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Solving Integer Interval Transportation Problem with Mixed Constraints

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Abstract: *In this paper the transportation problem with mixed constraints having all parameters as integer intervals is considered. Here we solve the fully integer interval transportation problem without converting it to the crisp transportation problem. Numerical example is illustrated to validate the argument and the results are compared with the existing methods.*

Keywords:*Interval numbers, Interval Transportation problem, ranking methods.*

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I. Introduction

The transportation problem is a linear programming problem which deals with the distribution of single commodity manufactured at different areas transported from various sources of supply to various destination of demand. The objective of the fuzzy transportation problem is to minimize the total transportation cost. An interval transportation problem constructs the data of supply, demand, and objective functions such as cost, time, etc. in some intervals. Many researchers have proposed different types of interval transportation problems. A. Akilbasha et al [1] solved the integer interval transportation problem in rough nature by split and separation method. P. Pandian et al. [8] proposed a new method for finding an optimal solution of fully interval integer transportation problems. A. Akilbasha et al.[7] solved the transportation problem with mixed constraints in Rough Environment by using rough slice sum method.G.Ramesh et al. [10] proposed a method for solving interval linear programming problem without converting it to a classical linear programming problem.Juman et al., [5] proposed a heuristic technique for solving transportation problems with interval numbers. Das et al. [3] solved interval transportation problems using the right bound and the midpoint of the interval. Sengupta et al.[12] developed a method to solve interval transportation problems by considering the midpoint and width of the interval in the objective function. Safi et al [11] solved a fixed charge transportation problem by converting the interval fuzzy constraints into multiobjective fuzzy constraints. Pandian et al.,[7] applied the separation method for solving fully interval integer transportation problems. Purushothkumar et al [9] developed a diagonal optimal algorithm to solve interval integer transportation problems.

II. Preliminaries

Definition: 2.1

Let $\bar{a} = [a_L,a_R] = \{x \in \Re, a_L \le x \le a_R \,\&\, a_L,a_R \in \Re\}$ be an interval on the real line $\Re$. If $\bar{a} = a_L = a_R = a$ then $\bar{a} = [a,a] = a$ is a real number. The midpoint and half width of an interval number $\bar{a} = [a_L, a_R]$ is definedas $m(\bar{a}) = \left(\frac{a_L + a_R}{2}\right)$ and $w(\bar{a}) = \left(\frac{a_R - a_L}{2}\right)$. Now the interval number $\bar{a}$ can be expressed as $\bar{a} = \langle m(\bar{a}), w(\bar{a})\rangle$.

Ranking of interval numbers:

Sengupta et al [12] proposed a simple and efficient index by comparing any two intervals on IR through decision maker's satisfaction.

Definition: 2.2

Let$\le$ be an extended order relation between the interval numbers $\bar{a} = [a_L,a_R]$, $\bar{b} = [b_L,b_R]$ in IR, then for $m(\bar{a}) < m(\bar{b})$ we construct a premise $(\bar{a} < \bar{b})$which means that $\bar{a}$ inferior to $\bar{b}$.

An acceptability function $A_{\leq}$: IR x IR → [0,∞) is defined as

$$A_{\leq}\left(\bar{a},\bar{b}\right) = A\left(\bar{a} \leq \bar{b}\right) = \frac{m(\bar{b})-m(\bar{a})}{w(\bar{b})+w(\bar{a})}$$, where $w(\bar{b}) + w(\bar{a}) \neq 0$

$A_{<}$ be interpreted as the grade of acceptability of the first interval number to be inferior to the second interval number. For any two interval numbers $\bar{a}$ and $\bar{b}$ in IR either A($\bar{a} \leq \bar{b}$) ≥ 0 (or) A($\bar{b} \geq \bar{a}$)≥ 0 (or) A($\bar{a} \leq \bar{b}$)=0 (or) A($\bar{b} \geq \bar{a}$) = 0 (or) A($\bar{a} \leq \bar{b}$) + A($\bar{b} \geq \bar{a}$) =0.A($\bar{a} \leq \bar{b}$)=0 and A($\bar{b} \geq \bar{a}$) = 0 then we say that interval numbers $\bar{a}$ and $\bar{b}$ are equivalent and it is denoted as $\bar{a} \approx \bar{b}$.

Also if A ($\bar{a} \leq \bar{b}$) ≥ 0, then $\bar{a} \leq \bar{b}$ and if A ($\bar{b} \geq \bar{a}$)≥0 then $\bar{b} \leq \bar{a}$.

Definition: 2.3

Ming et al [6] have proposed a new fuzzy arithmetic based on both location index and fuzziness index function. The location index number is taken in the ordinary arithmetic. Whereas the fuzziness index functions follows lattice concept.

Let us define for any two intervals $\bar{a}$ = [a_L, a_R] , $\bar{b}$ = [b_L,b_R] in IR and for * ∈ { +,-,.,÷}the arithmetic operations on $\bar{a}$ and $\bar{b}$ defined as follows.

Addition:

$$\tilde{a} + \tilde{b} = < m(\tilde{a}), w(\tilde{a}) > + < m(\tilde{b}), w\,(\widetilde{b}) > = < m(\tilde{a}) + m(\tilde{b}), \max\{w(\tilde{a}), w(\tilde{b}) >$$

Subtraction:

$$\tilde{a} - \tilde{b} = < m(\tilde{a}), w(\tilde{a}) > - < m(\tilde{b}), w\,(\widetilde{b}) > = < m(\tilde{a}) - m(\tilde{b}), \max\{w(\tilde{a}), w(\tilde{b}) >$$

Multiplication:

$$\tilde{a} \times \tilde{b} = < m(\tilde{a}), w(\tilde{a}) > \times < m(\tilde{b}), w\,(\widetilde{b}) > = < m(\tilde{a}) \times m(\tilde{b}), \max\{w(\tilde{a}), w(\tilde{b}) >$$

Division:

$$\tilde{a} \div \tilde{b} = < m(\tilde{a}), w(\tilde{a}) > \div < m(\tilde{b}), w\,(\widetilde{b}) > = < m(\tilde{a}) \div m(\tilde{b}), \max\{w(\tilde{a}), w(\tilde{b}) >$$

Provided m ($\bar{b}$)≠ 0.

III. Proposed Algorithm

The alternate algorithm proposed involves the following steps to solve IBTP.

Step 1: Write down the interval transportation problem in tabular form.

Step 2: Express all the interval parameters in terms of midpoint and half-width interval values.

Step 3: Check whether the problem is balanced or not. Otherwise, balance it by adding a dummy row/column.

Step 4: Select the first row (source) and verify which column (destination) minimum unit has cost. Write that source under column 1 and the corresponding destination under column 2. Continue this process for each source. However, if any source has more than the same minimum value in the different destinations then write all these destinations under column 2.

Step 5: Select those rows under column-1 which have a unique destination. In that row find minimum cost and allocate the value. Next delete that row/column where supply/demand exhausted. However, if destinations are not unique then follow step 7

Step 6: If the destination under column-2 is not unique then select those sources where destinations are identical. Next find the difference between the minimum and next minimum unit cost for all those sources where destinations are identical.

Step 7: Check the source which has a maximum difference. For that source allocate a minimum of supply or demand to the corresponding destination. Delete the corresponding row/column where supply/demand got exhausted.

Remark 1: For two or more than two sources, if the maximum difference happens to be the same then, in that case, find the difference between the minimum and next to next minimum unit cost for those sources and selects the source having a maximum difference. A minimum of supply and demand should be allocated to that cell. Next delete that row/column where supply/demand exhausted.

Step 8: Repeat steps 5 and 6 for the remaining sources and destinations till (m+n-1) cells are allocated.

Step 9: Total cost is calculated as the sum of the product of cost and corresponding allocated value of supply/ demand.

IV. Numerical Examples

Example 4.1

Consider a balanced interval integer transportation problem discussed by P. Pandian et al [8]

	D_1	D_2	D_3	D_4	supply
S_1	[3,5]	[2,6]	[2,4]	[1,5]	[7,9]
S_2	[4,6]	[7,9]	[7,10]	[9,11]	[17,21]
S_3	[4,8]	[1,3]	[3,6]	[1,2]	[16,18]
demand	[10,12]	[2,4]	[13,15]	[15,17]	[40,48]

Solution:

Let us define all the interval parameters as $\bar{a}=[a_L,a_R]$ in terms of midpoint and width $\bar{a} =< m(\bar{a}),w(\bar{a})>$ .Now the given interval transportation problem is as follows

	D_1	**D_2**	**D_3**	**D_4**	**supply**
S_1	<4,1>	<4,2>	<3,1>	<3,2>	**<8,1>**
S_2	<5,1>,	<8,1>	<8.5,1.5>	<10,1>	**<19,2>**
S_3	<6,2>	<2,1>	<4.5,1.5>	<1.5,0.5>	**<17,1>**
demand	**<11,1>**	**<3,1>**	**<14,1>**	**<16,1>**	**<44,4>**

Applying proposed method we have

	D_1	**D_2**	**D_3**	**D_4**	**supply**
S_1	<4,1>	<4,2>	**(8,1)** <3,1>	<3,2>	**<8,1>**
S_2	**(11,1)** <5,1>	**(2,1)** <8,1>	**(6,1)** <8.5,1.5>	<10,1>	**<19,2>**
S_3	<6,2>	**(1,1)** <2,1>	<4.5,1.5>	**(16,1)** <1.5,0.5>	**<17,1>**
demand	**<11,1>**	**<3,1>**	**<14,1>**	**<16,1>**	**<44,4>**

The optimum interval transportation cost is

= (8,1) <3,1> + (11,1) <5,1> +(2,1) <8,1>+(6,1) <8.5,1.5>+(1,1) <2,1> +(16,1) <1.5,0.5>

= <172, 1.5>

= [170.5, 173.5]

It is to be noted that our solution is very much sharper than the solution obtained by Pandian et al. [8]

Comparison Table: 1

Methods	**Interval value**	**Mid width value**
Zero suffix method[9]	[92,335]	<213.5,121.5>
Zero point method[8]	[119,232]	<175.5,56.5>
VAM,MODI method	[118,232]	<175,57>
Proposed method	[170.5,173.5]	<170.5,173.5>

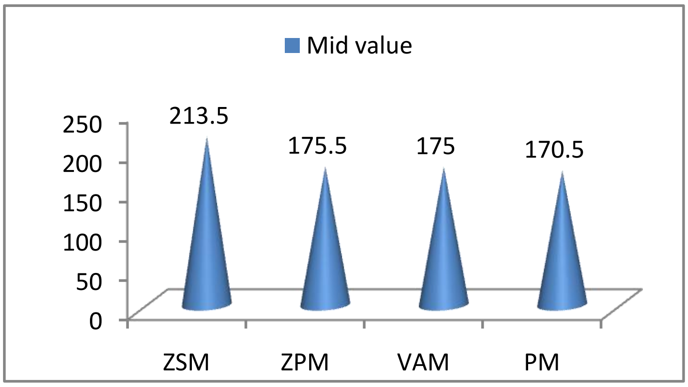

Fig:1

Example 4.2: Consider an unbalanced interval integer transportation problem given by [4]

	D_1	D_2	D_3	D_4	supply
S_1	[1,4]	[1,6]	[4,12]	[5,11]	[1,12]
S_2	[0,4]	[1,4]	[5,8]	[0,3]	[0,3]
S_3	[3,8]	[5,12]	[12,19]	[7,12]	[5,16]
demand	[5,10]	[1,10]	[1,6]	[1,4]	

Solution:

Let us define all the interval parameters as $\bar{a}$ =[a_L,a_R] in terms of midpoint and width $\bar{a}$ =< m($\bar{a}$),w($\bar{a}$)> .
Now the given interval transportation problem is as follows

	D_1	D_2	D_3	D_4	D_5	supply
S_1	<2.5,1.5>	<3.5,2.5>	<8,4>	<8,3>	<0,0>	**<6.5,5.5>**
S_2	<2,2>,	<2.5,1.5>	<6.5,1.5>	<1.5,1.5>	<0,0>	**<1.5,1.5>**
S_3	<5.5,2.5>	<8.5,3.5>	<15.5,3.5>	<9.5,2.5>	<0,0>	**<10.5,5.5>**
S_4	<0,0>	<0,0>	<0,0>	<0,0>	<0,0>	**<0.5,0>**
demand	**<7.5,2.5>**	**<5.5,4.5>**	**<3.5,2.5>**	**<2.5,1.5>**	**<0,1.5>**	**<19,12.5>**

Applying proposed method we have

	D_1	D_2	D_3	D_4	D_5	supply
S_1	<2.5,1.5>	**(5.5,4.5)** <3.5,2.5>	**(1,5.5)** <8,4>	<8,3>	<0,0>	**<6.5,5.5>**
S_2	<2,2>,	<2.5,1.5>	<6.5,1.5>	**(1.5,1.5)** <1.5,1.5>	<0,0>	**<1.5,1.5>**
S_3	**(7.5,2.5)** <5.5,2.5>	<8.5,3.5>	**(2,5.5)** <15.5,3.5>	**(1,1.5)** <9.5,2.5>	**(0,1.5)** <0,0>	**<10.5,5.5>**
S_4	<0,0>	<0,0>	**(0.5,0)** <0,0>	<0,0>	<0,0>	**<0.5,0>**
demand	**<7.5,2.5>**	**<5.5,4.5>**	**<3.5,2.5>**	**<2.5,1.5>**	**<0,1.5>**	**<19,12.5>**

The optimum interval transportation cost is
=(5.5,4.5) <3.5,2.5> +(1,5.5)<8,4> +(1.5,1.5)<1.5,1.5>+(7.5,2.5)<5.5,2.5>+(2,5.5)<15.5,3.5>+(1,1.5) <9.5,2.5>+(0,1.5)<0,0>+(0.5,0)<0,0>
= < 111.25, 5.5>
= [105.75, 116.75]
It is to be noted that our solution is very much sharper than the solution obtained by VAM method.

Comparison Table: 2

Methods	Interval value	Mid, width value
VAM,MODI method	[16,254]	<135,119>
Proposed method	[105.75, 116.75]	<111.25, 5.5>

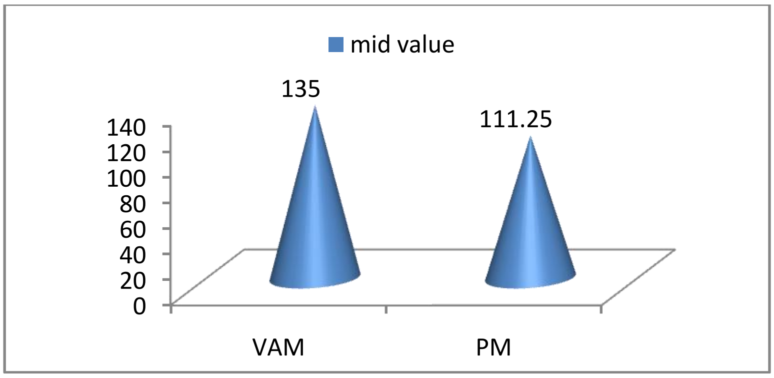

Fig: 2

V. Conclusion

The Transportation problem with mixed constraints having all parameters as integer intervals is considered. An alternate method is proposed to solve fully integer interval transportation problem without converting it to the classical transportation problem so that the total cost is minimized. The above method is a systematic procedure that is easy to apply. This method provides an optimal solution which is sharper than the other methods.

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