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Three-Step Method For Finding Root of Non-Linear Equations

Apichat Neamvonk[1]

[1](*Department of Mathematics, Faculty of Science, Burapha University, Thailand)*
Corresponding Author: Apichat Neamvonk

Abstract : *In this paper, we proposed a three step method for approximating roots of non-linear equations. This method has three evaluations of function and first derivative which is modified from McDougall and Wotherspoon [1]. Numerical examples are tested to compare the method with other methods and demonstrate the efficiency of the proposed method.*

Keywords: *Non-linear equation, Iterative method, Newton's method, Multiple roots, Order of convergence.*

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I. Introduction

Many mathematical models arise in the problems of physics, engineering and science, are applied with non-linear equations and system of nonlinear equations. Moreover, the efficient methods for finding the solutions or roots of non-linear equations has been developed in recent year [2-7]. Consider a function having at least a root of non-linear equation in the form of

$$f(x)=0, \tag{1}$$

where $f: I \subseteq R \to R$ is a scalar function for an open interval I and continuously differentiable function.

Newton's method is one of the most famous in producing a sequence $\{x_n\}$ which is approximation root of equation (1) with initial point x_0. The Newton's method has second order of convergence and is given as

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}, \quad n=0,1,2,... \tag{2}$$

Many modifications of Newton's method have been developed and analyzed by various techniques [1-9], which show better performance than the Newton's method.

II. Iterative Method

In 1864, Traub [8] developed Newton's method with predictor-corrector method was given as

$$\begin{aligned} x_{n+1}&=y_n-\frac{f(y_n)}{f'(x_n)},\\ y_n&=x_n-\frac{f(x_n)}{f'(x_n)}, \quad f'(x_n)\neq 0. \end{aligned} \tag{3}$$

McDougall and Wotherspoon [1] presented a simple modification of Newton's method which converged faster than the Newton's method with a convergence order $1+\sqrt{2}$ and obtain the following scheme:

Set the initial starting point x_0 and $n=0$

$$x_0^*=x_0,$$

$$x_1=x_0-\frac{f(x_0)}{f'\left(\frac{1}{2}(x_0+x_0^*)\right)}=x_0-\frac{f(x_0)}{f'(x_0)}$$

for $n\geq 1$

$$\begin{aligned} x_n^*&=x_n-\frac{f(x_n)}{f'\left(\frac{1}{2}(x_{n-1}+x_{n-1}^*)\right)},\\ x_{n+1}&=x_n-\frac{f(x_n)}{f'\left(\frac{1}{2}(x_n+x_n^*)\right)}, \quad n\geq 1. \end{aligned} \tag{4}$$

Kang et al. [9] modified equation (4) with using (3) given as:

$$x_0^* = x_0,$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

for $n \geq 1$

$$x_n^* = x_{n-1}^* - \frac{f(x_{n-1}^*)}{f'(x_{n-1}^*)},$$

$$x_{n+1} = x_n^* - \frac{f(x_n^*)}{f'(x_n)}, \quad n \geq 1. \tag{5}$$

Here, we use Secant method in equation (4) and obtain the following modified scheme:

Set the initial starting point x_0 and $n = 0$

$$x_0^* = x_0,$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

for $n \geq 1$

$$x_n^* = x_{n-1}^* - \frac{f(x_{n-1}^*)}{f'(x_{n-1}^*)},$$

$$x_{n+1} = x_n^* - \frac{f\left(\frac{1}{2}(x_n + x_n^*)\right)}{f'(\frac{1}{2}(x_n + x_n^*))}, \quad n \geq 1. \tag{6}$$

III. Algorithm For The Propose Method

STEP 1 : Given $x_0 \in R$ and stopping tolerance ε , for $n = 0$ set $x_0^* = x_0$

STEP 2 : Compute $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

STEP 3 : For $n \geq 1$ compute $x_n^* = x_{n-1}^* - \dfrac{f(x_{n-1}^*)}{f'(x_{n-1}^*)}$ and $x_{n+1} = x_n^* - \dfrac{f\left(\frac{1}{2}(x_n + x_n^*)\right)}{f'(\frac{1}{2}(x_n + x_n^*))}$

STEP 4 : Check the stopping condition, if $\left|\dfrac{x_{n+1} - x_n}{x_n}\right| \leq \varepsilon$ then STOP.

STEP 5 : Set $n = n+1$ and goto STEP 3

IV. Numerical Results

Numerical examples of the methods are performed using MATLAB R2017a. We compare our developed method (DM) with Newton's method (NM), McDougall and Wotherspoon method (MW) and Kang et al. (KM) using 8 test functions. The stopping condition used for these comparisons are $\left|\frac{x_{n+1} - x_n}{x_n}\right| \leq \varepsilon$ where $\varepsilon = 1 \times 10^{-8}$.

Table 1: Example and comparison of various iterative schemes

Function	x_0	Approximate roots	Number of iteration			
			DM	NW	MW	KM
$f_1(x) = xe^x - 1$	3	0.567143290410	5	9	6	5
$f_2(x) = x^2 - (1-x)^5$	5	0.345954815848	6	12	9	6
$f_3(x) = \cos(x) - x^3$	8	0.865474033102	6	10	7	5
$f_4(x) = x^3 - e^{-x}$	6	0.772882959149	6	10	7	5
$f_5(x) = (e^x + x - 20)^3$	4	2.842438975957	10	44	33	14
$f_6(x) = x^{30} - 1$	1.2	1.000000000000	7	10	8	5
$f_7(x) = -20x^5 - \frac{x}{2} + \frac{1}{2}$	1.5	0.427677296931	6	10	8	5
$f_8(x) = x - 3\ln(x)$	0.5	1.857183860208	5	7	5	4

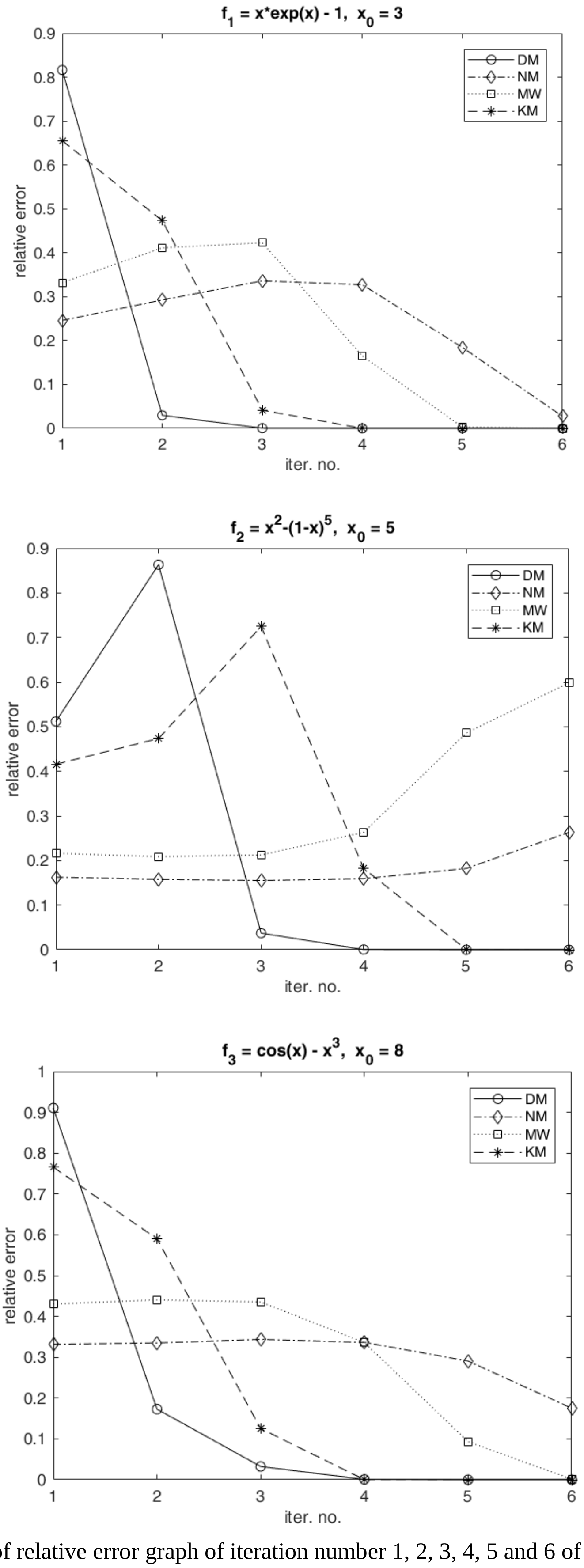

Figure 1: reduction of relative error graph of iteration number 1, 2, 3, 4, 5 and 6 of function f_1, f_2 and f_3

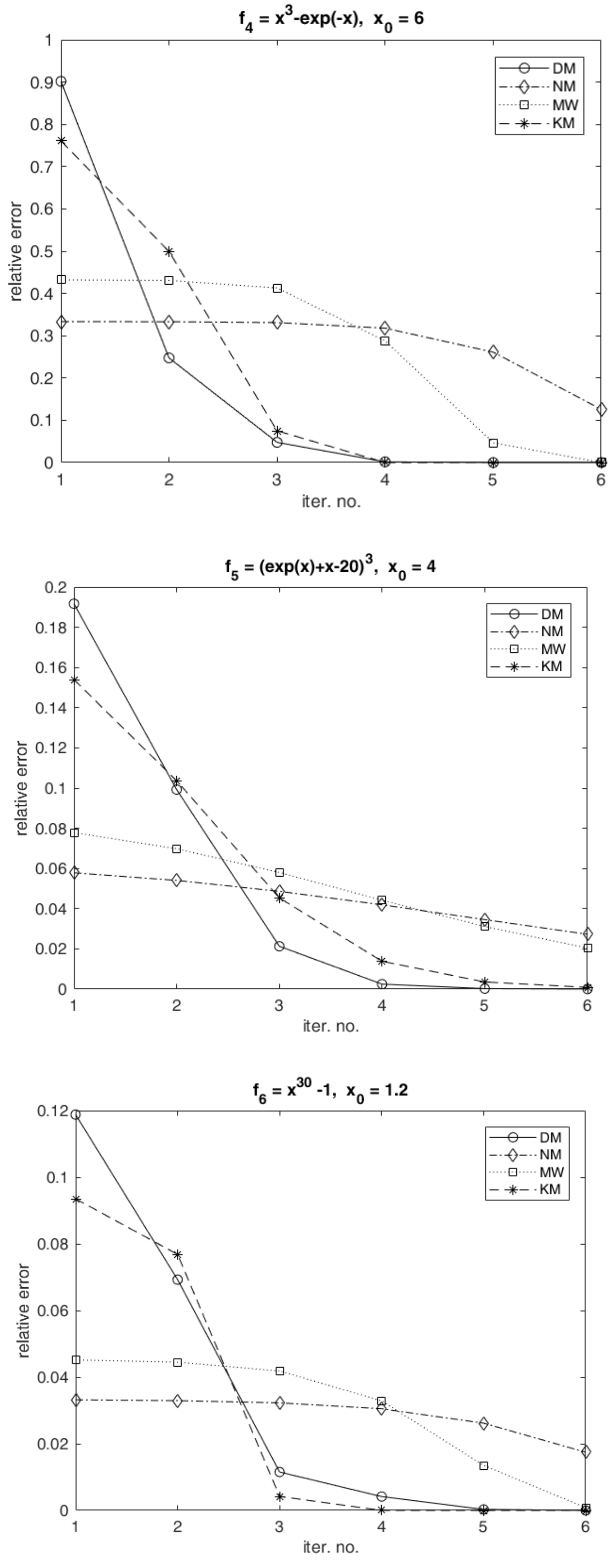

Figure 2: reduction of relative error graph of iteration number 1, 2, 3, 4, 5 and 6 of function f_4, f_5 and f_6

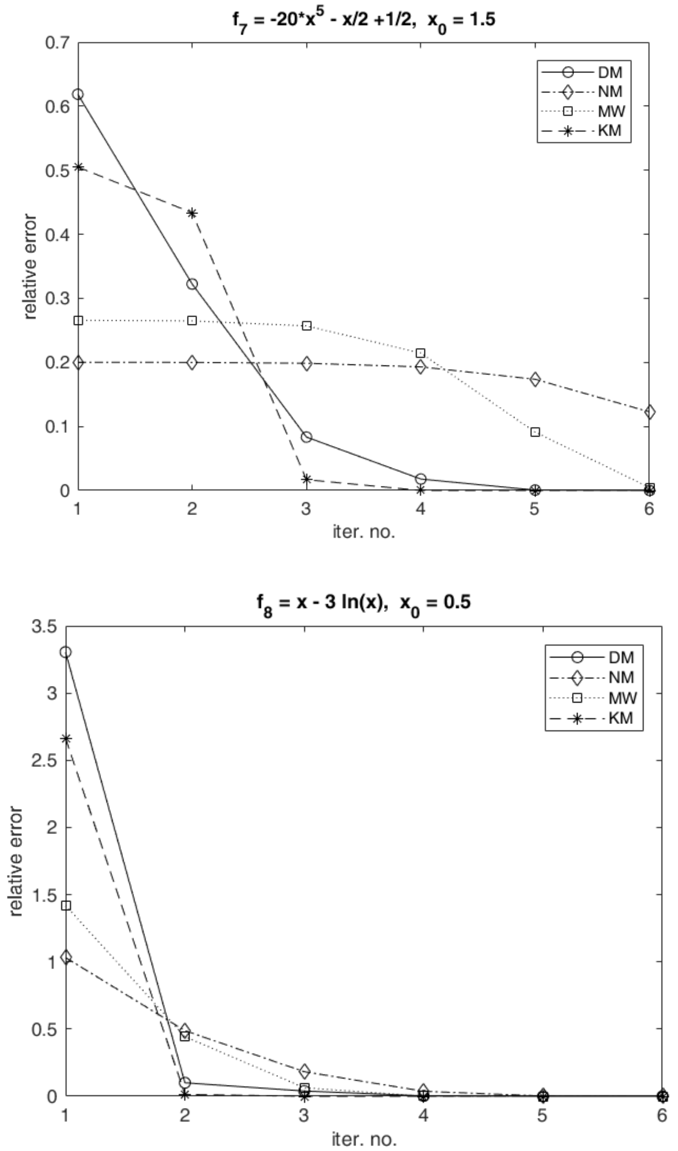

Figure 3: reduction of relative error graph of iteration number 1, 2, 3, 4, 5 and 6 of function f_7 and f_8

V. Conclusion

In this paper, we proposed three step predictor-corrector iterative method for finding approximation solutions of nonlinear equations $f_1, f_2, ..., f_7$ and f_8. The new method has the lowest number of iteration shown in Table 1. Also relative error reduces fastest among other methods as shown in Figure 1, 2 and 3. These can be explained that the new method has a good rate of convergence for approximating solution of the functions. The approximate solution of the nonlinear equations which have multiple solutions can be considered as a new topic for the future works.

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