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Relation between Indutive Limits and Barrelled Spaces

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***Abstract**: In this paper we define inductive limits of locally convex spaces and relation between inductive limits and Barrelled Spaces.*
***Key words:** Topological vector spaces, locally convex spaces, inductive limits, Barrelled spaces.*

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Some useful difinition :

(i) **Topological vector spaces** – A set E on which a structure of vector space over k and a topoloty are difined is a topological vector space if

(a) The map $(x, y) \rightarrow x + y$ from E × E into is continuous.

(b) The map $(\lambda, x) \rightarrow \lambda x$ from K × E into E is continuous.

(ii) **Locally convex spaces** – A topological vector space E said to be a locally convex topological vector space or simply locally convex space or convex space, if there is a fundamental system of convex neighbourhoods of the origin of E.

A topological vector space is locally convex if each point has a fundamental system of convex neighbourhoods

(iii) **Barrelled space** – A locally conves space E is said to be a barrelled, if every barrel in E is a neighbourhood of 0.

(iv) **Inductive limits of locally convex spaces** – Let $\{X_i\}_{i\in I}$ be a family of locally convex spaces and for each i $\in$ I let f_i be a linear mapping of X_i into a vector spaces X such that $\bigcup_{i\in I} f_i(x_i)$ span X. Then there is finest.

localy convex topology on X under which all the mapping f_i are continous.

The locally convex space X, with this topoloty is called the inductive limit of locally convex spaces X_i by the mapping f_i

Proof – Suppose U is a balanced nhd of origin in any topoloty on $f_i^{x-1}(\bigcup)$ for which all the f_i's is care continous. Then each X is nhd of origin in X_i.

Let u the family of all balanced, convex subsets V of X such that for each $i \in I, f_i^{-1}(v)$ is a nhd of zero in X_i. Then $U \in u$ and $f_i^{-1}(U)$ is absorbent in X_i and so U absorbs all thepts $f_i(x_i)$. Now, since $\bigcup_{i\in I} f_i(X_i)$ spans X. U is absorbent in X. It is clear that for every $\alpha = 0. \alpha U \in \mu.$

Also if U, $V \in u$ then $f_i^{-1}(U) \bigcap f_i^{-1}(V) = f_i^{-1}(u \bigcap V)$ is a nhd of zero in X_i and $U \bigcap V$ is balanced and convex. Hence $U \bigcap V \in u.$ Thus there exits a locally convex topoloty on X for which u is a fundamental system of nhds.of origin. This is therefore the finest locally convex topolot making each f_i continous.

If for each $i \in I, \upsilon_i$ is a fundamental system of balanced, convex nhds of origin in X_i then the set u of balanced, convex envelopes of sets of the form $\bigcup_{i\in I} f_i(V_i)(\text{with } V_i \in \upsilon_i)$ form a fundamental system of nhds of origin for the inductive limit topology on X.

In fact, the sets υ are nhds of origin in X. Moreover, if U is any balanced, convex nhd of origin in $X, f_i^{-1}(U)$ is a nhd of origin in X_i and hence $f_i^{-1}(U)$ contains a nhd. $V_i \in \upsilon_i$. Hence the balance convex envelope of $\bigcup_{i\in I} f_i(V_i)$ is a set of υ contained in U. Thus υ is a fundamental system of nhds of origin in X for the inductive limit topoloty of X.

RESULTS (1) :
An inductive limit of barrelled spaces is barrelled.
Proof :
of a barrelled space E. Therefore X is a barrelled space. Let X be the inductive limit of the barrelled spaces $X_i\,(i \in I)$ by the linear mappings f_i and let D be a barrel in X. Then D is absolutely convex, absorbent and closed in X. Since each f_i is continous for the inductive limit topoloty on X, $f_i^{-1}(D)$ is closed in X_i.
In order to see that $f_i^{-1}(D)$ is absolutely convex in X_i let $x_i, y_i \in f_i^{-1}(D)$ and $|\alpha| + |\beta| \le 1.$
Then $f_i(x_i), f_i(y_i) \in D.$
Since D is absolutely convex.
$\alpha f_i(x_i) + \beta f_i(y_i) \in D.$

Since f_i is linear.
$\alpha f_i(x_i) + \beta f_i(y_i) = f_i(\alpha x_i + \beta y_i) \in D.$
Thus $\alpha x_i + \beta y_i \in f_i^{-1} D$
Thus $f_i^{-1}(D)$ is absolutely convex.
Finally, to show that $f_i^{-1}(D)$ is absorbent let $x \in X_i$ be given. Then $f_i(x) \in X.$ Since D is absorbent in X there exists $\alpha > 0$ such that $f_i(x) \in \alpha D.$
Then $x \in \alpha\ f_i^{-1}(D).$ Thus $f_i^{-1}(D)$ is absorbent in X_i.

We have thus shown that $f_i^{-1}(D)$ is absolutely convex, absorbent and closed in X_i for each $i \in I.$ Thus $f_i^{-1}(D)$ is a barrel in X_i for each $i \in I.$ Since X_i a barrelled space, each barrel in X_i is a nhd of origin in X_i. Thus $f_i^{-1}(D)$ is nhd. of origin in X_i. Hence D is a nhd of origin in X (by definition iv). Thus every barrel in X i s a nhd of origin. Therefore X is a barrelled space.

RESUTLS (2) :
A quotient space of a barrelled space is barrelled.
Proof. : Let X = E/M be the quotient space of barrelled space E with respect to a linear subspace M and let $\phi : E \to X$ be the canonical mapping of E onto X = E/M defined by $\phi(x) = x + M$ for all $x \in E.$
The quotient topoloty on X is the finest locally convex topology making ϕ continous. Hence the quotient topoloty is an inductive limit topoloty. Thus X is an inductive limit

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