# Numerical Solution for Weather Forecasting Using Finite Difference Scheme.

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**Abstract:** In this Paper, some problems associated with numerical weather prediction are discussed. we have been able to simulate some finite difference schemes to predict weather trends of Abuja. By analyzing the results from these schemes, it has shown that the best scheme in the finite difference method that gives a close accurate weather forecast is the trapezoidal scheme when comparing with sunshine, Rainfall and windspeed. We use the trapezoidal scheme to stimulate the numerical weather data obtained from federal Airports Authority. Finally using Matlab (2012a) to acquire subsequent numerical tendency.

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## I. Introduction

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Numerical weather prediction uses mathematical models of the atmosphere and oceans to predict the weather based on current weather conditions. Post-processing techniques such as model output statistics have been developed to improve he handling of errors in numerical predictions. The possibility of forecasting the weather by numerical methods was recognized half a century ago. ((Richardson 1922). It is only the recent more years that the rapid development meteorology theory, finite difference methods and computer technology has advanced the subject to such a level that forecasts that forecasts can be confidently based on computer predictions.

Present day numerical methods give acceptable accurate forecast 2 or 3 days ahead and a general guide of the weather pattern for as long as 6days ahead. Early numerical models of the atmosphere were only capable of predicting the pressure pattern at a few levels in the vertical,however the advent of considerably more powerful computers will enable forecasts to be made in rainfall, cloud, convection showers, as well as wind and pressure in greater details than before. Weather forecasting is one of the most complex and remarkably problems of modern science. In spite of evident advancement in the few decades and shift from manual forecasting methods to numerical ones, there are some significant problems that are yet to be solved either by manual methods or methods based on computer simulation posing interesting challenges for all those engaged in the field. An interminably need for in depth information on the actual meteorological conditions and problems associated to the use of traditional methods are responsible from intensive development of numerical weather prediction (NWP).

However, the vast range of available finite difference scheme is both a blessing and a curse, and many different combinations have been proposed, analyzed and used for large scale geophysical fluid dynamics applications, particularly in the ocean modeling community Le Roux et al. (2005, 2007); Le Roux and Pouliot (2008); Danilov (2010); Cotter et al. (2009); Cotter and Ham (2011); Rostand and Le Roux (2008); Le Roux (2012); Comblen et al. (2010), whilst many other combinations have been used in engineering applications where different scales and modeling aspects are important. In geophysical applications, the stability properties of compatible finite difference have long been recognized, leading to various choices being proposed and analyzed on triangular meshes, Walters and Casulli (1998); Rostand and Le Roux (2008). However, no explicit use was made of the compatible structure beyond stability until Cotter and Shipton (2012) used it, which proved that all compatible finite difference methods have exactly steady geotropic modes; this is considered a crucial property for numerical weather prediction Staniforth and Thuburn (2012).

## II. Advection Equation

The advection equation is the partial differential equation that governs the motion of a conserved scalar field as it is advected by a known velocity vector field. It is derived using the scalar field's conservation law, together with Gauss's theorem, and taking infinitesimal limit

Let us consider a continuity equation for the one-dimensional drift of incompressible fluid. In the case that a particle density u(x,t) changes only due to convection processes one can write

 $\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = 0$ (1)where c is a constant. We divide the (x, t) -plane into a series of discrete points  $(i\Delta x, n\Delta t)$  and denote the approximate solution for u at this point by  $u_i^n$ . The possible finite-difference scheme for the equation is  $\frac{u_i^{n+1} - u_i^n}{\Delta t} + c \frac{u_i^n - u_{i-1}^n}{\Delta x} = 0$ We may rewrite (3) as (2) $u_i^{n+1} = (1-\mu)u_i^n + \mu u_{i-1}^n,$ (3)where  $\mu = c\Delta t / \Delta x$ . The advection equation Eq. (1) has a possible finite-difference scheme given by Eq. (2) and hence an analytic solution of the advection equation in the form of a single harmonic is  $u(x,t) = Re[U(t)e^{ikx}]$ (4) Here U(t) is the wave amplitude and k the wavenumber. Substituting this result into Eq. (2) gives  $\frac{dU}{dt} + ikcU = 0,$ (5)which has the solution  $U(t) = U(0)e^{-ikct}.$ (6)U(0) which is the initial amplitude. Hence  $u(x,t) = Re[U(0)e^{-ik(x-ct)}]$ (7)as expected. The solution is finally expressed in Eq. (8). However, in the von Neumann method we looked for an analogous solution of the finite-difference equation Eq. (4) which after substituting  $u_i^n = Re[U^{(n)}e^{ikj\Delta x}]$ , this reduces the entire scheme to the amplitude equation;  $U^{(n+1)} = \lambda U^{(n)}$ (8) which properly defines the amplification factor  $|\lambda|$  and hence we can now study the behavior of the amplitude  $U^{(n)}$  as n increases, the stability of the scheme and the frequency of the stability is given by;  $p = \omega \Delta t$ (9)  $\Delta t \leq \frac{1}{|\omega|}$ (10)where p is the stability of the scheme,  $\lambda$  is the wavelength  $\omega$  is the frequency and  $\Delta t$  the time interval and  $\omega = 1, 2, ..., n.$ For Euler Scheme  $\lambda = 1 + ip, \quad |\lambda| = (1 + p^2)^{\frac{1}{2}}.$ (11)at p = 1, we have  $\lambda = 1 + i$ This scheme is unstable  $|\lambda| > 1$  for any p > 0For Backward Scheme  $\lambda = \frac{\left(1 + \frac{1}{4}ip\right)}{(1 + p^2)}, \quad |\lambda| = (1 + p^2)^{-\frac{1}{2}}$ (12)at p = 1, we have  $\lambda = 0.5 + 0.125i$ This scheme is stable For Trapezoidal Scheme  $\lambda = \frac{\left(1 + \frac{1}{4}p^2 + ip\right)}{\left(1 + \frac{1}{4}p^2\right)},$  $|\lambda| = 1.$ (13)at p = 1, we have  $\lambda = 1 + i/1.25$ This scheme is always neutral. For Matsuno Scheme  $\lambda = 1 - p^2 + ip$ ,  $|\lambda| = (1 - p^2 + p^4)^{\frac{1}{2}}$  (14) at p = 1, we have  $\lambda = i$ This scheme is stable, if  $|p| \leq 1$ . For Heun Scheme  $\lambda = 1 - \frac{1}{2}p^2 + ip, \ |\lambda| = \left(1 + \frac{1}{4}p^4\right)^{\frac{1}{2}}.$ (15)at p = 1, we have  $\lambda = 0.5 + i$ This is always > 1 so that the Heun scheme is always unstable. However, we select the real part minus the product of the imaginary part of the deduced wavelength with itself for the resultant solution of

as  

$$U^{(n+1)} = \lambda U^{(n)}$$

$$U^{(n+1)} = Re[\lambda U^{(n)}].$$
(16)

#### III. Numerical Solutions Summary of Weather Data Set From Federal Airport Authority of Nigeria, Abuja Station

Table 1: Dataset from the Federal Airport Authority of Nigeria for Abuja Station

Annual	Climato	logical Su	ımmary		Year: 2016									
Station	: ABUJA,	NG				E	Elev: 343.1ft. Lat: 09.15°N Lon: 07.00°E							
STATI ON NUMB ER	STATI ON NAME	ELEV	LAT	LONG	DATE	RelH um	TM AX	TM IN	RAINF ALL	SUNS HINE HRS	WIND SPEED	WIND DIREC TION		
65125	Abuja	343.1	09.15'N	07.00'E	201601	43	35.5	19.3	0	7.3	2.9	N		
65125	Abuja	343.1	09.15'N	07.00'E	201602	50	37.4	23.2	0.6	7.5	3.7	NE		
65125	Abuja	343.1	09.15'N	07.00'E	201603	62	37.7	25	7.5	8.2	3.5	NE		
65125	Abuja	343.1	09.15'N	07.00'E	201604	62	36.6	25.7	74.2	7.5	5	Е		
65125	Abuja	343.1	09.15'N	07.00'E	201605	76	35.8	24.6	109.2	7.4	4.9	SW		
65125	Abuja	343.1	09.15'N	07.00'E	201606	81	30.2	23.2	267.2	7.5	4.7	S		
65125	Abuja	343.1	09.15'N	07.00'E	201607	86	28.7	22.3	314.8	4.5	3.7	SW		
65125	Abuja	343.1	09.15'N	07.00'E	201608	87	28.7	22.5	278.3	5.2	4.2	NW		
65125	Abuja	343.1	09.15'N	07.00'E	201609	83	29.5	22.2	258.4	5.2	4.1	W		
65125	Abuja	343.1	09.15'N	07.00'E	201610	78	30	21.8	238.2	6.8	3.3	NW		
65125	Abuja	343.1	09.15'N	07.00'E	201611	64	33.7	21.6	Trace	9.2	3	E		
65125	Abuja	343.1	09.15'N	07.00'E	201612	36	35	17.2	0	8.8	3.2	NE		

Source: FAAN

#### Solution of Sunshine Hours Prediction Using Finite Difference Scheme

Using Eq. (16) and sunshine hours value from Table 1 for the first month, we compute the predicted values for the different schemes.

$$U^{(n+1)} = Re[\lambda U^{(n)}]$$

For Heun Scheme where  $\lambda = 0.5 + i$  and  $U^{(n)} = 7.3$  for sunshine hours then

$$U^{(n+1)} = Re[7.3(0.5+i)] = Re[3.65+7.3i] = 3.65 - 1 = 2.65$$

For Matsuno Scheme where  $\lambda = i$  and  $U^{(n)} = 7.3$  for sunshine hours

$$U^{(n+1)} = Re[7.3(i)] = Re[7.3i] = 0$$

For Trapezoidal Scheme where  $\lambda = 1 + i/1.25$  and  $U^{(n)} = 7.3$  for sunshine hours then

 $U^{(n+1)} = Re[7.3(1 + i/1.25)] = 7.66 - 1 = 6.66$ 

For Backward Scheme where  $\lambda = 0.5 + 0.125i$  and  $U^{(n)} = 7.3$  for sunshine hours

then

$$U^{(n+1)} = Re[7.3(0.5 + 0.125i)] = 4.6 - 1 = 3.6$$

For Euler Scheme where  $\lambda = 1 + i$  and  $U^{(n)} = 7.3$  for sunshine hours then

 $U^{(n+1)} = Re[7.3(1+i)] = 7.3 - 1 = 6.3$ 

The results of predicted sunshine hours for all the 12 months of the year are shown in Table 2

Months	Wavelength $\lambda$					Amplitude U <sup>(n)</sup>	Schemes U <sup>(n+1)</sup> (SHRs 2016)				
ω	Heun	Matsuno	Trapezoidal	Backward	Euler	Sunshine 2015	Heun	Matsuno	Trapezoidal	Backward	Euler
1	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	7.3	2.65	0	6.66	3.6	6.3
2	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	7.5	2.75	0	6.86	3.7	6.5
3	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	8.2	3.1	0	7.56	4.08	7.2
4	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	7.5	2.75	0	6.86	3.7	6.5
5	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	7.4	2.7	0	6.76	3.68	6.4
6	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	7.5	2.75	0	6.86	3.7	6.5
7	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	4.5	1.25	0	3.86	2.2	3.5
8	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	5.2	1.6	0	4.56	2.58	4.2
9	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	5.2	1.6	0	4.56	2.58	4.2
10	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	6.8	2.4	0	6.16	3.38	5.8
11	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	9.2	3.6	0	8.56	4.58	8.2
12	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1+i	8.8	3.4	0	8.16	4.38	7.8

Table 2:	Sunshine	Hours 2	2017	(SHRs 201	9)

From the above table, the selection of the scheme to represent the model forecasting for the sunshine hours for 2017 is based on the trend of the scheme whose result is closest to the previous year 2016 and hence among all five schemes in the table it is very obvious that aside Euler's (Forward) Scheme which is the second closest, the Trapezoidal Scheme is the closest to the given sunshine hours in 2018. Hence we use the Sunshine Hours predicted using the Trapezoidal Scheme.



Figure 1: A Comparative Chart Showing the Sunshine Hours Deduced by Various Schemes in one year

Observing our choice Trapezoidal Scheme from Figure 1 above it is obviously showing that the sunshine hours between January and May will be relatively high and will begin to decrease from June and start to rise again around September and falls again in December.

### Solution of Wind Speed Prediction Using Finite Difference Scheme

Using equation (16) and wind speed value from Table 1 for the third month, we compute the predicted values for the different schemes.

 $U^{(n+1)} = Re[\lambda U^{(n)}]$ For Heun Scheme where  $\lambda = 0.5 + i$  and  $U^{(n)} = 3.5$  for wind speed then  $U^{(n+1)} = Re[3.5(0.5+i)] = Re[1.75+3.5i] = 1.75-1 = 0.75$ For Matsuno Scheme where  $\lambda = i$  and  $U^{(n)} = 3.5$  for wind speed then  $U^{(n+1)} = Re[3.5(i)] = Re[3.5i] = 0$ For Trapezoidal Scheme where  $\lambda = 1 + i/1.25$  and  $U^{(n)} = 3.5$  for wind speed then  $U^{(n+1)} = Re[3.5(1 + i/1.25)] = 3.86 - 1 = 2.86$ For Backward Scheme where  $\lambda = 0.5 + 0.125i$  and  $U^{(n)} = 3.5$  for wind speed then  $U^{(n+1)} = Re[3.5(0.5 + 0.125i)] = 2.7 - 1 = 1.7$ For Euler Scheme where  $\lambda = 1 + i$  and  $U^{(n)} = 7.3$  for wind speed then  $U^{(n+1)} = Re[3.5(1+i)] = 3.5 - 1 = 2.5$ 

The results of predicted wind speed for all the 12 months of the year are shown in Table 3

Table 3: Wind Speed 2017 (WS 2017)

Months	Waveleng	th λ				Amplitude U <sup>(n)</sup>	Schemes U <sup>(n+1)</sup> (WS 2016)				
ω	Heun	Matsuno	Trapezoidal	Backward	Euler	Wind Speed'15	Heun	Matsuno	Trapezoidal	Backward	Euler
1	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1+i	2.9	0.45	0	2.26	1.4	1.9
2	0.5 + i	i	1 + i/1.25	0.5 + 0.125 <i>i</i>	1+i	3.7	0.85	0	3.06	1.8	2.7
3	0.5 + i	i	1 + i/1.25	0.5 + 0.125 <i>i</i>	1+i	3.5	0.75	0	2.86	1.7	2.5
4	0.5 + i	i	1 + i/1.25	0.5 + 0.125 <i>i</i>	1+i	5	1.5	0	4.36	2.48	4
5	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1+i	4.9	1.45	0	4.26	2.4	3.9
6	0.5 + i	i	1 + i/1.25	0.5 + 0.125 <i>i</i>	1+i	4.7	1.35	0	4.06	2.3	3.7
7	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1+i	3.7	0.85	0	3.06	1.8	2.7
8	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	4.2	1.1	0	3.56	2.08	3.2
9	0.5 + i	i	1 + i/1.25	0.5 + 0.125 <i>i</i>	1+i	4.1	1.05	0	3.46	2	3.1
10	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1+i	3.3	0.65	0	2.66	1.49	2.3
11	0.5 + i	i	1 + i/1.25	0.5 + 0.125 <i>i</i>	1+i	3	0.5	0	2.36	1.48	2
12	0.5 + i	i	1 + i/1.25	0.5 + 0.125 <i>i</i>	1+i	3.2	0.6	0	2.56	1.58	2.2

From the above table, the selection of the scheme to represent the model forecasting for the Wind Speed for 2017 is based on the trend of the scheme whose result is closest to the previous year i.e. 2016 and hence among all five schemes in the table it is very obvious that aside Euler's (Forward) Scheme which is the second most closest, the Trapezoidal Scheme is the most closest to the given Wind Speed in 2016. Hence we use the Wind Speed predicted using the Trapezoidal Scheme.



Figure 2: A Comparative Chart Showing the Wind Speed Deduced by Various Schemes in one year

Observing our choice Trapezoidal Scheme from Figure 2 above it is obviously showing that the wind speed will increase from January to May and will begin to decrease from June and start to rise again around September and falls again in November.

#### Solution of Rainfall Prediction Using Finite Difference Scheme

Using equation (17) and rainfall value from Table 1 for the second month, we compute the predicted values for the different schemes.

$$U^{(n+1)} = Re[\lambda U^{(n)}]$$

For Heun Scheme where  $\lambda = 0.5 + i$  and  $U^{(n)} = 0.6$  for rainfall then  $U^{(n+1)} = Re[0.6(0.5 + i)] = Re[0.3 + 0.6i] = 0.3 - 1 = -0.7$ For Matsuno Scheme where  $\lambda = i$  and  $U^{(n)} = 0.6$  for rainfall then  $U^{(n+1)} = Re[0.6(i)] = Re[0.6i] = 0$ For Trapezoidal Scheme here  $\lambda = 1 + i (1.25 \text{ mod } U^{(n)}) = 0.6$  for each of U

where  $\lambda = 1 + i/1.25$  and  $U^{(n)} = 0.6$  for rainfall then

$$U^{(n+1)} = Re[0.6(1 + i/1.25)] = 0.36 - 0.8 = -0.04$$

For Backward Scheme where  $\lambda = 0.5 + 0.125i$  and  $U^{(n)} = 0.6$  for rainfall then  $U^{(n+1)} = Re[0.6(0.5 + 0.125i)] = 0.3 - 0.015628 = 0.28$ For Euler Scheme where  $\lambda = 1 + i$  and  $U^{(n)} = 0.6$  for rainfall then

 $U^{(n+1)} = Re[0.6(1+i)] = 0.6 - 1 = -0.4$ 

The results of predicted rainfall for all the 12 months of the year are shown in Table 4

Months	Waveleng	gth λ			Amplitude $U^{(n)}$ Schemes $U^{(n+1)}$ (RF 2016)						
ω	Heun	Matsuno	Trapezoidal	Backward	Euler	Rain Fall '15	Heun	Matsuno	Trapezoidal	Backward	Euler
1	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	0	-1	0	-0.64	-0.02	-1
2	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	0.6	-0.7	0	-0.04	0.28	-0.4
3	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	7.5	2.75	0	6.86	3.7	6.5
4	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	74.2	36.1	0	73.56	37	73.2
5	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	109.2	53.6	0	108.56	54.58	108.2
6	0.5 + i	i	1 + i/1.25	0.5 + 0.125 <i>i</i>	1 + i	267.2	132.6	0	266.56	133.58	266.2
7	0.5 + i	i	1 + i/1.25	0.5 + 0.125i	1 + i	314.8	156.4	0	314.16	157.38	313.8
8	0.5 + i	i	1 + i/1.25	0.5 + 0.125 <i>i</i>	1 + i	278.3	138.15	0	277.66	139	277.2
9	0.5 + i	i	1 + i/1.25	0.5 + 0.125 <i>i</i>	1 + i	258.4	128.2	0	257.76	129	257.4
10	0.5 + i	i	1 + i/1.25	0.5 + 0.125 <i>i</i>	1 + i	238.2	118.1	0	237.56	119	237.2
11	0.5 + i	i	1 + i/1.25	0.5 + 0.125 <i>i</i>	1 + i	Trace	Trace	Trace	Trace	Trace	Trace
12	0.5 + i	i	1 + i/1.25	0.5 + 0.125 <i>i</i>	1 + i	0	-1	0	-0.64	-0.02	-1

Table	4.RainFall	2017	(RF 2016)	١
Lanc	T. Kann an	2017	$(\mathbf{R}^{-2} - 2010)$	,

From the above table, the selection of the scheme to represent the model forecasting for the Rain Fall for 2017 is based on the trend of the scheme whose result is closest to the previous year 2016 and hence among all five schemes in the table it is very obvious that aside Euler's (Forward) Scheme which is the second most closest, the Trapezoidal Scheme is the most closest to the given Rain Fall in 2016. Hence we use the Rain Fall predicted using the Trapezoidal Scheme.



Figure 3: A Comparative Chart Showing the Rain Fall Deduced by Various Schemes in one year

Observing our choice Trapezoidal Scheme from Figure 3 above it is obviously showing that the rain fall will start around late February and be very high in June till around October then will begin to reduce and dry season will set in from November.

		ee ee eomp			• tron	10						
Station: ABUJA, NG						Elev: 343.1ft. Lat: 09.15°N Lon: 07.00°E						
	STATION	STATION	STATION ELEV		LONG	DATE SUNSHINE		RAINFALL	WIND	WD		
	NUMBER	NAME					HRS		SPEED			
	65125	Abuja	343.1	09.15'N	09.24'W	201601	-0.64	6.66	2.26	NE		
	65125	Abuja	343.1	09.15'N	09.24'W	201602	-0.04	6.86	3.06	Ν		
	65125	Abuja	343.1	09.15'	09.24'W	201603	6.86	7.56	2.86	NW		
				Ν								
	65125	Abuja	343.1	09.15'N	09.24'W	201604	73.56	6.86	4.36	NE		
	65125	Abuja	343.1	09.15'N	09.24'W	201605	108.56	6.76	4.26	NE		
	65125	Abuja	343.1	09.15'N	09.24'W	201606	266.56	6.86	4.06	Ν		
	65125	Abuja	343.1	09.15'N	09.24'W	201607	314.16	3.86	3.06	NE		
	65125	Abuja	343.1	09.15'N	09.24'W	201608	277.66	4.56	3.56	NW		
	65125	Abuja	343.1	09.15'N	09.24'W	201609	257.76	4.56	3.46	W		
	65125	Abuja	343.1	09.15'N	09.24'W	201610	237.56	6.16	2.66	Е		
	65125	Abuja	343.1	09.15'N	09.24'W	201611	Trace	8.56	2.36	W		
	65125	Abuja	343.1	09.15'N	09.24'W	201612	-0.64	8.16	2.56	NE		

Summary of Predicted Weather Data Set From Compatible Finite Difference Scheme Table 5: Compatible FDM Numerical Weather Prediction Year: 2017

**Table 5** Shows the values of the predicted weather data values obtained by using the trapezoidal scheme. This compared favorably with the real weather data values collected from Federal Airport Authority of Nigeria (FAAN) Abuja Station shown on Table 1.

#### **IV. Conclusion**

Weather prediction for windspeed, sunshine and rainfall in a particular station are mostly accurate in the advent of recursive use of previous predictions or measurement. This research has unveiled that studying the weather trends helps in predicting future weather attenuation using numerical solutions deduced by finite difference method. The finite difference method has been used to deduce compatible models for automated attenuation of various parameters involved in the weather formation with the use of MATLAB in predicting future weather trends. The derivation of the models based on the finite difference method gives a high level of significance. In conclusion, the weather prediction for a station (i.e. Abuja, Nigeria) was flexibly obtained accurately prior to the use of previous determined or forecasted data and a compatible C-grid staggered finite difference method.

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