulum with string length 8x and speed v if the radius of the circular motion is 5x find,

i. The string tension (assuming g=10ms^{-1})

ii. V in terms of x ,g

Solution:

i. l = 8x

m = 20g= 0.02 kg

r = 5x

g = 10 ms^{-1}

cosθ= $\frac{5}{8}$

θ=51.3°

T sinθ = mg (by conical pendulum)

T = $\frac{mg}{sin\theta}$

T = $\frac{0.02}{0..7804}$

T= 0.2563

The string tension T is 0.26 N

ii. Resolution horizontally

T cosθ= $\frac{\mathrm{m}v^2}{r}$

Substituting for T, from T = $\frac{mg}{sin\theta}$ above,

mg $\frac{\cos\theta}{\sin\theta}$ =$\frac{\mathrm{m}v^2}{r}$

$\frac{gr}{tan\theta}$ $= v^2$

V=$\sqrt{\frac{gr}{tan\theta}}$

Substituting for r = 5x, θ=51.3°

V=$\sqrt{\frac{5gx}{\tan(51.3^{\circ})}}$

V=$\sqrt{\frac{5gx}{1.24820}}$

V=$\sqrt{\frac{5gx}{4.0057}}$

V=$\sqrt{4gx}$

V=2$\sqrt{gx}$

Velocity v in terms of gx is 2$\sqrt{gx}$

RESULT:

The string tension T is 0.26 N

Velocity v in terms of gx is 2$\sqrt{gx}$

PROBLEM: 3

A 50g mass suspended at the end of a light inextensible string performs vertical motions of radius 2m if the mass has a speed of 5ms^{-1} when the string makes an angle of 30° with the vertical. What is the tension?

Solution:

m= 50g = 0.05 kg

v= 5 ms^{-1}

θ= 30°

r = 2m

g = 10 ms^{-1}

The centripetal force is the sum of the tension in the string and the component of the weight along the string

$mg\cos\theta + T = \frac{mv^2}{r}$

$T = \frac{mv^2}{r} - mg\cos\theta$

$T = \frac{(0.05)(5^2)}{2}$ - (0.05) (10) $\cos 30^\circ$

$T = \frac{(0.05)(25)}{2}$ – (0.05)(10) 0.8660254038

$T = \frac{1.25}{2}$ – 0.4330127019

T= 0.625 – 0.4330127019

T= 0.192

T=0.2

RESULT:
The tension of string is 0.2 N

PROBLEM 4:
A 30g mass suspended at the end of light inextensible string performs vertical motions of radius 2m if the mass has a speed of 5ms^{-1} when the string makes an angle of 45° with the vertical what is the tension?

Solution:

m= 30g =0.03kg

v=5ms^{-1}

θ=45°

r=2m

g=10ms^{-1}

The centripetal force is the sum of the tension in the string and the component of the weight along the string.

$mg\cos\theta + T = \frac{mv^2}{r}$(0.03)($5^2$)

$T = \frac{mv^2}{r}$ - mg $\cos\theta$

$T = \frac{(0.03)(5^2)}{2}$ - (0.03)(10)cos45°

$T = \frac{(0.03)(25)}{2}$ - (0.03)(10)(0.7071067812)

$T = \frac{0.75}{2}$ - 0.2121318

T = 0.375 – 0.2121318

T=0.1628682

T=0.163N

RESULT:
The tension of string 0.163 N

APPLICATION PROBLEM
PROBLEM: 5
A Space shuttle moving at 1000 ms^{-1} in a circular orbit around a distant moon. If the radius of the circle followed by the satellite is 1000km find?

i. The acceleration of the space shuttle.
ii. The time for the space shuttle to complete one full orbit of the moon in minutes

Solution:

i. a is acceleration , v speed and r orbit radius

$a = \frac{v^2}{r}$

Substituting for v = 1000 ms^{-1}

r=1000 km (10^6)

$a = \frac{(1000)\ (1000)}{1000000}$

a=1

Acceleration of the space shuttle is 1ms^{-1}

ii. distance travelled by space shuttlei one orbit = circumference of orbit circle

$= 2\pi r = 2\pi \times 10^6$

Time for one orbit = $\frac{distance\ \ travelled\ \ in\ one\ orbit}{speed\ \times v}$

$=\frac{2\pi\times10^6}{2\times10^3} = 2\pi\times10^3$

= 6285sec

=104.76 min

Time for one orbit is104.76 min

RESULT:

Acceleration of the space shuttle is 1ms^{-1}

Time for one orbit is104.76 min

PROBLEM: 6

A 900kg truck moving at 20m/s takes a turn around a circle with a radius of 25.0m determine the acceleration and the net forces acting upon the car?

Solution:

m= 900 kg

R=25.0

V=20.00m/s

Acceleration =?

f_{net} = ?

To determine the acceleration of the truck, use the equation a = $\frac{v^2}{R}$

a = $\frac{v^2}{R}$

a = $\frac{(20.0\frac{m}{s})^2}{25.0m}$

a= $16m/s^2$

To determine the net force acting upon the truck , use the equation

I_{net} = ma

f_{net} = (900) ($16m/s^2$)

= 900 ×16

= 14400 N

f_{net} = 14400 N

Hence proved

PROBLEM: 7

A 95kg halfback makes a turn on the football field. The halfback sweeps out a path that is a portion of a circle with a radius of 12meters. The halfback makes a quarter of a turn around the circle in 2.1seconds. Determine the speed, acceleration and net force acting upon the halfback?

Solution:

m = 95.0 kg

R = 12.0m

V=?

a =?

f.net =?

Travelled $1/4^{th}$ of the circumference in 2.1s

To determine the speed of the halfback use the equation v=d/t where the d is $1/4^{th}$ of the circumference and the time is 2.1s The solution is ,

V= d/t

V = $\frac{(0.25\times2\times\pi\times R)}{t}$

V = $\frac{(0.25\times2\times\pi\times R)}{2.1s}$

V = $\frac{\mathbf{18.84}}{\mathbf{2.1}s}$

V = 8.97 m/s

To determine the acceleration of the half back , use the equation a= $\frac{v^2}{r}$

a = $\frac{(8.97\ m/s)^2}{12.0m}$

a = $\frac{8.05\ m^2/s^2}{12.0m}$

a = 6.71 m/s^2

To determine the net force acting upon the half back, use the equation f_{net} =m×a

The solution is,

f_{net} = (95.0 kg)×(6.71m/s^2)

f_{net} =637 N

RESULT:

The speed v=8.97 m/s

Acceleration = 6.71 m/s^2

Net force = 637 N

References

[1]. Gowri.P,Abhinandana.R, Madhu Gokul.B, Saranya.B, Mrithula.R" A Case Study on Path of Projectile Motion and ItsApplication", International Journal of Engineering Science Invention Research andDevelopment, vol-4, issue-4,dec-2017.

[2]. Gowri.P,Deepika. Dand Krithika.S,"A Case Study on Simple Harmonic Motionand Its Applications", International Journal ofLatest Engineering and Management Research (IJLEMR)

[3]. Gowri.P,Asst. Prof, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore, Tamil Nadu,India. Glory Bebina.E, IIIBsc Mathematics, Sri Krishna Arts and Science College, Coimbatore, Tamil Nadu, India,"A CaseStudyon Elastic Oscillationsandits Applications in Bungee Jumping by Differential Equation**"**, International Journal of Trend in Scientific Research and Development(IJTSRD).

[4]. Gowri.P[1],Priyadharshini.S[2],Maheswari.T[3], [1]Asst.Prof. Sri Krishna Arts and Science College,[2,3]Sri Krishna Arts and Science College, "A Case Study onRudgeKutta4thOrder Differential Equations and Its Application", Imperial Journal of Interdisciplinary Research(IJIR).

[5]. JohnVaughan, "Static andDynamic balancing using portable measuring equipment

[6]. S.Priyadharshini, ShajeenaM Basheer, KaushalyaM and SnehaM,"A Study on Motion of a Free Falling Body In Kinematic Equation", International JournalforResearchinAppliedScience&EngineeringTechnology,Vol.6, Issue 1, 2018. ISSN 2321-9653

[7]. http://what-when-how.com/automobile/wheel-balancing-automobile/

[8]. https://www.google.com/searchq=WHEEL+BALANCER&ie=utf-8&oe=utf-8&client=firefox-b

[9]. http://fmpmotorfactors.com/news/importance-of-wheel-alignment-and-tyre-balancing/

Gowri. P. " A Study on Circular Motion and Its Applications." IOSR Journal of Mathematics (IOSR-JM) 14.6 (2018): PP-18-25.