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Prime Labeling of Split Graph of Star $K_{1,n}$

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Abstract: *A graph G = (V, E) with p vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not exceed p such that the label of each pair of adjacent vertices are relatively prime. A graph which admits prime labeling is called a prime graph. In this paper, we investigate prime labeling of Spliting graph of star $K_{1,n}$ .We also present an algorithm which enable us to find the chromatic number of $Spl(K_{1,n})$.*

Keywords: *Star Graph, Split Graph, Graph Labeling, Prime Labeling*

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I. Introduction

In labeling of graphs, we consider only simple, undirected, connected and non-trivial graph G = (V, E) with the vertex set V and the edge set E. The number of elements of V, denoted as |V|, called the order of the graph while the number of elements of E, denoted as |E|, called the size of the graph. The notion of a prime labeling originated with Entringer and was introduced in a paper by Tout, Dabbouchy and Howalla [2]. Entringer conjectured that all trees have a prime labeling. Many researchers have studied prime graph. Haxell, Pikhuriko and Taraz[7] proved that all large trees are prime graph. In [4], Ganesan V et al discuss prime labeling of Split graph of Path Graph Pn and in [5] Ganesan V and Lavanya S proved that Spl(Cn) is a prime graph. For various graph theoretic notations and terminology, we follow Bondy. J. A and U. S. R. Murthy [1]. For latest survey on graph labeling we refer to [3] (Gallian. J. A. 2017) we will give brief summary of definitions and other information which are useful for the present task.

II. Preliminary Definitions

Definition 2.1: Let G = (V, E) be a graph with p vertices. A bijection $f : V(G) \rightarrow \{1, 2, \ldots, p\}$ is called a prime labeling if for each edge e = uv, $\gcd(f(u),f(v)) = 1$. A graph which admits prime labeling is called a prime graph.
Definition 2.2: A tree containing exactly one vertex is not a pendent vertex is called a star graph $K_{1,n}$
Definition 2.3: For a graph G, the splitting graph, Spl(G) of a graph G is obtained by adding a new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$
Definition 2.4: A k–coloring of a graph G = (V, E) is a function c : V → C, where |C| = k. Vertices of the same color form a color class. A coloring is proper if adjacent vertices have different colors. A graph G is k-colorable if there is a proper k-coloring. The chromatic number χ (G) of a graph G is the minimum k such that G is k-coloring.

III. Main Results

Theorem 3.1: The Split graph of star $K_{1,n}$ is a prime graph.
Proof: Let $K_{1,n}$ be the star graph and let $V(K_{1,n}) = \{v\} \cup \{v_i : 1 \leq i \leq n\}$ where v be the apex vertex and $v_i : i = 1, 2, \ldots, n$ are the pendent vertices of $K_{1,n}$.

$$E(K_{1,n}) = \{vv_i : 1 \leq i \leq n\}$$

Now, Construct the Split graph of $K_{1,n}$ by adding the new $\{v'\}$ and $\{v'_i : 1 \leq i \leq n\}$ corresponding to the vertices $\{v\}$ and $\{v'_i : 1 \leq i \leq n\}$ of $K_{1,n}$ which are added to obtain $Spl(K_{1,n})$
In $Spl(K_{1,n})$, the vertex set and edge set are given by

$$V(Spl(K_{1,n})) = \{v_i\} \cup \{v'_i\} \cup \{v\} \cup \{v'\} \quad \text{for } 1 \leq i \leq n$$

and $$E(Spl(K_{1,n})) = \{vv_i\} \cup \{vv'_i\} \cup \{v\} \cup \{v'v_i\} \quad \text{for } 1 \leq i \leq n$$

Clearly, $|V(Spl(K_{1,n}))| = 2n + 2$.
Therefore, we define a bijection $f : V(Spl(K_{1,n})) \rightarrow \{1, 2, \ldots, 2n+2\}$
as follows,

Let
$$f(v) = 1$$
$$f(v') = 2$$
$$f(v_i) = 2i + 2 \text{ for } i = 1, 2, \ldots, n$$

$f(v'_i) = 2i + 1$ for $i = 1, 2, \ldots, n$

Based on the above labeling pattern, the Split graph of star $K_{1,n}$ admits prime labeling.
Hence, $Spl(K_{1,n})$ is a prime graph.

Illustrations:

Illustration 3.1 Let n = 5, $Spl(K_{1,5})$ is a prime graph

$K_{1,5}$:

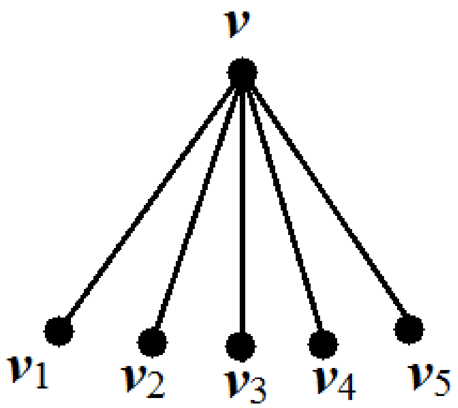

Fig: 1 Star graph $K_{1,5}$

$Spl(K_{1,5})$:

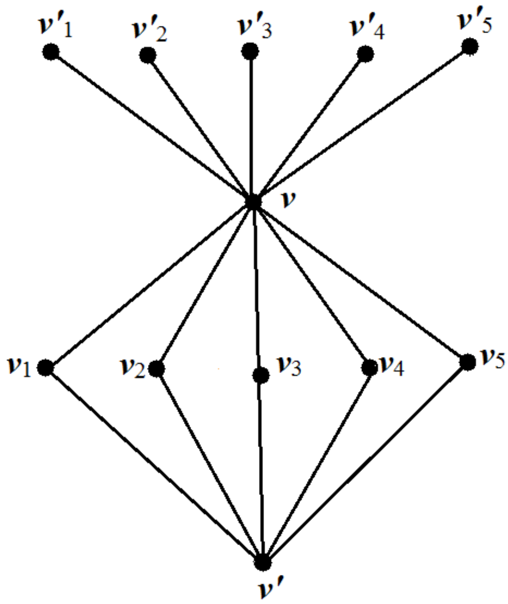

Fig: 2 Split graph of $K_{1,5}$

Let

$f(v) = 1$ and $f(v') = 2$

$f(v_i) = 2i + 2$ for $i = 1, 2, 3, 4, 5$

$f(v'_i) = 2i + 1$ for $i = 1, 2, 3, 4, 5$

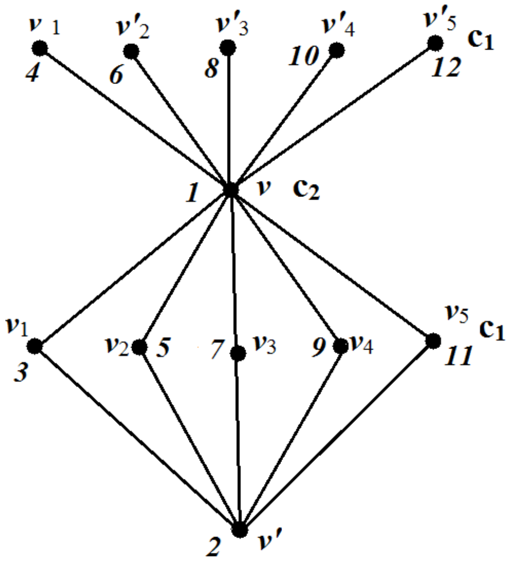

Fig: 3 Prime labeling of Spl($K_{1,5}$)

Preposition 3.1 Algorithm to find the chromatic number of Spl($K_{1,n}$)
Step 1: Let $K_{1,n}$ be the given star graph.
$V(K_{1,n}) = \{v\} \cup \{v_i : 1 \leq i \leq n\}$ where v be the apex vertex and v_i for $i = 1, 2, \ldots, n$ are the pendent vertices of $K_{1,n}$.
Step 2: Let G = Spl($K_{1,n}$) be the split graph of $K_{1,n}$. Obliviously $|V(Spl(K_{1,n})| = 2n + 2$
Therefore, define a bijection
$f : V(Spl(K_{1,n})) \rightarrow \{1, 2, \ldots, 2n+2\}$
as follows

$f(v) = 1$ and $f(v') = 2$
$f(v_i) = 2i + 1$ for $i = 1, 2, \ldots, n$
$f(v'_i) = 2i + 2$ for $i = 1, 2, \ldots, n$

Let G* be the newly labeled prime graph of Spl($K_{1,n}$)
Step 3: Assign the color C_1 to the vertices v'_i for $i = 1, 2, \ldots, n$ and the vertices v_i for $i = 1, 2, \ldots, n$. Assign the color C_2 to the vertex v and v' .
Step 4: Now, G* is properly colored with 2-colors. Hence χ (Spl($K_{1,n}$)) = 2.
Illustration 3.2 Find the chromatic number of Spl($K_{1,6}$)
Spl($K_{1,6}$):

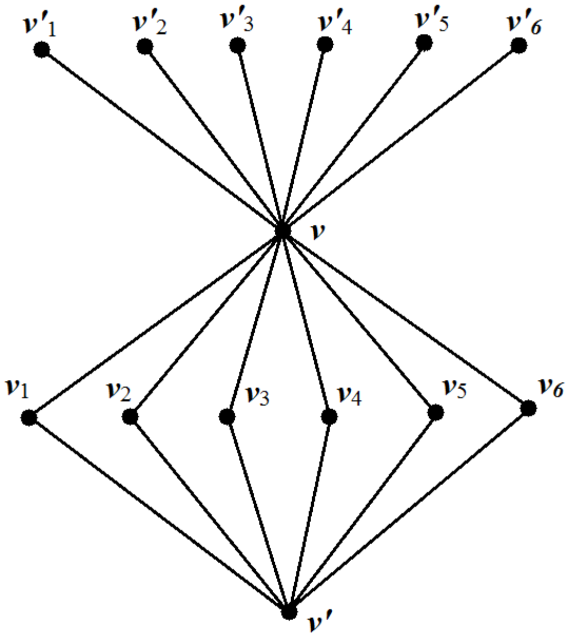

Fig: 4 Split graph of $K_{1,6}$

G*:

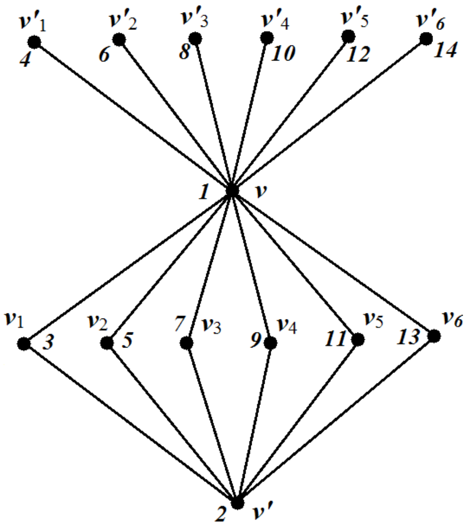

Fig: 5 Prime labeling of Spl($K_{1,6}$) = G*

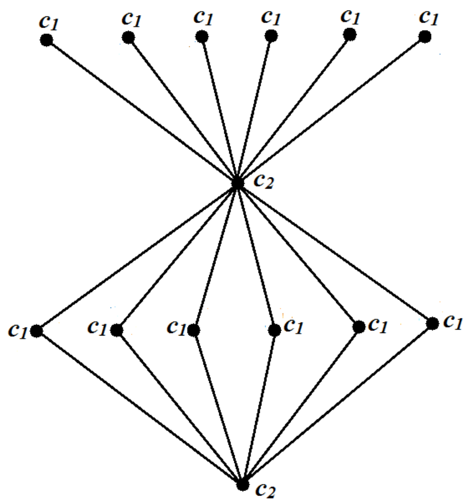

Fig: 6 Proper coloring of G*

IV. Conclusion

Here we investigate prime labeling of Split graph of star graph $K_{1,n}$ and gave an algorithm for application of prime labeling to proper coloring of some classes of graphs namely Split graph of star graph. To investigate similar results for other graph families and in the context of different graph labeling techniques is an open area of research.

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