IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 15, Issue 2 Ser. I (Mar – Apr 2019), PP 33-41
www.iosrjournals.org

Controllability Result for Nonlinear Integrodifferential Equation

Eli Innocent Cleopas[1] and Godspower C. Abanum
Department of Mathematics/ Statistics, Federal University Otuoke, Nigeria
Department of Mathematics, University of Port Harcourt, Nigeria
Corresponding Author: Eli Innocent Cleopas

Abstract: *In this paper, the controllability of a class of nonlinear integro-differential equation with implicit derivatives is investigated. We employ the Darbo fixed point theorem to investigate our result. Also , we give example to illustrate to obtained result.*

Keyword: *Nonlinear integrodifferential equation, Darbo fixed point theorem, controllability*

Date of Submission: 11-03-2019 Date of acceptance: 27-03-2019

I. Introduction

Controllability is one of the essential concepts in mathematical control theory.Controllability is a strong characteristic of dynamical control systems and it is of great importance in control theory. Klamka (2000). Dacka (1980) observed that many authors effectively applied Schauder's fixed-point theorem in solving the local and global controllability of nonlinear systems since the paper by Davison and Kunze (1980). It was also observed that there is a generalization of Schauder's theorem based on the notion of measure of noncompactness of a set. The author then introduced a new method of analyzing for the controllability of nonlinear dynamical systems which consists of using the measure of noncompactness of a set and the Darbo fixed-point theorem. This method was used in obtaining sufficient conditions for the global and local controllability of certain types of nonlinear time-varying systems with implicit derivative. See Aghajani*et al*.(2013) for some generalization of the Darbo fixed point theorem and its applications.

Dynamical systems theory deal with long-term qualitative behavior of dynamical systems. The focus is not finding precise solutions to the equations defining the dynamical system (which is often hopeless) but rather to some problem like "will the system settle down to a steady state in a long term? And if so, what ate the possible steady states or does the long term behavior of the system defend on its initial condition. The study of dynamical system is the focus of dynamical system theory, which has applications to wide variety of fields such as mathematics, physics, chemistry, biology, economics and medicine. Dynamical systems are a fundamental part chaos theory, logistic map dynamics, bifurcation theory, the self-assembly process, and the edge of chaos concept.

Real life example of dynamical systems that are found mostly in system surrounding us are **in Mechatronic System, Temperature System, Biological Science, In Business System.** dynamical system as a system that evolves in time through the iterated application of an underlying dynamical rule. It is a mathematical model that one usually construct in order to investigate some physical phenomenon that evolves in time. This model usually involves mainly are ordinary differential equation, partial differential equations or functional differential equations which describe the evolution of the process under study in mathematical terms. Patrice (2016). Davies and Jackreece (2005) examined the controllability and null controllability of linear systems. The authors integrated the concept of null controllability into a generalized system with delay in the state and control. Sufficient conditions was obtained for the assumption of relative controllability for the null controllability with constraint. The result showed that if the uncontrolled system is asymptotically stable and the controlled system is relatively controllable then the system is null controllable with constraints. For a survey on controllability of dynamical systems, see Klamka (2008, 2013).

Controllability of dynamical systems has been applied in various areas such as spacecraft, (Liu and Wilms, 1996), mechanical systems (Klamka, 2005), 2D linear systems (Klamka, 1999; Kaczorek, 2000), chemical reactors in electric systems containing long lines and in the case of heat exchangers and acoustic systems (Campbell, 1962; Bienkowska-Lipinska, 1974; Luyben, 1990). Park *et al.* (2009) studied the controllability of impulse neutral integrodifferential systems with infinite delay in Banach spaces. The authors obtained sufficient conditions for the controllability of the system using Schauder's fixed point theorem. See Klamka and Niezabitowski (2014) and Klemka*et al.*(2017) for a survey on the controllability of switched infinite-dimensional linear dynamical systems and some results in the exact controllability of second order infinite dimensional semilinear deterministic systems.

Sikora (2003) studied the constrained controllability of dynamical systems with multiple delays in the state. In the paper, relative and approximate controllability properties with constrained controls were also examined and it was shown that approximate relative controllability is a weaker notion than relative controllability although it appears sufficient for many controllability tasks. In this study, we consider existence and the uniqueness of the controllability of dynamical system. The system under consideration is nonlinear integro-differential equation defined as

$$\dot{x} = A(t)x(t) + \int_{to}^{t} K(t,s)x(s)ds + B(t)u(t) + f\left(u(t), x(t-h(x(t)t), x^1(t), t\right)) \qquad 1$$

Where the state x(t) is an n-vector and the control u(t) is an m-vector.

$A: J \to R^{n2} B: J \to R^{nm}, K: D \to R^{n2}$, f: $JXR^{2n+m} \to R^n$,

Where A is a matrix, B is a matrix as well, K is the kernel of a matrix, and F is continuous vector function and using measure of noncompactness of set to formulate conditions for Darbo's fixed point theorem which is used to established existence and uniqueness of a solution.

1. Preliminaries

In this section, we present some basic definition which are useful for our discussion.

Definition 2.1: Condensing map: Let *X*be a subset of a Banach space. An operator $T: X \to X$ is called condensing if for any bounded subset E in X $\varsigma(E) \neq O$, we have $\varsigma(T(E)) \angle \varsigma(E)$ where $\varsigma(E)$ denotes the measure of non-compactness of the set E.

Definition 2.2: Lipschitz condition: suppose f is defined in a domain D of the (t,x)plane. If there exists a constant K>O such that for every $(t, x_1)$and(t, x_2) in D.

$$\left| f(t,x_1) - f(t,x_2) \right| \leq K \left| X_1 - X_2 \right|$$

Then f is said to satisfy a lipschitz condition (with respect to x). (Earl and Norman 1955).

Definition 2.3. Open set: Let (x, p) be a metric space and Let E be an arbitrary subset of X, then the set E is said to be an open set in X if for each given point, xϵEthere exists a positive real number r(i.e, r>o) such that $B_r(x) \subseteq E$.

Definition 2.4. Matix: Matix can be define as an array of numbers in rows and columns.

Definition 2.5. Transpose of a Matrix: This is the process whereby the elements in the rows and columns inter-change.

Definition 2.6. Nonsingular Matrix:This is a matrix in which its determinant is greater than zero.

Definition 2.7. Closure of set A: The closure $\overline{A}$ is the smallest closed set containing A.

Definition 2.8. Mathematically, let $G(t_{o,} t) = \int_{to}^{t} \varphi(t,s)B(s)B^1(s)\varphi^1(t,s)ds$ be defined as the controllability Matrix.

Where

The kernel φ(t,s) is a matrix, B(s) is a square matrix, $B^1(s)$ is a matrix transpose.

Definition 2.9. Boundedness: A linear functional on a normed linear space N is said to be bounded if there exist a constant M ≥0 such that |f(x)| ≤ M||x|| for all x ϵ N.

THEOREM 2.1 : DARBO'S FIXED POINT THEOREM

Let C be a nonempty, bounded, closed and convex subset of a Banach space E and T: C→C be a continuous operator such that

$$O(F: \varsigma(T(X))) + \varphi((TX)) \leq [O(F, \varsigma(X)) + \varphi((X))], \qquad 2$$

For any subset X of C, $O(\bullet,\cdot)\varepsilon\theta$ and $\varphi: R_+ \to R_+$ is a continuous function, where ζ is an arbitrary measure of noncompactness, $\phi: [O,\infty] \to [O,\infty]$is a non-decreasing functions such that $\lim_{n\to\infty} \phi^n(t) = o$ for each $t \geq o.$Then T has at least one fixed point in C.

Proof

Let $C_o = C,$ we construct a sequence $\{C_n\}$such that $C_{n+1} = Conv(TC_n)$

For $n \geq O.Tc_o = Tc \subseteq C = C_o,$

$C_1 = conVC(TC_o) \subseteq C = C_o$, Therefore by continuing this manner we get

$C_o \supseteq C_1 \supseteq \ldots \supseteq C_n \supseteq C_{n+1} \supseteq \ldots.$

If there exists a natural number N such that $\varsigma(C_N) = O$

Then C_N is compact.

In this case, if C be a closed, convex subset of a Banach space E then every compact continuous map T: C→C has at least one fixed point, then T has a fixed point, so we assume that

$\varsigma(C_n) \neq O$ for $n = 0,1,2\ldots$

Also by eqn (2) we have

$$\begin{aligned} O(F;\varsigma(C_{n+1})) + \varphi(\varsigma(C_{n+1})) &= O(F;\varsigma(ConV(TC_n))) + \varphi(\varsigma(ConV(TC_n))) \\ &= O(F,\varsigma(TC_n)) + \varphi(\varsigma(TC_n)) \\ &\leq \phi[O(F;\varsigma(C_n)) + \varphi(\varsigma(C_n))] \\ &\leq \phi^2[O(F;\varsigma(C_{n-1})) + \varphi(\varsigma(C_{n-1}))] \\ &\vdots \\ &\leq \phi^n[O(F;\varsigma(C_o)) + \varphi(\varsigma(C_o))] \\ &= \phi^n[O(F;\varsigma(C)) + \varphi(\varsigma(C))]\ldots. \end{aligned} \quad 3$$

Taking the limit of eqn (2), as $n \to \infty$, we get

$\lim_{n\to\infty}[O(f;\varsigma(C_{n+1})) + \varphi(\varsigma(C_{n+1}))] = O$

Hence

$\lim_{n\to\infty} O(f;\varsigma(C_{n+1})) = O\left(F;\lim_{n\to\infty}\varsigma(C_{n+1})\right) = O,$

And if $F([O,\infty)$ be a class of all function $F:[O,a) \to [O,\infty]$ and if θ be class of all operators $O(\circ;.) : F([O,\infty]) \to F([O,\infty]), F \to O(F;.)$

Satisfying

$O(F;t) > O$ For $t > O$ and $O(f;O) = O$

Then, it implies that

$\lim_{n\to\infty} \varsigma(C_{n+1})) = O$

Since $C_n \supseteq C_{n+1}$ and $TC_n \subseteq C_n$ for all n=1, 2, ----

Then from f of definition of noncompactness of set we have

$X_\infty = n_{n=1}^{\infty} X_n$ Is a nonempty convex closed set.

Therefore, if C be closed, convex subset of a Banach space E. Then every compact, continuous map T: C→C has at least one fixed point (Reza Arab, 2015).

Lemma 3.1 Let X be a uniform space. A regular measure of noncompactness of X is an arbitrary function ϕ: $P(X) \to [o,\infty]$ which satisfies the following conditions.

1. $\phi(A) = \infty$ if an only if the set A is unbounded
2. $\phi(A) = \phi(\overline{A})$

Where – is the closure of A

3. A $\phi(A) = O$ it follows that A is a totally bounded set

4. If X is a complete space, and if $\{B_n\}_{n\epsilon N}$is a sequence of closed subsets of X such that $B_{n+1} \subseteq B_n$ for each $_{n\epsilon N}$and $\lim_{n\to\infty} \phi(B_n) = 0$, then $k = n_{n\epsilon N} B_n$ is a nonempty compact set.

5. From $A \subseteq B$ it follows that $\phi(A) \le \phi(B)$

Theorem 3.4: If f: K→R is continuous and K<R is compact, then F (K) is compact

Proof

We show that f (k) is sequentially compact.

Let (y_n) be a sequence in f (k). Then. y_n= f (X_n) for some $X_n \epsilon K$. Since k is compact, the sequence (X_n) has a convergent subsequence (X_{ni}) such that $\lim_{i\to\infty} X_{ni=X}$

Where x ϵK-since f is continuous on k.

$\lim_{i\to\infty} f(X_{ni}) = f(x)$.

Putting y = f (x), we have y ϵf (K) and $\lim_{i\to\infty} y_{ni} = y$

Therefore every sequence (y_n) in f (K) has a convergent subsequence whose limit belongs to f (k), so f (k) is compact. (Reza 2015).

4. Controllability Result

For we to prove our controllability result, we make the following assumptions

i. $\lim_{|x|\to\infty} sup \frac{|f(t,x,y,u)|}{|x|} = 0$

ii. There exist a continuous non decreasing function

w: $R^+ \to R^+$, with $w(r) < r \quad such\ that$

$|f(t,x,y,u) - f(t,x,z,u)| < w(|y - z|)$

$for\ all\ (t,x,y,u) \epsilon J \times R^{2n} \times R^m$

iii. The controllability matrix

$G(t_{o,} t_1)$ is nonsingular for some t > t_o

Where

$$G(t_{o,} t) = \int_{to}^{t} \varphi(t,s) B(s) B^1(s) \varphi^1(t,s) ds$$

.

Where is B(s) is a matrix and $B^1(s)$ is the transpose of the matrix B(s).If equation (4.1) satisfies conditions i to iii, then equation (4.1) is completely controllable

Proof

Let us define the nonlinear transformation

$$Q: c_m(J) X c_n^1(J) \to c_m(J) X c_n^1(J) \ \ as$$

$$Q(u,x)(t) = (Q_1(u,x)(t),\ Q_2(u,x(t))$$

Where the pair of operators Q_1 and Q_2 are define by

$$Q_1(u,x)(t) = -B^1(t)\varphi^1(t_1,t) G^{-1}(t_0,t_1) \times [\int_{t_o}^{t} \varphi(t_1,s) f(u(s), x(s-h(x(s),s)), x^1(s),s)) ds - x_1 + \varphi(t_1,t_o) x_0]$$

$$Q_2(u,x)(t) = \varphi(t,t_0)x_0 + \int_{t_o}^{t} \varphi(t,s) B(s) Q_1(u,x)(s) ds + \int_{t_0}^{t} \varphi(t,s) f(u(s), x(s),s)), x^1(s),s), Q_1(u,x)(s)) ds$$

since all the functions involved in the definition of the operator Q are continuous, Q is continuous. Reason because all continuously differentiable function are continuous. Besides by direct differentiation with respect to t, a fixed point for the operator Q leads to a control u and a corresponding function $\boldsymbol{x}$=$\boldsymbol{x}$(u), solution of the equation(4.1) satisfying that $\boldsymbol{x}(t_o) = \boldsymbol{x}_o$, $\boldsymbol{x}(t_1) = \boldsymbol{x}_1$.can be seen in the last part of this **proof.**

$$Let\,\eta^0 = (u^0, x^0) \in C_m(J) \times C_n^1(J)$$

$$\eta = (u,x) \ne 0 \in C_m(J) \times C_n^1(J)$$

And consider the system

$$\eta^0 = (\eta - \propto Q(\eta) \qquad 4$$

Where $\propto \in [0, 1]$. This equation(4.3) can be equally written as

$$u = u^0 + \propto Q_1(u, \boldsymbol{x}) \qquad 5$$

$$\boldsymbol{x} = \boldsymbol{x}^0 + \propto Q_2(u, \boldsymbol{x}) \qquad 6$$

From condition (ii) for any $\mathcal{E}>0$ there exists R>0 such that if $|\boldsymbol{x}|>R$ then $|f(t, \boldsymbol{x}, y, u)| < \mathcal{E}|\boldsymbol{x}|$.

Then from (5) we get

$$|u| \leq |u^0| + |\propto||B||\varphi||G^{-1}|\{|\varphi|\varepsilon|x|1\delta + |x_1| + |\varphi||x_0|\}$$

$$\leq |u^0| + K_1 + |B||\varphi|^2|G^{-1}|\varepsilon\delta|x| \qquad 7$$

Where $\delta = t_1 - t_0$ and

$$K_1 = |B||\varphi||G^{-1}|(|x_1| + |\varphi||x_0|)$$

From this inequality and from (6), and by applying the gronwall lemma, we obtain

$$|x| \leq [|x^0| + |\varphi||x_0||Q_1(u,x)||\varphi||B|\delta]\exp(|\varphi|\epsilon\delta)$$

$$\leq [|x^0| + |\varphi||x_0| + (K_1 + |B||\varphi|^2|G^{-1}|\epsilon\delta|x|)]\exp(|\varphi|\epsilon\delta) \qquad 8$$

It should be noted that

$$\frac{d}{dt} Q_2(u,x)(t) = A(t)\,Q_2(u,x)(t) + \int_{t_o}^{t} k(t,s)\,Q_2(u,x)(s)ds +$$

$$B(t)Q_1(u,x)(t) + f(T_1(u,x)(t), x(t-h)(x(t),t), x^1(t), t)$$

By application of the Gronwall lemma and by using change of order of integration we have

$$Q_2(u,x) \leq [|B||Q_1(u,x)|\delta + \varepsilon\delta|x|]\exp(A_0) \qquad 9$$

Where

$$A_0 = \int_{t_o}^{t} \left| A(s) + \int_{s}^{t_1} k(\eta, s)d\eta \right| ds.$$

Differentiating with respect to t, we obtain from (6)

$$\chi^1 = \frac{dx^0}{dt} + \propto \frac{d}{dt}(Q_2(u,x)(t))$$

And that yields

$$|\dot{x}| \leq [|x^0| + |A||Q_2(u,x)| + |K||Q_2(u,x)|\delta + |B||Q_1(u,x)|\delta] + \varepsilon|x|$$

$$\leq |(\dot{x})^0| + |Q_1(u,x)||(A + |K|\delta)||B|\delta\exp(A_0 + |B|$$

$$+|x|[|A| + |(A + |K|)\delta|\delta\varepsilon\exp(A_0 + \varepsilon)]$$

$$= |(\dot{x})^0| + K_2 + |x|[|B|^2|\varphi|^2G^{-1}]\delta\epsilon\{(|A| + |K|\delta)\exp(A_0) + \varepsilon\}$$

$$+(|A| + |K|\delta)\exp(A_0) + \varepsilon \qquad 10$$

where

$$K_2 = K_1[|B|(|B| + |K|\delta)\delta\exp(A_0 + 0)]$$

from equation (6), *we have*

$$|u| - |B||\varphi^2|G^{-1}\epsilon\delta|x| \leq |u^0| + K_1$$

and from (6), (7) *and* (8). *we have*

$$|x|[exp(-|\varphi|\epsilon\delta - |B|^2|\varphi|^3|G^{-1}|)\delta\epsilon\delta] \leq K_3 + |x^0|$$

where

$$K_3 = |\varphi||x_0| + K_1|B||\varphi|\delta$$

And

$|\dot{x}| - |x|[|B|^2|\varphi|^2|G^{-1}|\epsilon\delta\{(|A| + |K|\delta)\delta\exp(A_0) + 1\}]$

$$= (|A| + |H|\delta)\epsilon\delta\exp(A_0) + \varepsilon) \le K_2 + |(\dot{x})^0|$$

Adding all the above quantities, we have

$$|u| - |x|\{|B||\varphi|^2|G^{-1}|\epsilon\delta - \exp(-|\varphi|\epsilon\delta) + |B|^2|\varphi|^3|G^{-1}|\delta\epsilon\delta + (|B|)^2(\varphi)^2|G^{-1}|\epsilon\delta\}[(|A| + |K|\delta)\delta\exp(A_0) + 1]$$
$$+(|A| + |K|\delta)\epsilon\delta\exp(A_0) + \varepsilon)$$

$= |u| - \lambda|x| + |x^1| \leqq |u^0| + K_1 + K_3 + |x^0| + K_2 + |(x^1)^0|$

Where

$\lambda = |B||\Psi|^2|G^{-1}|\mathcal{E}\delta\{1 + |B||\Psi|\delta + |B|[(|A| = |K|\delta)\delta\exp(A_0) + |]\}$

$+ \mathcal{E} + (|A||K|\delta)\mathcal{E}\delta\exp(A_0) - \exp(-|\Psi|\mathcal{E}\delta)$

Then, for convenient nonnegative constants a, b, v, we can write

$|u| - |\mathcal{E}a - \exp(-\mathcal{E}b)]\,|x| + |x^1| \leqq |u^0| + |x^0| + |(x^{1})^0| + V.$

So if we divide by $|u|+|x|+|(x^1)^0|$ and, from the arbitrariness of $\mathcal{E}$, we obtain the existence of a ball S in $C_m(J) \times C^1_n(J)$ sufficiently big such that

$|\eta - \alpha Q(\eta)| > 0$ *for* $\eta = (u, x) \in \delta s.$

Next we want to show that Q is a condensing map. To this purpose, we note that $Q_1: C_m(J) \longrightarrow C_m(J)$ is a compact operator and then, if E is bounded set, $\zeta(Q_1E) = 0$. Then it will be sufficient to show that Q_2 is a condensing operator. For that, let us consider the modulus of continuity of $DQ_2(u,x)(\bullet)$. Now, for t, s $\in$J, we have.

$|DQ_2(u,x)(t) - DQ_2(u,x)(s)| \leqq A(t)Q_2(u,x)(t) - A(s)Q_2(u,x)(s)|$

$$+\left|\int_{t_o}^{t} K(t,\eta)Q_2(u,x)(\eta)d\eta - \int_{t_o}^{t} K(s,\eta)Q_2(u,x)(\eta)d\eta\right| +$$

$|B(t)Q_1(u,x)(t) - B(s)Q_1(u,x)(s)| + |f(Q_1(u,x)(t), x(t-h(x(t),t)), x^1(t),t)| - |fQ_1(u,x)(s), x(s),s)), x^1(s),s)|$

Considering the first three terms of the right hand side of the inequality we may give the upper estimate as $\boldsymbol{\beta}_0(|t-s|)$ with $\lim_{n\to o}\boldsymbol{\beta}_0(h) = 0$ and it may be chosen independent of the choice of (u,x). For the fourth term we can give the following estimate.

$\omega(|x^1(t) - |x^1(s)|$ whereas for second term we may find an estimate

$\boldsymbol{\beta}_1(|s-t|)$ with

$$\lim_{h\to o} \beta_1(h) = 0$$

hence

$\theta(DQ_2(u,x), h) \leqq ((DE,h)) + \beta(h)$

But $\beta_0 + B_1 = \beta$. Therefore, by lemma 2, we get

$\theta_0(DQ_2(E) < \theta_0(DE)$

Hence from

$2\zeta_1(Q_2(E)) = 2\zeta(DQ_2(E)) = \theta_0(DQ_2(E)) < \theta_0(DE) = 2\zeta(DE) = 2\zeta_1(E)$

It follows that$\zeta(Q_2(E)) < \zeta(E))$. Then there exist the existence of a fixed point of the operator Q following from lemma 2, that is, there exist.

Functions $u^* \in C_m(J)$ and $x^* \in C_n^1(J)$ such that $Q(u^*, x^*) = (u^* x^*)$.

That is,

$U^*(t) = Q_1(u^*, x^*)(t),\ x^*(t) = Q_2(u^*, x^*)$

These functions are the required solutions. Similarly from equation (1) $(t_o) = x_0$ and$x(t_1) = x_1$. hence the equation (1) is globally controllable .

<u>**Note**</u>: To show that $(xt_1) = x_1$.

If

$u_1(t) = Q_1([u,])(t)$ 11

$u_2(t) = Q_2([u,x])(t)$ 12

Extending the function (12) by the $R(t,t_o)\, x_0\, t_o$ a $\leqq$ t $\leqq t_o$ we get

$$u_2(t) = -B^1(t)G^{-1}(t_0,t_1)[\varphi(t,t_0)x_0 + \int_{t_o}^{t_1} f(u(s), x(\bullet), x^1(s), s)ds] \quad 13$$

And

$$x(t) = \varphi(t,t_o)x_0 + \int_{t_o}^{t} \varphi(t,s)B(s)u(s)ds + \int_{t_o}^{t} \varphi(t,s) f(u(s), x(s-h(x(s),s)), \dot{x}(s), s) \qquad 14$$

$t \in (t_0, t_1)$

$$x(t) = \varphi(t,t_0)x_0$$

From equation (13)

$$x(t) = \frac{u_2(t)}{-B^1(t)G^{-1}(t_0,t_1)} = \varphi(t,t_0)x_0 + \int_{t_o}^{t} f(u(s), x(\bullet); x^1(s), s)ds + \int_{t_o}^{t} \varphi(t,s) f(u(s), x(s-h(x(s),s)), \dot{x}(s), s) \quad 15$$

Taking arbitrary value of u(t) and u(s) ie u(t) = u(s) = 0. Then (15) reduces to
x(t) = 0 = ⪧x(t_o)=$\boldsymbol{x}_0$, $\boldsymbol{x}(t_1) = \boldsymbol{x}_1$. Hence equation (1) is controllable.

EXAMPLE

The modified system is

$$\boldsymbol{x}\ (t) = \exp(-3(t[1]-t)-3). \int_{t[0]}^{t} (\exp(-5(t-s)).\, x(s)ds + \exp(-2t).u(t)$$

$$+\frac{\log(t)}{sqrt(1+u^2)} \qquad 16$$

Let J[$t_{0,}t_1$] such that $t_1 > t_0$.
If (.) = exp (.) and u (.) = sin (.).
Then for every u>o, we have

$$R := e^{-6+3t}e^{t} - \frac{1}{2}e^{-5t} + \frac{1}{2}e^{t} + e^{-2t}\sin(t) + \frac{1}{5}In(t)\sqrt{5}.$$

Also,

$$\frac{\partial}{\partial t}R + R.\exp(-3)(t[1]-3) + \int_{t[0]}^{t} R.\exp(-5(t-s))ds, \qquad 17$$

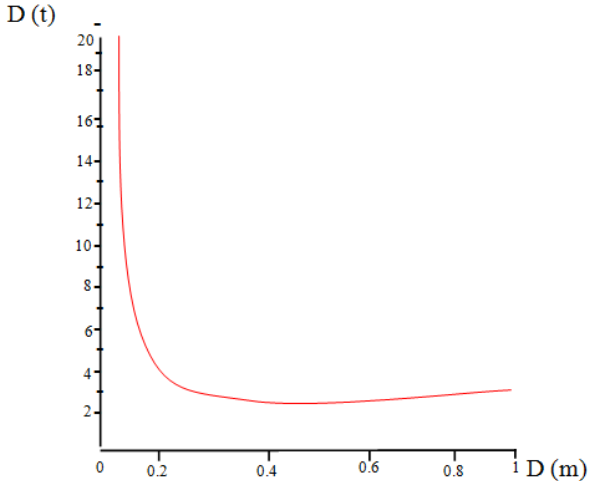
Which gives

$$4e^{-6+3t}e^{t} + \frac{5}{2}e^{-5t} + \frac{1}{2}e^{t} - 2e^{-2t}\sin(t) + e^{-2t}\cos(t) + \frac{1}{5}\frac{\sqrt{5}}{t} + \left(e^{-6+3t}e^{t} - \frac{1}{2}e^{-5t} + \frac{1}{2}e^{-2t}\sin(t) + \frac{1}{5}In(t)\sqrt{5}\right)$$

$$e^{-6+3t} + \frac{1}{50}10e^{-6+14t} + 2e^{10t}In(t)\sqrt{5} + 5e^{11t} - 10e^{-6+9t} + 10e^{8t}\sin(t) - 2In(t)\sqrt{5\ e^{5t}} - 5e^{6t} - 5e^{5}$$

$$-10\sin(t)e^{3t} + 5)e^{-10t}$$

For t=$t_{0=}$.$t_1$in(3), we have
G(t_0,t_1)=2.608643414>0.
Below is a graphical stimulation for the system in example



This implies that the system is not exploding but bounded, which itself is Lipchitz continuous At t=3, the system is stably controllable.

II. Conclusion

We considered a class of nonlinear integro-differential system with implicit derivatives of the form

$$\dot{x}(t) = \mathrm{A(t)x(t)} + \int_{t_0}^{t} k(t,s)x(s)ds + B(t)u(t) + f(u(t), x(t-h(x(t),t)), x^1(t), t \ for t \geq t_o$$

$x(t)= \varphi(t,t_0)x_0$ for $t \leq t_0$

By using measure of non compactness of set, we formulate conditions for Darbo's fixed point theorem which is used to established controllability result. We concluded by solving a numerical example.

References

[1]. Aghajani, A., Banas, J., andSabzali, N. (2013). Some generalizations of Darbo fixed point theorem and applications. *Bulletin of the Belgian Mathematical Society-Simon Stevin*, *20*(2); 345-358.

[2]. Balachandran, K., and Dauer, J. P. (2002). Controllability of nonlinear systems in Banach spaces: a survey. *Journal of Optimization Theory and Applications*, *115*(1): 7-28.

[3]. Balachandran, K., andSomasundaram, D. (1983). Controllability of a class of nonlinear systems with distributed delays in control. *Kybernetika*, *19*(6): 475-482.

[4]. Balachandran, K., and Somasundaram, D. (1985). Relative controllability of nonlinear systems with time varying delays in control. *Kybernetika*, *21*(1): 65-72.

[5]. Balachandran, K., andSomasundaram, D. (1986). Controllability of nonlinear delay systems with delay depending on state variable. *Kybernetika*, *22*(5): 439-444.

[6]. Bienkowska-Lipinska, K. (1974). Introduction to the Theory of Differential Equations with Perturbed Argument.

[7]. Dacka, C. Z. E. S. L. A. W. (1980). On the controllability of a class of nonlinear systems. *IEEE Transactions on Automatic Control*, *25*(2): 263-266.

[8]. Davies, I., andJackreece, P. (2005). Controllability and null controllability of linear systems. *Journal of Applied Sciences and Environmental Management*, *9*(3): 31-36.

[9]. Davison, E. J., andKunze, E. G. (1970). Some sufficient conditions for the global and local controllability of nonlinear time-varying systems. *SIAM Journal on Control*, *8*(4): 489-497.

[10]. Joshua E.S. Socolar (2006). Nonlinear dynamical system. Physics Department, Duke University, Durhan, North Carolina. Durhan, NC 27708; 919-660-2557.

[11]. Kaczorek, T. (2000). Reachability and controllability of 2D positive linear systems with state feedbacks. *IFAC Proceedings Volumes*, *33*(14): 745-749.

[12]. Klamka, J. (1999). Controllability of 2D linear systems. *Advances in Control. Highlights of ECC*, *99*: 319-326.

[13]. Klamka, J. (2000). Schauder's fixed-point theorem in nonlinear controllability problems. *Control and Cybernetics*, *29*, 153-165.

[14]. Klamka, J. (2005). Approximate constrained controllability of mechanical system. *Journal of Theoretical and Applied Mechanics*, *43*(3): 539-554.

[15]. Klamka, J. (2008). Controllability of dynamical systems. *MatematykaStosowana*, *9*: 57-75.

[16]. Klamka, J. E. R. Z. Y. (1993). Constrained controllability of linear retarded dynamical systems. *Applied Mathematics and Computer Science*, *3*(4): 647-672.

[17]. Klamka, J., andNiezabitowski, M. (2014, September). Controllability of switched infinite-dimensional linear dynamical systems. In *Methods and Models in Automation and Robotics (MMAR), 2014 19th International Conference on* (pp. 171-175). IEEE.

[18]. Klamka, J., Zawiski, R., andNiezabitowski, M. (2017, July). On some recent results in the exact controllability of second order infinite dimensional semilinear deterministic systems. In *AIP Conference Proceedings* (Vol. 1863, No. 1, p. 080004). AIP Publishing.

[19]. Sikora, B. (2003). On the constrained controllability of dynamical systems with multiple delays in the state. *International Journal of Applied Mathematics and Computer Science*, *13*: 469-479.

Eli Innocent Cleopas. "Controllability Result for Nonlinear Integrodifferential Equation." IOSR Journal of Mathematics (IOSR-JM) 15.2 (2019): 33-41.