Restriction of Soft Sets and Some of its Real Life Applications

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Abstract
In this search we define the restriction of soft set in two methods, first method, the restriction of soft set \((F,E)\) on \(A \subseteq E\) (\(E\) is the set of parameters), second method, the restriction of soft set \((F,E)\) with respect to \(A \subseteq X\) \((X\) is the universal set), introduce some practical examples. Then, we introduce and study restriction of soft mappings.

Key words: restricted soft set, soft topology, image processing, restriction of soft mappings.

Date of Submission: 18-07-2020
Date of Acceptance: 03-08-2020

I. INTRODUCTION


In this search we will define the restriction of soft set, give an examples in restriction of a matrix of pixels. Then, introduce the restriction mappings over a soft set, study the restriction of mapping on soft continuous, soft open, soft close maps, we would like to mention that all soft sets in this search are on the same set of parameters \(E\), denoted by \(S(X)_E\), through this search simply \((A,E)\) equals to \(\overline{A}\), \((F,E)\) equals to \(\overline{F}\).

Definition 1.1,[5],[7]
1. The soft set \((F,E)\) over \(X\) is said to be a (null soft set) denoted by \(\overline{F}\), if \(\forall e \in E \, F(e) = \Phi\) (\(\Phi\) the null set).
2. The soft set \((F,E)\) over \(X\) is said to be an (absolute soft set) denoted by \(\overline{X}\) if \(\forall e \in E \, F(e) = X\).
3. Let \((F,A),(G,B)\) be two soft sets in \(S(X)_E\). we say that \((F,A)\) is a soft subset of \((G,B)\) denoted \((F,A) \subseteq (G,B)\) if (i) \(A \subseteq B\) and (ii) \(F(e) \subseteq G(e) \forall e \in A\).
4. The soft difference \((H,E)\) of two soft sets \((F,E)\) and \((G,E)\) over \(X\) denoted by \((F,E) \setminus (G,E)\) is defined as \(H(e) = F(e) \setminus G(e)\) for all \(e \in E\).
5. The union of two soft sets of \((F,A)\) and \((G,B)\) over the common universe \(X\) is the soft set \((H,C)\), where \(C = A \cup B\), and \(\forall \, e \in C\) we write \((F,A) \cup (G,B) = (H,C)\).

\[
H(e) = \begin{cases} 
F(e) & \text{if } e \in A \setminus B \\
G(e) & \text{if } e \in B \setminus A \\
F(e) \cup G(e) & \text{if } e \in A \cap B
\end{cases}
\]

6. The intersection of two soft sets \((F,A)\) and \((G,B)\) over a common universe set \(X\) is the soft set \((H,C)\), where \(C = A \cap B\) and \(\forall \, e \in C\), \(H(e) = F(e) \cap G(e)\), we write \((F,A) \cap (G,B) = (H,C)\).

Definition 1.2,[10]
Let \(\mathcal{T}\) be the collection of soft sets over \(X\) then \(\mathcal{T}\) is soft topology on \(X\) if:
1. \(\Phi\), \(X\) belong to \(\mathcal{T}\).
2. The union of any number of soft sets in \(T\) belongs to \(\mathcal{T}\).
3. The intersection of any two soft sets in \(T\) belongs to \(\mathcal{T}\).
The triple \((X,\mathcal{T},E)\) is called a soft topological space over \(X\).
II. The restriction of soft set on A ⊂ E (E is the set of parameters)

Definition 2.1.
Let X the universal set , (F,E) is soft set where F:E→P(X) the restriction of (F,E) with respect to the set of parameters A ⊂ E is (G,E)where G:E\A→P(X), F(e) = G(e) ∀ e ∈ A.

Example 2.2.
Let X = {Mathematics , Physics , Geology} represent the scientific information's that each book b1, b2 and b3 consist , E the set of parameters represent the set of three books = { b1, b2, b3 } , the soft set (F,E) represent the scientific information's that each book consist 
(F,E) = { (b1, {Mathematics}), (b2, {Mathematics , Physics}), (b3, {X}) } 
Means that the book1 contains Mathematics only , b2 contains Mathematics and physics , b3 contains all Mathematics , Physics and Geology.
Let A = { b1, b2 } ⊂ E , G:E\A→P(X).Then, (G,A) = { (b1, {Mathematics}), (b2, {Mathematics , Physics})} is the restriction of (F,E) with respect to A which represent only two books b1, b2.

Application 2.3.
A Pixel is a sets of three numbers that together to represent a particular color.
In this example we will restrict an image on 3×3 matrix of Pixels by using the concept " restriction of soft sets "
Let X represent the three colors read , green and blue the degree of each color n equals 0 to 255,
X = {Rn, Gn, Bn} for some n , E the set of parameters represent the set of n-Pixels
E = { [p0, i, j = 1, 2, 3] } , the soft set (F,E) represent the image that contain 9 Pixels
(F,E) = { (p11, { R255, G0, B0 }), (p12, { R102, G102, B255 }), (p13, { R0, G204, B153 }), (p21, { R255, G51, B0 }), (p22, { R255, G0, B204 }), (p23, { R51, G102, B255 }), (p31, { R102, G51, B0 }), (p32, { R255, G51, B102 }), (p33, { R255, G51, B153 }) } .
Let A = { p11, p21, p31, p32 } ⊂ E , C:E\A→P(X).
Then, (C,A) = { (p11, { R255, G0, B0 }), (p21, { R102, G102, B255 }), (p31, { R102, G51, B0 }) }, (p32, { R255, G51, B102 }), (p33, { R255, G51, B153 }) is the restriction of (F,E) with respect to A.

Application 2.4.
Let X represent the three colors read , green and blue the degree of each color n equals 0 to 255,
X = {Rn, Gn, Bn} for some n , E the set of parameters represent the set of n-Pixels
E = { [p0, i, j = 1, 2, 3] } , the soft set (F,E) represent the image that contain 9 Pixels
(F,E) = { (p11, { R255, G0, B0 }), (p12, { R102, G102, B255 }), (p13, { R0, G204, B153 }), (p21, { R255, G51, B0 }), (p22, { R255, G0, B204 }), (p23, { R51, G102, B255 }), (p31, { R102, G51, B0 }), (p32, { R255, G51, B102 }), (p33, { R255, G51, B153 }) } .
Let B = { p11, p22, p31, p32, p33 } ⊂ E , C:E\B→P(X).
Then, (C,B) = { (p11, { R255, G51, B0 }), (p11, { R102, G51, B0 }) }, (p32, { R51, G102, B255 }), (p33, { R255, G51, B153 }) is the restriction of (F,E) with respect to B.

Figure 1. This figure show the restriction of 3×3 matrix of Pixels in to a of 2×2 matrix of Pixels.
Proposition 2.5.
Let X be the universal set, for a given soft set the restricted soft set (with respect to A ⊆ E) is a special case of soft subset.
Proof:
Let (F,E) is soft set where F:E→P(X), (G,A) is the restriction of (F,E) with respect to the set of parameters A ⊆ E where G:E\A→P(X), since (i) A ⊆ E and (ii) G(e) ⊆F(e) ∀ e ∈ A. Then, (G,A)⊆(F,E).

Proposition 2.6.
Let X be the universal set, the soft union of a soft set with restricted soft set (with respect to A ⊆ E) is a soft set.
Proof:
Let (F,E) is soft set where F:E→P(X), (G,A) is the restriction of (F,E) with respect to the set of parameters A ⊆ E where G:E\A→P(X), since e ∈ A∩E and F(e) ∪G(e) = F(e). Then, (F,E) ∪ (G,A) = (F,E).

Proposition 2.7.
Let X be the universal set, the soft intersection of soft set with the restricted soft set (with respect to A ⊆ E) is the restricted soft set.
Proof:
Let (F,E) is soft set where F:E→P(X), (G,A) is the restriction of (F,E) with respect to the set of parameters A ⊆ E where G:E\A→P(X), since F(e) = G(e) ∀ e ∈ A. Then, (F,E) ∩ (G,A) = (G,A).

III. The restriction of soft set with respect to A ⊆ X (X is the universal set)

Definition 3.1. Let X be a universal set let (F,E) be a soft set where F:E→P(X) the restriction of soft set (F,E) with respect to A ⊆ X is a soft set (G,E) where G : E→F(e) ∩A, ∀F(e) ∈ p(X) where G(e) = F(e) ∩A, ∀ e ∈ E.

Example 3.2. Let X = {a, b, c} be a set of three fonts, E the set of parameters represent the set of three words = {w₁, w₂, w₃} where (F,E) = {(w₁, {a, b, c}), (w₂, {a, c, b}), (w₃, {Φ, Φ, Φ})}
(F,E) is a statement contain three words with fonts a, b, c, arranged in different ways. So (G,E) = {(w₁, {a, b, c}), (w₂, {a, Φ, Φ}), (w₃, {Φ, Φ, Φ})} is a restricted soft set with respect to A.

Application 3.3. Let X = {x, y, z}, x, y, z are arbitrary three numbers belongs to the set of natural numbers, X represent the set of point in three dimension, E the set of parameters represent the set of three points in time = {t₁, t₂, t₃}
(F,E) = {(x₁, y₁, z₁), (x₂, y₂, z₂), (x₃, y₃, z₃)}
represent a particle in a specific three dimension space moves through a specific three points in time, Φ = {(t₁,0,0,0), (t₂,0,0,0), (t₃,0,0,0)} represents a particle in a three dimension space does not exist in the three discrete points of time,
X = {x₁, x₂, x₃} represent a particle in a three dimension space exist at three discrete points of time, T = {Φ, X, (F,E)} represent several probabilities of a particle in three dimension through three points in time. Let A = {x₁, y₁, 0} ⊆ X, for a special case (for numbers we replace the intersection with minimum) where x₄ = min {x₁, x₇}, y₄ = min {y₁, y₇}.
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Propositions 3.4.
Let X be the universal set, for a given soft set the restricted soft set (with respect to A ⊆ X) is a special case of soft subset.

Proof:
Let (F,E) is soft set where F:E→P(X), (G,E) is the restriction of (F,E) with respect to A ⊆ X where G : E → F(e) ∩ A, ∀ F(e) ∈ P(X), since (i) E ⊆ E and (ii) G(e) ⊆ F(e) ∀ e ∈ A. Thus, (G,A) ⊆ (F,E).

Propositions 3.5.
Let X be the universal set, the soft union of a soft set with restricted soft set (with respect to A ⊆ X) is a soft set.

Proof:
Let (F,E) is soft set where F:E→P(X), (G,E) is the restriction of (F,E) with respect to A ⊆ X where G : E → F(e) ∩ A, ∀ F(e) ∈ P(X), since E ∈ E and F(e) ∪ G(e) = F(e). Then, (F,E) ∪ (G,E) = (F,E).

Propositions 3.6.
Let X be the universal set, the soft intersection of soft set with the restricted soft set (with respect to A ⊆ X) is the restricted soft set.

Proof:
Let (F,E) is soft set where F:E→P(X), (G,E) is the restriction of (F,E) with respect to A ⊆ X where G : E → F(e) ∩ A, ∀ F(e) ∈ P(X), since G(e) = F(e) ∩ A = F(e) ∀ e ∈ A. Then, (G,E) ∩ (F,E) = (G,E), ∀ e ∈ E.

IV. Restriction of soft mappings

Definition 4.1. [11]
Let (F,E),(G,E) are soft sets in S(X)E. A soft relation  is called a soft mapping from (F,E) to (G,E) [denoted by (F,E)→ (G,E)] if the following two conditions are satisfied:
(i) for each soft element x ∈ (F,E); there exists only one soft element y ∈ (G,E) such that xRy which will be noted as  (x) = y.
(ii) for each empty soft element x ∈ (F,E), (x) is an empty soft element of (G,E).

Definition 4.2.[3]
Let (X,T,E) be a soft topological space over X, (F,E) be a non-empty soft subset of (X,T,E). Then, (F,E)* = (F,E) ∩ (G,E), ∀ (G,E) ∈ T is the soft relative topology on (F,E) and ((F,E), (G,E)) is called soft subspace topology of (X,T,E).

Definition 4.3.[3]
Let (X,T,E) be a soft mapping between two soft topological spaces if the image (F,E) of each soft open set (soft close set) (F,E) over X is a soft open set (soft close set) in Y. Then, (F,E) is said to be a soft open mapping (soft close mapping).

Theorem 4.4.
The restriction of soft open (soft close) map over a soft open (soft close) set is soft open (soft close) map.

Proof:
Let  : (X,T,E) → (Y,T,Y) be a soft mapping between two soft topological spaces, (A,E) be a soft open set in (X,T,E) to prove (A,E) ∈ (Y,T,Y). Then, (A,E) ∩ (G,E) ∈ T is soft open in T Y by soft relative topology definition, since (F,E) is soft open map then (F(E)) is soft open in T Y so prove (F(E)) is soft open map, similarly for close case.

Example 4.5.
Let $X = \{h_1, h_2\}$, $Y = \{k_1, k_2\}$, $E = \{e_1, e_2\}$, the soft map $\tilde{f} : (X, \tilde{T}_X, E) \Rightarrow (Y, \tilde{T}_Y, E)$ defined as follow:

$\tilde{f}(e_i, \{h_j\}) = (\tilde{e}_i, \{k_j\}), \forall i, j = 1, 2$ is soft open and soft closed map.

**Definition 4.6[11]**

$\tilde{f} : (X, \tilde{T}_X, E) \Rightarrow (Y, \tilde{T}_Y, E)$ is a soft continuous mapping if for each soft open set $(G, E)$ over $Y$, $f^{-1}((G, E))$ is a soft open set over $X$.

**Theorem 4.7.**

The restriction of soft continuous mapping over a soft open set (soft close set) is soft continuous.

**Proof:** Let $\tilde{f} : (X, \tilde{T}_X, E) \Rightarrow (Y, \tilde{T}_Y, E)$ be a soft continuous mapping between two soft topological spaces, $(A, E)$ be a soft open set in $(X, \tilde{T}_X, E)$, the restriction of $\tilde{f}$ on $(A, E)$ is the mapping

$\tilde{f}^{-1}((A, E), \tilde{T}_A, E) \Rightarrow (Y, \tilde{T}_Y, E)$, to prove

$\tilde{f}^{-1}((A, E), \tilde{T}_A, E)$ is soft continuous map, $(G, E)$ be a soft open set in $\tilde{T}_Y$.

Then, $\tilde{f}^{-1}((A, E), \tilde{T}_A, E)  \cap ((G, E)) = \tilde{f}^{-1}((G, E)) \cap (A, E)$ since $\tilde{f}$ is soft continuous then $\tilde{f}^{-1}((G, E))$ is soft open set in $\tilde{T}_A$

by soft relative topology definition $\tilde{f}^{-1}((G, E)) \cap (A, E)$ is soft open in $\tilde{T}_A$, so $\tilde{f}^{-1}((A, E), \tilde{T}_A, E)$ is soft continuous map, similarly for close case.

**Example 4.8.**

Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and

$\tilde{T} = \{(\tilde{X}, \{(e_1, \{h_2\}), (e_2, \{h_1\})\}, \{(e_1, \{h_2\}), (e_2, \{h_1\}), \{(e_1, \{h_1\}), (e_2, \{h_2\}))\}$

$\tilde{T} = \{(\tilde{X}, (e_1, \{h_2\}), (e_2, \{h_1\})\}, \{(e_1, \{h_2\}), (e_2, \{h_1\}), \{(e_1, \{h_1\}), (e_2, \{h_2\}))\}$

$\tilde{T} = \{(\tilde{X}, \{h_1\}, \{h_2\})\}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}$

$\tilde{f} : (X, \tilde{T}_X, E) \Rightarrow (X, \tilde{T}_Y, E)$ is a mapping defined as

$\tilde{f}(e_i, h_i) = (e_i, h_i)$, $i = 1, 2, j = 1, 2, 3$. Then, $\tilde{f}$ is a soft continuous.

Let $A = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$ be a soft subset of $\tilde{T}$. Then $\tilde{T}_A = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$ is a soft continuous.

**V. Conclusions**

Any soft set $(F, E)$ can be restricted in two ways:

First method - The restriction of soft set $(F, E)$ on $A \subseteq E$ (E is the set of parameters),

as example we restricted a matrix of pixels which can be used in image processing in computer or printers.

Second method - The restriction of soft set $(F, E)$ with respect to $A \subseteq X$ (X is the universal set)

as example suppose a soft set $(F, E)$ represent a particle in a specific three dimension space moves through a specific three points in time. Then, the restriction of $(F, E)$ with respect to a set $A \subseteq X$ will give the soft set $(G, E)$ which is a particle in two dimension moves through the same three points in time $t_1, t_2, t_3$.

For a soft map $\tilde{f} : (X, \tilde{T}_X, E) \Rightarrow (Y, \tilde{T}_Y, E)$ and $[(A, E), \tilde{T}_A, E]$ is subspace of $(X, \tilde{T}_X, E)\Rightarrow (Y, \tilde{T}_Y, E)$, the restriction of $\tilde{f}$ with respect to $(A, E)$ is the map:

$\tilde{g} : (A, E), \tilde{T}_A, E)$ as $(A, E), \tilde{T}_A, E)$ $\Rightarrow (Y, \tilde{T}_Y, E)$, we prove that the restriction of soft open map over a soft open set is soft open map, the restriction of soft close map over a soft close set is soft close map and the restriction of soft continuous mapping over a soft open set (or soft close set) is soft continuous.

DOI: 10.9790/5728-1604033383 www.iosrjournals.org 37 | Page
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