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Conformal Mapping and Its Applications to Heat Flow in Solids.

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Abstract: *In this paper, a simple but efficient method of solving harmonic Dirichlet problems of heat flow in solids which can be reduced to two dimensional problems using conformal mapping is presented. The method employs an appropriate mapping function to transform the domain and boundary of the given problem in the w plane onto onein the upper half of the z plane and the appropriate portions of the x axis where its solution for steady state temperature is easily identified as the imaginary part of some branch of the logarithmic function. The required solution in the w plane was then obtained from the mapping function by simply substituting u and v for x and y, respectively. Furthermore, the isothermal lines or level curves of the solution were also obtained to show the lines/surfaces of constant temperature within the given solid. This method gave exact analytical solutions for the steady state temperature within the given solid and can therefore be a suitable alternative method for solving such problems in two dimensions.*
Keywords and Phrases: Harmonic function, Conformal Map, Analytic Function, Schwarz-Christoffel Map,Joukowski map, Steady State Temperature, Isothermal lines.

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I. Introduction

Inthe theory of heat conduction through solids having a temperature distribution that is constant or may be varying, we are often interested in the heat conduction per unit area per unit time across a surface located in the solid. This quantity, which is often called the heat flux across the surface is expressed mathematically by Spiegel (1974) as

$\hbar = -k\nabla\phi$ (1)

where ϕ is the temperature and k, assumed to be a constant, is called the thermal conductivity and depends on the material of which the solid is made. For two dimensional types of problems in heat flow, which is the concern of this paper, equation (1) takes the form

$\hbar = -k\left(\frac{\partial\phi}{\partial x} + i\frac{\partial\phi}{\partial y}\right) = Q_x + iQ_y$ (2)

where

$$Q_x = -k\frac{\partial\phi}{\partial x}\text{and}Q_y = -k\frac{\partial\phi}{\partial y}$$

If it is assumed that steady state conditions prevail so that there is no net accumulation of heat across any simple closed curve C in the solid which contains no sources and sinks, then the temperature function satisfies Laplace's equation

$\nabla^2\phi = 0$ (3)

together with some prescribed conditions on the boundary $\partial\Omega$ of Ω(Spiegel (1974) and Churchill and Brown (1984)). In this paper, we will only focus on Dirichlet harmonic problems in which the function ϕ satisfies equation (3) and takes prescribed values on the boundary.It is known in the theory of functions of a complex variable that a function such as $\phi(x,y)$ which is harmonic in a domainΩ must have a harmonic conjugate $\psi(x,y)$ such that the function$\Omega(z) = \phi(x,y) + i\psi(x,y)$ is analytic throughoutΩ. In the theory of heat flow, the function Ω is called the complex temperature. The solution to problem (3) using complex variable methods therefore reduces to finding the real and imaginary parts of analytic functions in Ω which satisfy the boundary conditions. The complex variable method of conformal mapping is a useful intermediate step in the solution and analysis of two dimensional harmonic Dirichlet problems in theory of heat flow as well as other Dirichlet problems in ideal fluid flows, electromagnetism, and thermal physics as is evident in the works of Churchill and Brown (1984), Spiegel (1974), Tobin and Lloyd (2002), Ganzolo*et al*. (2008), Tao *et al.* (2008), Anders (2008), Weiman*et al*. (2016), Yariv and Sherwood (2015), Andreas and Yorgos (2004), Xu *et al.* (2015), Wesley *et al*. (2008), Suman (2008) . The technique involves the transformation of the problem from a domain with an inconvenient geometry in one complex plane into a domain with a simpler geometry in another complex plane by means of an appropriate mapping function which preserves the magnitude of the angles between curves as well as their orientation. Amongst a variety of conformal transformations, the most commonly used ones in problems of heat floware the Schwarz-Christoffel map, the Joukowski map, the bilinear transformation, and the transformation $w = \frac{1}{z}$ and this paper will focus on them.

TheSchwarz-Christoffel transformation which is given by Churchill and Brown (1984) as

$w = f(z) = A \int_{z_0}^{z} \prod_{j=1}^{n-1} (s - x_j)^{-k_j} ds + B$ (4)

or

$\frac{dw}{dz} = f'(z) = A \prod_{j=1}^{n-1} (z - x_i)^{-k_j}$ (5)

is one that conformally maps the upper half $\text{Im } z > 0$ of the z plane and the entire x axis except for a finite number of points $x_1, x_2, \ldots, x_{n-1}, \infty$in a one-to-one correspondence onto the interior of a given simple closed polygon and its boundary, respectively, such that $w_j = f(x_j)(j = 1,2, \ldots, n-1)$ and $w_n = f(\infty)$ are the vertices of the polygon. The point $z = x_j$ $(j = 1,2, \ldots, n-1)$ are arranged such that the order relation$x_1 < x_2 < \cdots < xn-1$ is satisfied. The complex constants $A$and$B$ in formula (1) determine the size, orientation and position of the polygon, the k_j's are real constants between -1 and 1 determined from the relation$-\pi < k_j \pi < \pi$, where $k_j \pi$ $(j = 1,2, \ldots, n-1)$ are the exterior angles at the verticesw_j $(j = 1,2, \ldots, n-1)$ of the polygon, while the limits of integration z_0 and z are respectively fixed and variable points in the region $\text{Im } z \geq 0$ of analyticity of the Schwarz-Christoffel function. In order to make the function $f'(z)$ in (5) analytic everywhere in the region $\text{Im } z \geq 0$ except at the $n-1$ points $z = x_j$ $(j = 1,2, \ldots, n-1)$, we introduce branch lines or cuts extending below those points and normal to the real axis and write

$(z - x_j)^{-k_j} = |z - x_j|^{-k_j} e^{-ik_j \theta_j} \left(|z - x_j| > 0, -\frac{\pi}{2} < \theta_j < \frac{3\pi}{2} \right)$(6)

where $\theta_j = \arg(z - x_j)$ and $j = 1,2, \ldots, n-1$. It then follows that the function

$G(z) = \int_{z_0}^{z} f'(z) dz$ (7)

is analytic in the region $\text{Im } z \geq 0$ and that $G'(z) = f'(z)$. Furthermore, the function $G(z)$ is defined at the points $z = x_j$ $(j = 1,2, \ldots, n-1)$ such that it continuous there (Churchill and Brown 1984) so that the Schwarz-Christoffel transformation (4) is continuous throughout the region $\text{Im } z \geq 0$ and conformal there except for the points $z = x_j$ $(j = 1,2, \ldots, n-1)$.

The second conformal mapping to be considered in this paper is the Joukowski transformation which is defined by Kapania*et al*; 2008 as

$z = w + \frac{c^2}{w}$(8)

where $c$is the transformation parameter. This transformation is extensively used in aerodynamics (see for instance, Theodorsen (1931), Zedan (1990), National Aeronautics and Space Administration Website (2019) to simplify the oblong shape of an airfoil and the flow around it onto a pseudo circle and the flow exterior to it, respectively. It has critical points at $w = \pm c$ which map into the critical points $z = \pm 2c$, respectively. The transformation maps circles with centres offset from the origin of the z plane and made to pass through one of the critical points of the transformation and envelops the other into profiles that resemble real airfoils called Joukowskiairfoils. In addition, it maps the circle$|w| = c$or $w = ce^{i\theta}$ $(0 \leq \theta < 2\pi)$ into a line segment$-2c \leq x \leq 2c$ of length $4c$.We shall be interested in the case of transformation (8) when $c = 1$and the semi-circle with equation $w = e^{i\theta}$ $(0 \leq \theta \leq \pi)$, whose image is still the line $-2c \leq x \leq 2c$ but the mapping is in this case one-to-one.

The next transformation we shall use in the paper is described by the equation

$\zeta = \frac{1}{w}$ $(w \neq 0, \zeta = \xi + i\eta, w = u + iv)$(9)

and establishes a one-to-one correspondence between nonzero points of the $w$andζ planes. The transformation (9) is an inversion with respect to the unit circle$w = e^{i\theta}$ $(0 \leq \theta < 2\pi)$ followed by a reflection in the u axis. A detailed presentation of the manner in which this transformation maps curves and regions from one complex plane to another can be found in Churchill and Brown (1984), Spiegel (1974) and Ahlphors (1979).

Finally, we shall also be interested in the special formof the bilinear transformation

$z = i \left(\frac{1-\zeta}{1+\zeta} \right)$ $(\zeta = \xi + i\eta, z = x + iy)$(10)

which maps the interior of the unit circle $|\zeta| = 1$ onto the upper half of the z plane. It also maps the upper and lower semi-circular arcs of the circle in the upper and lower halves of the ζ plane onto the positive and negative real axis of the z plane, respectively. A detailed presentation on bilinear transformation and the manner in which it maps regions from one complex plane to another can be found in Churchill and Brown (1984) and Spiegel (1974).

II. Methodology

Consider a solid which can be represented in two dimensions by a domainΩ with boundary $\partial\Omega$in the w plane whose temperature on the different portions of its inconvenient boundary are kept constant but not necessarily having the same magnitude. The problem of determining the steady state temperature withinΩ requires solving the mathematical problem (3) subject to some conditions on the boundary $\partial\Omega$ for which ϕ takes prescribed values. In order to solve this problem using complex variable methods, the appropriate conformal mapping function is used to transform the domain Ω of the given problemfrom the w plane onto the upper half $\mathrm{Im}z > 0$of the z plane. The various portions of the boundary $\partial\Omega$ of Ω with their respective temperatures are also mapped onto the appropriate portions of the x axis (see figures 1 for the type of boundary conditions on the x axis), respectively, since the transformation of a harmonic function via a conformal map remains harmonic (Spiegel (1974) and Churchill and Brown (1984)). The mapping function thus simplifies the given harmonic Dirichlet problem to one in the upper half $\mathrm{Im}\, z > 0$ of the z plane and which satisfies the boundary conditions on the x axis.

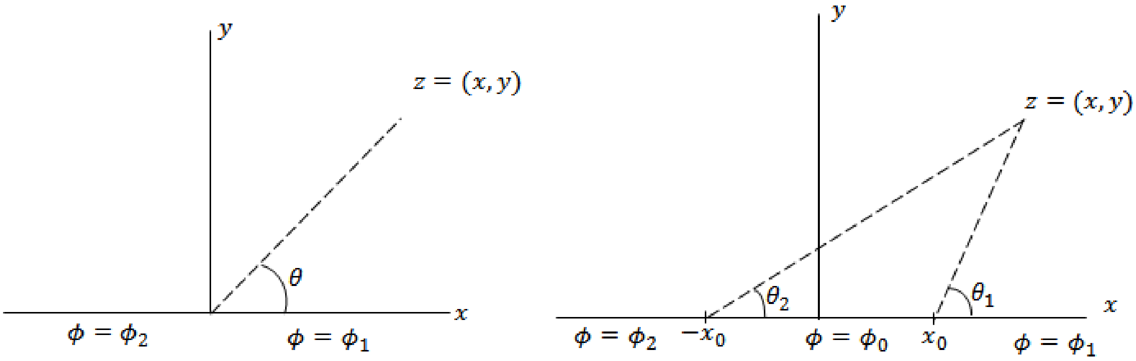

Figure 1: The Two Types of Boundary Conditions on the x axis

In the first diagram of figure 1, the function $\phi = A\theta + B$, where $A$and$B$ are real constants is harmonic in the upper half $\mathrm{Im}\, z > 0$ of the z plane since it is the imaginary part of the logarithmic function $w = A\ln z + iB$, where the branch of $\ln z$ is given as

$\ln z = \ln r + i\theta \; (r > 0, -\frac{\pi}{2} < \theta < \frac{3\pi}{2})$ (11)

The constants $A$and$B$ are then determined using the boundary conditions along the x axis

$$\phi = \phi_1 \text{when} x > 0 (\text{i.e} \theta = 0) \text{ and } \phi = \phi_2 \text{when} x < 0 (\text{i.e} \theta = \pi)$$

to obtain the solution for the steady statetemperature in the z plane. The required solution in the w plane is then obtained using the mapping function by replacing $u$and$v$ for $x$and$y$, respectively in the expression for $\phi(x, y)$.

In the second case for which the boundary condition is given in the second diagram of figure1, the harmonic function in the upper of the z plane is$\phi = A\theta_1 + B\theta_2 + C$ since it is the imaginary part of the logarithmic function$w = A\ln(z - x_0) + B\ln(z + x_0) + iC$. Here too the real constants $A, B$and$C$ are determined using the boundary conditions and the solution ϕ is determined as before.

Finally, the isothermal lines of the heat flow are thengenerated byeither setting the steady state temperatureϕ in the w planeto a constant$c$ and then varying its values or plottingthe images of the steady state temperature in the z plane using the mapping function.

Problem 1: (Steady State Temperaturein a semi-infinite Slab)

We first consider the harmonic Dirichlet problem in equation (3) for the determination of the steady state temperature at any point interior to an infinite slab (represented in two dimensionsby an infinite strip in Figure 2(a)) whose boundaries are maintained at the indicated temperatures where T is a constant.

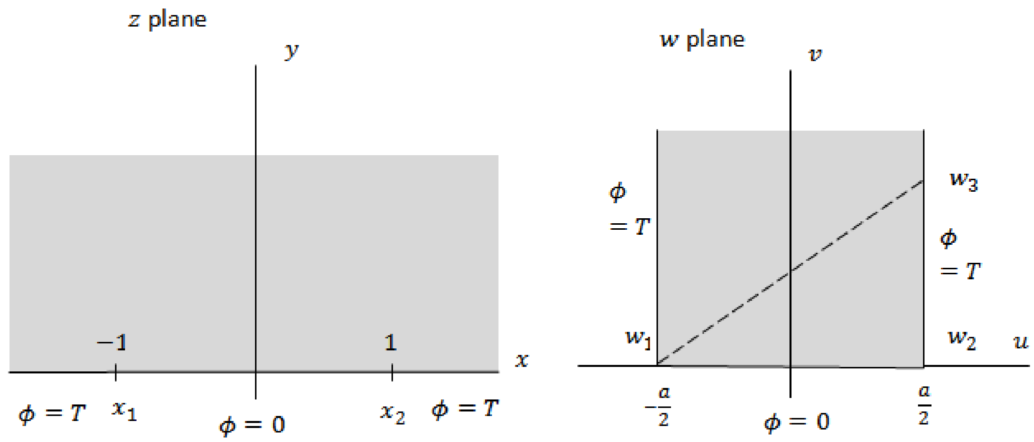

Figure 2(a):One-to-One Mapping of the Half Plane Imz ≥ 0 in the z plane Onto the Semi-Infinite Strip in the w plane

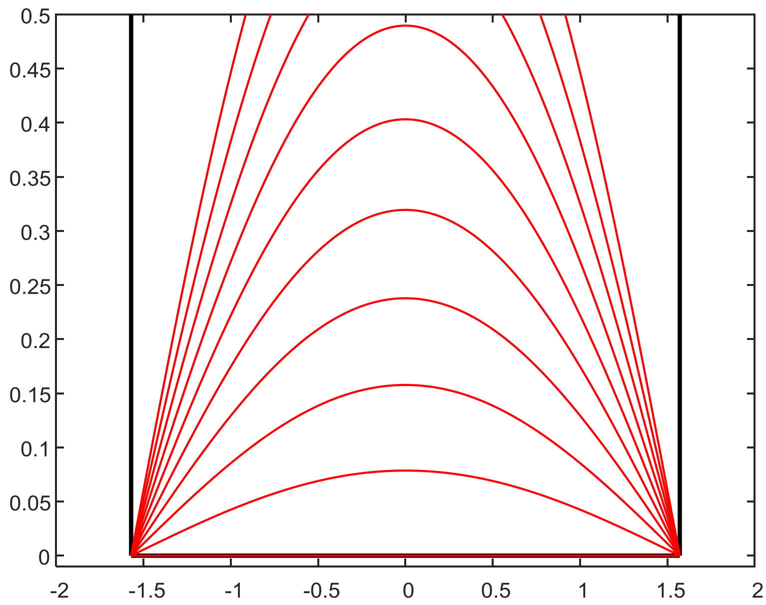

Figure 2(b):Isothermal Lines interior to the Semi-Infinite Strip in the w plane

Here, the Schwarz-Christoffel transformation $w = f(z)$ that maps the upper half Im$z \geq 0$ of the z plane in a one-to-one manner onto the semi-infinite strip of width a described by the equation

$$-\frac{a}{2} \leq u \leq \frac{a}{2}, \qquad v \geq 0$$

is found to be

$w = \frac{a}{\pi}\sin^{-1} z$(12)

by considering the semi-infinite strip as a limiting form of a triangle with vertices at $w_1 = -a/2$, $w_2 = a/2$, and w_3 as the imaginary part of w_3 tends to infinity or simply using the table of transforms given by Spiegel (1974). The part $x \leq -1$ of the x axis is mapped by the transformation (12) onto the vertical line $x = -a/2$, the part $-1 < x < 1$ maps onto the segment $-a/2 < u < a/2$ of the strip, the part $x \geq 1$ map onto the vertical line $x = a/2$, while the points $z = -1\ and z = 1$ map into the points $-\frac{a}{2}$and $\frac{a}{2}$, respectively. The inverse of the transformation (12) is

$z = \sin\left(\frac{w\pi}{a}\right) = g(w)$ (13)

and maps the interior of the semi-infinite strip and its boundaries together with their corresponding temperatures in a one-to-one manner onto the upper half Im$z \geq 0$ of the z plane and the x axis as already indicatedin figure 2(a). The inverse transformation (13) therefore simplifies the given boundary value problem to one in the upper half Im$z>0$ of the z plane subject to the following boundary conditions:$\Phi = T$when$x>1$($i.e\theta_1=\theta_2=0$) , $\Phi = 0$ when $-1<x<1$(i.e $\theta_1=\pi,\ \theta_2=0$), and $\Phi = T$when$x<-1$ (i.e $\theta_1=\theta_2=\pi$). Now, the function $\phi = A\theta_1 + B\theta_2 + C$, where $A, B,$ and$C$ are real constants is harmonic in the upper half Im$z>0$ of the z plane since it is the imaginary part of the analytic function $w = A\ln(z-1) + B\ln(z+1) + Ci$. The constants $A, B,$ and$C$ are determined using the boundary conditions and their values were obtained as $-T/\pi, T/\pi,$ and$T$, respectively. Hence

$$\phi = \frac{T}{\pi}\theta_2 - \frac{T}{\pi}\theta_1 + T$$

or

$(\phi - T)\frac{\pi}{T} = \theta_2 - \theta_1$ (14)

On taking tangents of both sides and simplifying yields

$\tan\left[(\phi - T)\frac{\pi}{T}\right] = \tan(\theta_2 - \theta_1) = \frac{-2y}{x^2+y^2-1}$(15)

Substituting the real and imaginary parts

$$x = \sin\frac{u\pi}{a}\cosh\frac{v\pi}{a}\text{and}y = \cos\frac{u\pi}{a}\sinh\frac{v\pi}{a}$$

of the inverse function (13) and simplifying yields

$$\tan\left[(\phi - T)\frac{\pi}{T}\right] = \frac{-2\cos\frac{u\pi}{a}\sinh\frac{v\pi}{a}}{\sinh^2\frac{v\pi}{a} - \cos^2\frac{u\pi}{a}}$$

or

$\phi = \frac{T}{\pi}\tan^{-1}\left(\frac{-2\cos\frac{u\pi}{a}\sinh\frac{v\pi}{a}}{\sinh 2\frac{v\pi}{a} - \cos 2\frac{u\pi}{a}}\right) + T$ (16)

as the required solution in the w plane.

Problem 2: (Steady State Temperature in a domain above the semicircle $w = e^{i\theta}$ ($0 \le \theta \le \pi$) and parts $u < -1$ and$u > 1$ of the u axis)

We next consider the harmonic Dirichlet problem in equation (3) for the determination of the steady state temperature at any point in the upper half Im$w>0$bounded by the semi-circular arc $w = e^{i\theta}$ $(0 \le \theta \le \pi)$ and parts $u<-1$ and$u>1$of the u axis which are maintained at the indicated temperatures where T is a constant (Figure 3(a)).

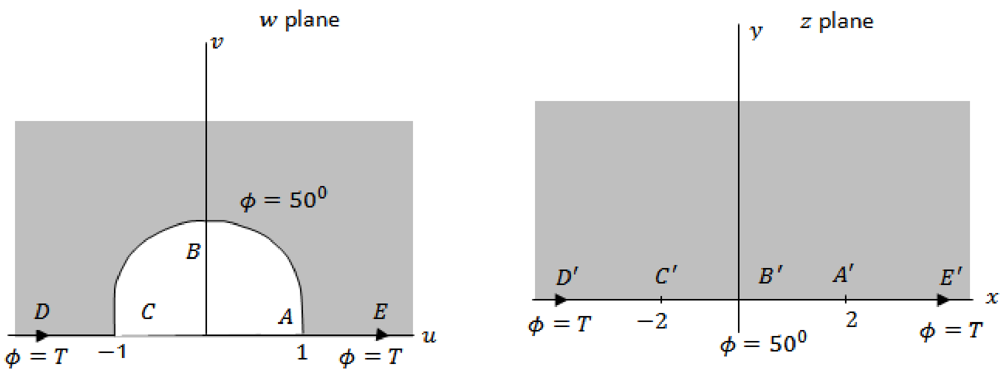

Figure 3(a):One-to-one Mapping of the DomainIm$w>0$and Exterior to the Unit circle $|w| = 1$ Onto the Upper Half of the z plane

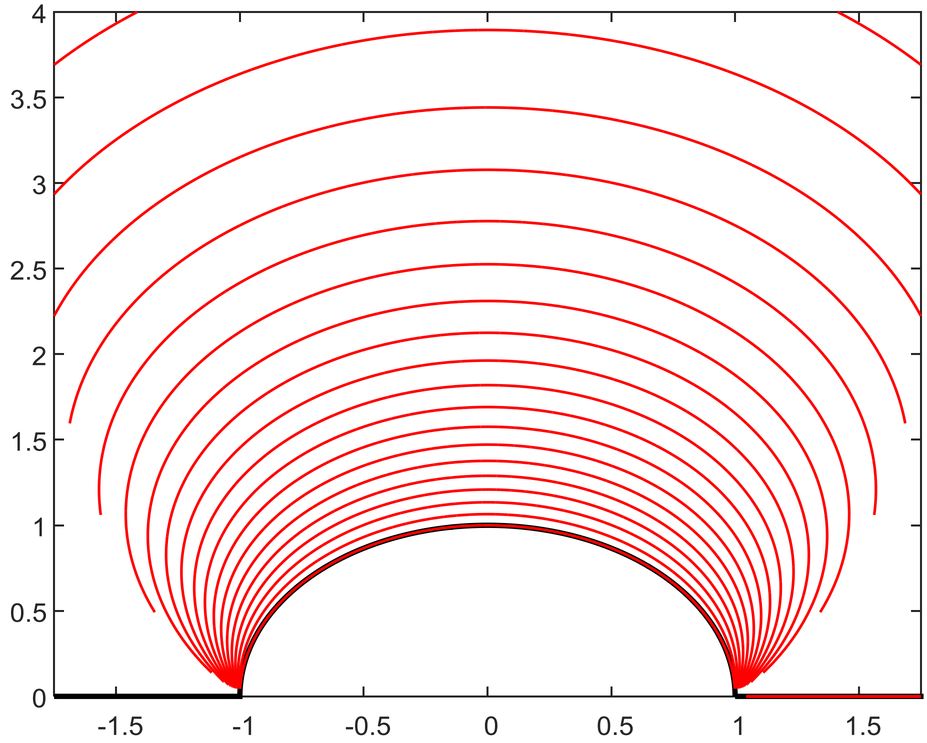

Figure 3(b):Isothermal Lines in the Domain Imw > 0 Exterior to the Unit circle $|w| = 1$

Here the mapping function is the Joukowski transformation and it maps the half circle$w = e^{i\theta}$ $(0 \leq \theta \leq \pi)$ (and its corresponding temperature of 50^0C) in a one-to-one manner onto the line segment $-2 \leq x \leq 2$, the parts $u < -1$ and $u > 1$ of the u axis (and their corresponding temperature T) in a one-to-one manner onto the portions $x < -2$ and$x > 2$ of the x axis, respectively. It also maps the entire domain abovethe semi-circular arc in a one-to-one manner onto the entire upper half of the$z$ plane (Spiegel (1974), Churchill and Brown (1984)). The Joukowski transformation therefore simplifies the given problem to one in the upper half Im$z > 0$ of the z plane subject to the boundary conditions:$\Phi = T$when$x > 2$($i.e\theta_1 = \theta_2 = 0$) , $\Phi = 50^0C$when $-1 < x < 1$(i. e $\theta_1 = \pi,\ \theta_2 = 0$), and $\Phi = T$when$x < -2$ (i. e $\theta_1 = \theta_2 = \pi$).

Clearly, the function

$\phi = \left(\frac{50-T}{\pi}\right)\theta_1 - \left(\frac{50-T}{\pi}\right)\theta_2 + T$(17)

is harmonic in the upper half Im$z > 0$ of the z plane since it is the imaginary part of the analytic function

$$w = \left(\frac{50-T}{\pi}\right)\ln(z-1) - \left(\frac{50-T}{\pi}\right)\ln(z+1) + T$$

Hence

$\tan\left[\left(\frac{\phi-T}{50-T}\right)\pi\right] = \tan(\theta_1 - \theta_2) = \frac{4y}{x^2+y^2-4}$(18)

is the solution of the problem in the z plane where

$\tan\theta_1 = \frac{y}{x-2}$ and $\tan\theta_2 = \frac{y}{x+2}$

The required solution in the w plane is obtained by substituting

$$x = u + \frac{u}{u^2+v^2} \text{ and} y = v - \frac{v}{u^2+v^2}$$

from equation (8) with$c = 1$into equation (18) and simplifying to have

$\tan\left[\left(\frac{\phi-T}{50-T}\right)\pi\right] = \frac{4v(u^2+v^2-1)}{(u^2+v^2)(u^2+v^2-4)+2(u^2-v^2)+1} = \frac{4\rho\sin\sigma(\rho^2-1)}{\rho^2(\rho^2-4)+2\rho^2\cos 2\sigma+1}$ (19)

where

$$u = \rho\cos\sigma \text{ and} v = \rho\sin\sigma$$

Problem 3: (Steady State Temperature in a Domain Exterior to the Circle $w = e^{i\theta}$ $(0 \leq \theta < 2\pi)$) With Constant Temperatures of $80^0C$and 20^0C on its Arcs in the Upper and Lower halves of the z planes

Finally, we now consider the harmonic Dirichlet problem in equation (3) for the determination of the steady state temperature at any point of an infinite conducting plate that has in it a circular hole $ABCD$ of unit radius, with temperatures of $20^0$and 80^0 applied to arc ABCandADC and maintained indefinitely (Figure 4(a)).First, we know from the table of transformations of regions given by Spiegel (1974) that the transformation (9) maps nonzero points in the w plane which are exterior to the unit circle$|w| = 1$ onto points interior to it and conversely, followed by a reflection in the u axis. Also each point of the unit circle map into itself and is then reflected in the u axis following the operation of complex conjugation. Consequently, the arc ABC with its corresponding temperature of 20^0 is mapped on to the arc $A'B'C'$, while the arc ADC with its corresponding temperature of 80^0 is mapped onto the arc $A'D'C'$ as shown in the second diagram of figure 4(a). Also the special bilinear transformation of equation (10) maps the interior of the unit circle $|\zeta| = 1$ onto the upper half $\text{Im}z > 0$ of the z plane. It also maps the arcs $A'B'C'$ and $C'D'A'$ onto the negative and positive real axis $A''B''C''$ and $C''D''A''$ of the z plane.

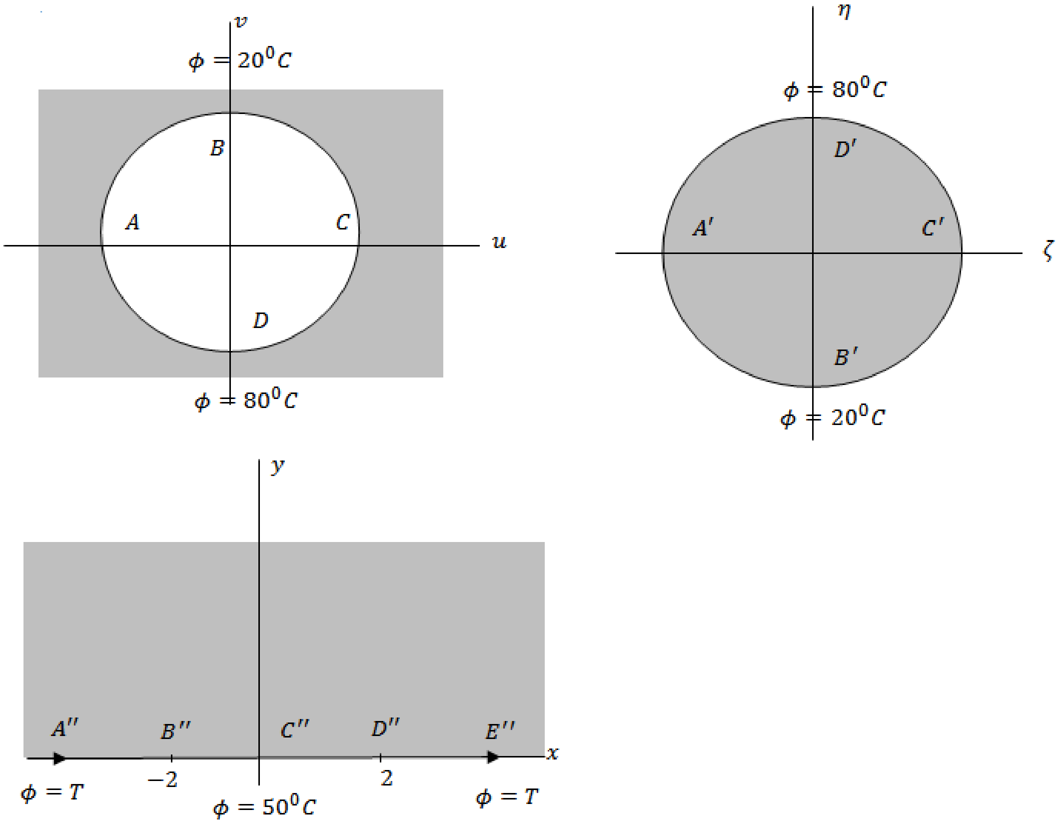

Figure 4(a):One-to-One Mapping of the Domain Exterior to the Circle $w = e^{i\theta}$ $(0 \leq \theta < 2\pi)$) With Constant Temperatures of $80^0C$and 20^0C on its Arcs in the Upper and Lower halves of the w plane onto the ζ plane and Finally z plane

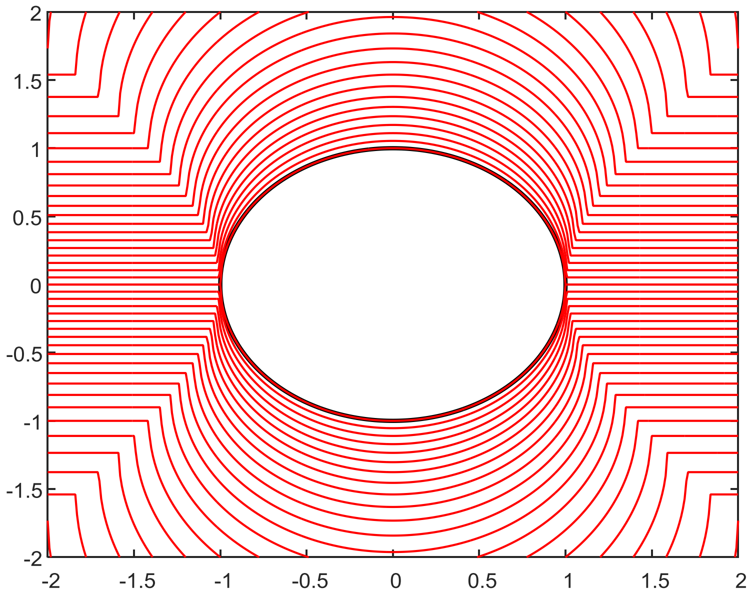

Figure 4(b):Isothermal Lines of Steady State Heat Flow in the Domain Exterior to the Circle$w = e^{i\theta}$ $(0 \le \theta < 2\pi)$) With Constant Temperatures of $80^0C$and 20^0C on its Arcs in the Upper and Lower halves of the w planes

Now, the function

$\phi = -\frac{60}{\pi}\theta + 80$ (20)

is harmonic in the upper half of the z plane since it is the imaginary part of the analytic function

$$w = -\frac{60}{\pi}\ln z + 80$$

From equation (20)

$\tan\left[\left(\frac{80-\phi}{60}\right)\pi\right] = \tan\theta = \frac{y}{x}$ (21)

is the solution of the problem in the z plane. Since

$$x = \frac{2\eta}{(1+\xi)^2+\eta^2} \text{ and } y = \frac{1-\xi^2-\eta^2}{(1+\xi)^2+\eta^2}$$

from equation (10), we have the solution in the ζ plane as

$\tan\left[\left(\frac{80-\phi}{60}\right)\pi\right] = \frac{1-\xi^2-\eta^2}{2\eta}$(22)

The required solution in the w plane is obtained by substituting

$$\xi = \frac{u}{u^2+v^2} \text{ and} \eta = -\frac{v}{u^2+v^2}$$

from equation (9) into equation (22) to have

$$\tan\left[\left(\frac{80-\phi}{60}\right)\pi\right] = -\left(\frac{u^2+v^2-1}{2v}\right)$$

or

$\phi = 80 - \frac{60}{\pi}\tan^{-1}\left[-\left(\frac{u^2+v^2-1}{2v}\right)\right]$ (23)

If $w = \rho e^{i\sigma}$ then the solution in (23) takes the polar form

$\phi = 80 - \frac{60}{\pi}\tan^{-1}\left[-\left(\frac{\rho^2-1}{2\rho\sin\sigma}\right)\right]$ (24)

III. Discussion

Steady State Temperature in Semi-Infinite Strip of Width a

In analysing the steady state heat flow through the semi-infinite strip of problem 1, we first note that the steady state temperature function $\phi(u, v)$ in equation (16) is indeed its solution since it satisfies Laplace's equation and the boundary conditions there. Figure 2(b) show MATLAB plots of isothermal lines generated in thesemi-infinite strip of width π units (that is with $a = \pi$ in the problem) by fixing the value of $T = 10\pi$ degrees centigrade, and setting$\phi = c$ where$c$ is varied from 0 to 4.5π degrees centigrade in steps of $\pi/2$.The isothermal lines are level curves of $\phi = c$ on which the temperature is constant on each line and varies from one line to another. More specifically, on the firstred line on the boundary $v = 0$ of the strip the steady state temperature $\phi = 0$ and increases in steps of $\pi/2$ on the subsequent lines.

Steady State Temperature in a domain above the semi-circle $w = e^{i\theta}$ $(0 \leq \theta \leq \pi)$ and parts$u < -1$ and$u > 1$ of the u axis).

In this problem too the steady state temperature$\phi(u, v)$ is also harmonic in the indicated domain and satisfies the boundary conditions. Figure 3(b) show isothermal lines generated by fixing the boundary temperature $T = 0^0$ centigrades and setting $\phi = c$ and varying c from 0 to 50. Observe from the isthermal lines that the temperature spreads out from the circular arc which is maintained at constant temperature of 50^0 into the solid. The first equipotential curve corresponding to the semi-circular arc has a constant temperature of 50^0 as expected, the second has a constant temperature of 48^0, and decreases in steps of 2. Observe that the temperature distribution agrees with reality since heat flows from a region of higher to that of cooler temperature.

Steady State Temperature in a Domain Exterior to the Circle $w = e^{i\theta}$ $(0 \leq \theta < 2\pi)$) With Constant Temperatures of $80^0C$and 20^0C on its Arcs in the Upper and Lower halves of the z planes

Here too the steady state temperature $\phi(u, v)$ is harmonic throughout the indicated domain and satisfy the prescribed conditionson the boundary. Figure 4(b) show isothermal lines exterior to the indicated domain in figure 4(b) and hence the temperature variation in the solid. As stated earlier, the steady state temperature on each of the line is constant but varies from one line to another.

IV. Conclusion

In this paper, a simple but efficient conformal mapping method in solving Dirichlet Harmonic problems in the theory of heat flow was presented and applied in the determination of the steady state temperature in domains of the z plane whose boundaries were maintained at constant but different temperatures on its various parts. Although the method gives exact general analytic solutions to the problems considered it is not without limitations. One obvious limitation has to do with the ability of identifying the mapping function to use for a particular problem. Unfortunately, there is no systematic way of knowing this function but depends largely on ones experience and familiarity with the manner in which curves and regions are mapped by most analytic functions.Another limitation has to do with the fact that the method is conformal based and hence limited to problems which can be reduced to ones in two dimensions and have a high degree of symmetry. This technique is often difficult to apply when the symmetry is broken.

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