

The Lattice Structure of the Subgroups of Order 16in the Subgroup Lattices Of 3 X 3 Matrices Over Z_3

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Abstract

Let \mathcal{G} be the set of all 3×3 non-singular matrices $\begin{pmatrix} a & bc \\ d & ef \\ ghi \end{pmatrix}$, where a,b,c,d,e,f,g,h,i are integers

modulo p . Then \mathcal{G} is a group under matrix multiplication modulo p , of order $(p^n - 1)(p^n - p)(p^n - p^2) \dots (p^n - p^{n-1})$. Let G be the subgroup of \mathcal{G} defined by $G = \{abcdeghi \in \mathcal{G} : abcdeghi = 1\}$. Then G is of order $\frac{(p^n - 1)(p^n - p)(p^n - p^2) \dots (p^n - p^{n-1})}{p-1}$. Let $L(G)$ be the lattice formed by all subgroups G . In this paper, we give the structure of the subgroups of order 16 of $L(G)$ in the case when $P= 3$.

Keywords: Matrix group, subgroups,Lagrange's theorem,Lattice, Atom.

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I. Introduction

In 1992, Karan M. Gragg and P.S Kung [12] have attempted to characterize the finite groups with a consistent lattice of subgroups. In that endeavor, they discovered that the lattice of subnormal subgroups of a finite group is consistent and dually semi modular (lower semi modular). A. Vethamanickam has cited one of their theorems and has given a counter example in his thesis [19]. Suzuki's [13] results are mainly concerned with L-isomorphic groups. That is, groups whose lattice of subgroups are isomorphic.

In 2012, R. Sulaiman [18] has given the structure of the subgroup lattice of the symmetric group S_4 and Bashir Humera and Zahid Raza [2] have given the structure of the subgroup lattice of Quasidihedral group. In 2015, Jebaraj Thiraviam. D [6], in his thesis, has given the structure of the lattice of subgroups of the group of 2×2 matrices over Z_p having determinant value 1, under matrix multiplication modulo p , where p is prime and studied their properties.

Let $L(G)$ be the Lattice of Subgroups of G , where G is a group of 3×3 matrices over Z_p having determinant value 1 under matrix multiplication modulo p , where p is a prime number.

$$\text{Let } \mathcal{G} = \left\{ \begin{pmatrix} a & bc \\ d & ef \\ ghi \end{pmatrix} : a, b, c, d, e, f, g, h, i \in Z_p, \begin{vmatrix} a & bc \\ d & ef \\ g & hi \end{vmatrix} \neq 0 \right\}$$

Then \mathcal{G} is a group under matrix multiplication modulo p .

$$\text{Let } G = \left\{ \begin{pmatrix} a & bc \\ d & ef \\ ghi \end{pmatrix} \in \mathcal{G} : \begin{vmatrix} a & bc \\ d & ef \\ g & hi \end{vmatrix} = 1 \right\}$$

Then G is a subgroup of \mathcal{G} .

$$\text{we have, } o(\mathcal{G}) = (p^n - 1)(p^n - p)(p^n - p^2) \dots (p^n - p^{n-1})$$

$$\text{and } o(G) = \frac{(p^n - 1)(p^n - p)(p^n - p^2) \dots (p^n - p^{n-1})}{p-1}$$

In this paper, we give the structure of the subgroups of order 16 of $L(G)$ in the case when $P= 3$.

II. Preliminaries

In this section we give the definition needed for the development of the paper.

Definition 2.1

A partial order on a non-empty set P is a binary relation \leq on P that is reflexive, anti-symmetric and transitive. The pair (P, \leq) is called a **partially ordered set or poset**. A poset (P, \leq) is totally ordered if every $x, y \in P$ are comparable, that is either $x \leq y$ or $y \leq x$. A non-empty subset S of P is a chain in P if S is totally ordered by \leq .

Definition 2.2

Let (P, \leq) be a poset and let $S \subseteq P$. An upper bound of S is an element $x \in P$ for which $s \leq x$ for all $s \in S$.

The least upper bound of S is called the **supremum or join** of S . A lower bound for S is an element $x \in P$ for which $x \leq s$ for all $s \in S$. The greatest lower bound of S is called the **infimum or meet** of S .

Definition 2.3

Poset (P, \leq) is called a **lattice** if every pair x, y elements of P has a supremum and an infimum, which are denoted by $x \vee y$ and $x \wedge y$ respectively.

Definition 2.4

For two elements a and b in P , a is said to **cover** b or b is said to be covered by a (in notation, $a > b$ or $b < a$) if and only if $b < a$ and, for no $x \in P$, $b < x < a$.

Definition 2.5

An element $a \in P$ is called an **atom**, if $a > 0$ and it is a dual atom, if $a < 1$.

Theorem 2.6

If G is a finite group and H is a subgroup of G , then the order of H is a divisor of the order of G .

Theorem 2.7

If G is a finite group and $a \in G$, then the order of ' a ' is a divisor of the order of G .

Theorem 2.8

Let G be a finite group and let p be any prime number that divides the order of G . Then G contains an element of order p .

Theorem 2.9

If p is a prime number and $p^\alpha \mid o(G)$, $p^{\alpha+1} \nmid o(G)$, then G has a subgroup of order p^α , called a p -sylow subgroup.

Theorem 2.10

The number of p -sylow subgroups in G , for a given prime p , is of the form $1 + kp$.

III. Arrangement of elements of G according to their orders

The number of elements of order 2 is 117. The number of elements of order 3 is 728. The number of elements of order 4 is 702. The number of elements of order 6 is 936. The number of elements of order 8 is 1404. The number of elements of order 13 is 1728.

IV. Subgroups of G of different orders

The number of subgroups of order 2 is 117. The number of subgroups of order 3 is 364. The number of subgroups of order 4 is 351. The number of subgroups of order 6 is 468. The number of subgroups of order 8 is 468. The number of subgroups of order 9 is 117. The number of subgroups of order 13 is 144. The number of subgroups of order 16 is 351. The number of subgroups of order 27 is 52.

V. Lattice structure of some lower intervals of subgroups of order 16 in $L(G)$ over $Z3$

Let R be an arbitrary subgroup of order 16. Then the elements of U must have orders 1, 3 or 9.

We tabulate the subgroups of order 16 in $L(G)$

Table 5.1: Intervals $[\{e\}, R_i]$ in $L(G)$, $i = 1, 2, \dots, 351$.

| Order | Subgroups | Order | Subgroups | Order | Subgroups |
|-------|-----------------------------------|-------|-----------------------------------|-------|-----------------------------------|
| 16 | R_1 | 16 | R_2 | 16 | R_3 |
| 8 | N_1 | 8 | N_2 | 8 | N_3 |
| 4 | L_{62}, L_{285} | 4 | L_{18}, L_{285} | 4 | L_{18}, L_{266} |
| 2 | $H_{27}, H_{83}, H_{92}, H_{113}$ | 2 | $H_{25}, H_{29}, H_{82}, H_{107}$ | 2 | $H_{19}, H_{69}, H_{82}, H_{150}$ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R_4 | 16 | R_5 | 16 | R_6 |
| 8 | N_4 | 8 | N_5 | 8 | N_6 |
| 4 | L_{248}, L_{317} | 4 | L_{234}, L_{316} | 4 | L_{272}, L_{292} |
| 2 | $H_{35}, H_{81}, H_{92}, H_{111}$ | 2 | $H_{31}, H_{50}, H_{60}, H_{104}$ | 2 | $H_{13}, H_{46}, H_{63}, H_{92}$ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R_7 | 16 | R_8 | 16 | R_9 |
| 8 | N_7 | 8 | N_8 | 8 | N_9 |

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| | | | | | |
|--------------|--|--------------|---|--------------|--|
| 4 | L ₂₄ , L ₁₄₂ | 4 | L ₂₅₂ , L ₃₁₈ | 4 | L ₅₈ , L ₁₀₄ |
| 2 | H ₂₅ , H ₃₆ , H ₇₅ , H ₉₃ | 2 | H ₆ , H ₅₀ , H ₆₅ , H ₁₁₃ | 2 | H ₄₁ , H ₇₂ , H ₉₃ , H ₁₀₃ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₀ | 16 | R ₁₁ | 16 | R ₁₂ |
| 8 | N ₁₀ | 8 | N ₁₁ | 8 | N ₁₂ |
| 4 | L ₄₄ , L ₃₁₉ | 4 | L ₁₂₅ , L ₁₅₈ | 4 | L ₁ , L ₂₆₃ |
| 2 | H ₂₄ , H ₄₀ , H ₄₇ , H ₁₁₁ | 2 | H ₃₈ , H ₇₆ , H ₉₃ , H ₉₄ | 2 | H ₁₉ , H ₅₀ , H ₆₇ , H ₉₈ |

| Order | Subgroups | Order | Subgroups | Order | Subgroups |
|--------------|--|--------------|---|--------------|--|
| 16 | R ₁₃ | 16 | R ₁₄ | 16 | R ₁₅ |
| 8 | N ₁₃ | 8 | N ₁₄ | 8 | N ₁₅ |
| 4 | L ₆ , L ₈₇ | 4 | L ₁₂₃ , L ₂₅₇ | 4 | L ₁₀₉ , L ₃₁₉ |
| 2 | H ₇ , H ₃₈ , H ₇₀ , H ₉₉ | 2 | H ₁₉ , H ₄₇ , H ₆₈ , H ₉₆ | 2 | H ₇ , H ₁₈ , H ₂₃ , H ₅₅ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₆ | 16 | R ₁₇ | 16 | R ₁₈ |
| 8 | N ₁₆ | 8 | N ₁₇ | 8 | N ₁₈ |
| 4 | L ₄₇ , L ₂₆₁ | 4 | L ₂₃ , L ₃₀₄ | 4 | L ₃₃ , L ₂₈₀ |
| 2 | H ₅₀ , H ₁₀₃ , H ₁₁₂ , H ₁₁₇ | 2 | H ₄₀ , H ₇₅ , H ₉₈ , H ₁₀₅ | 2 | H ₅₁ , H ₁₀₀ , H ₁₀₉ , H ₁₁₇ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₉ | 16 | R ₂₀ | 16 | R ₂₁ |
| 8 | N ₁₉ | 8 | N ₂₀ | 8 | N ₂₁ |
| 4 | L ₂₉ , L ₃₀₃ | 4 | L ₂₁ , L ₂₇₅ | 4 | L ₁₃₇ , L ₂₁₀ |
| 2 | H ₄₀ , H ₇₁ , H ₁₀₂ , H ₁₀₆ | 2 | H ₄₆ , H ₆₆ , H ₉₉ , H ₁₀₉ | 2 | H ₂₁ , H ₂₆ , H ₃₄ , H ₅₁ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₂ | 16 | R ₂₃ | 16 | R ₂₄ |
| 8 | N ₂₂ | 8 | N ₂₃ | 8 | N ₂₄ |
| 4 | L ₁₄₀ , L ₃₁₃ | 4 | L ₁₁₅ , L ₃₀₆ | 4 | L ₁₃ , L ₂₉₇ |
| 2 | H ₃₅ , H ₅₃ , H ₈₉ , H ₁₀₄ | 2 | H ₅ , H ₂₂ , H ₇₉ , H ₁₀₄ | 2 | H ₄₀ , H ₇₈ , H ₉₄ , H ₁₀₄ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₅ | 16 | R ₂₆ | 16 | R ₂₇ |
| 8 | N ₂₅ | 8 | N ₂₆ | 8 | N ₂₇ |
| 4 | L ₂₃₆ , L ₃₁₅ | 4 | L ₂₀₉ , L ₃₀₆ | 4 | L ₂₃₆ , L ₂₅₅ |
| 2 | H ₇ , H ₁₁ , H ₇₇ , H ₁₀₄ | 2 | H ₆ , H ₈₄ , H ₉₁ , H ₁₀₃ | 2 | H ₁₂ , H ₁₄ , H ₆₂ , H ₈₄ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₈ | 16 | R ₂₉ | 16 | R ₃₀ |
| 8 | N ₂₈ | 8 | N ₂₉ | 8 | N ₃₀ |
| 4 | L ₁₃ , L ₂₇₁ | 4 | L ₂₁₄ , L ₂₅₆ | 4 | L ₁₉₂ , L ₂₆₈ |
| 2 | H ₈ , H ₈₄ , H ₉₅ , H ₁₀₉ | 2 | H ₁ , H ₃₃ , H ₄₆ , H ₅₉ | 2 | H ₁₅ , H ₄₃ , H ₆₄ , H ₁₀₄ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₃₁ | 16 | R ₃₂ | 16 | R ₃₃ |
| 8 | N ₃₁ | 8 | N ₃₂ | 8 | N ₃₃ |
| 4 | L ₁₂₄ , L ₂₇₇ | 4 | L ₁₀₀ , L ₁₂₇ | 4 | L ₁₀₃ , L ₂₇₉ |
| 2 | H ₅ , H ₂₀ , H ₃₅ , H ₄₆ | 2 | H ₂₁ , H ₂₄ , H ₅₂ , H ₁₀₅ | 2 | H ₁₀ , H ₁₆ , H ₄₆ , H ₆₇ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₃₄ | 16 | R ₃₅ | 16 | R ₃₆ |
| 8 | N ₃₄ | 8 | N ₃₅ | 8 | N ₃₆ |
| 4 | L ₃₉ , L ₂₉₁ | 4 | L ₂₆ , L ₂₇₈ | 4 | L ₂₈ , L ₃₀₀ |
| 2 | H ₃₉ , H ₄₈ , H ₁₀₄ , H ₁₁₀ | 2 | H ₈ , H ₄₆ , H ₁₀₀ , H ₁₀₈ | 2 | H ₃₀ , H ₇₁ , H ₉₄ , H ₁₀₅ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₃₇ | 16 | R ₃₈ | 16 | R ₃₉ |
| 8 | N ₃₇ | 8 | N ₃₈ | 8 | N ₃₉ |
| 4 | L ₂₃₃ , L ₂₄₃ | 4 | L ₉ , L ₂₇₄ | 4 | L ₁₇₈ , L ₂₈₇ |
| 2 | H ₁₄ , H ₃₂ , H ₈₆ , H ₁₀₅ | 2 | H ₈₄ , H ₉₉ , H ₁₀₈ , H ₁₁₇ | 2 | H ₁₈ , H ₆₈ , H ₈₃ , H ₁₀₆ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₄₀ | 16 | R ₄₁ | 16 | R ₄₂ |
| 8 | N ₄₀ | 8 | N ₄₁ | 8 | N ₄₂ |
| 4 | L ₂₇₃ , L ₃₁₄ | 4 | L ₂₃₈ , L ₃₁₄ | 4 | L ₁₆₆ , L ₁₇₈ |
| 2 | H ₃₂ , H ₆₁ , H ₈₄ , H ₉₃ | 2 | H ₃₃ , H ₆₃ , H ₈₂ , H ₁₀₆ | 2 | H ₉ , H ₁₇ , H ₃₉ , H ₈₄ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₄₃ | 16 | R ₄₄ | 16 | R ₄₅ |
| 8 | N ₄₃ | 8 | N ₄₄ | 8 | N ₄₅ |
| 4 | L ₂₉ , L ₃₀₃ | 4 | L ₂₁ , L ₂₇₅ | 4 | L ₂₉ , L ₃₀₃ |
| 2 | H ₄₀ , H ₇₁ , H ₁₀₂ , H ₁₀₆ | 2 | H ₄₆ , H ₆₆ , H ₉₉ , H ₁₀₉ | 2 | H ₄₀ , H ₇₁ , H ₁₀₂ , H ₁₀₆ |

| Order | Subgroups | Order | Subgroups | Order | Subgroups |
|--------------|--|--------------|--|--------------|---|
| 16 | R ₄₆ | 16 | R ₄₇ | 16 | R ₄₈ |
| 8 | N ₄₆ | 8 | N ₄₇ | 8 | N ₄₈ |
| 4 | L ₂₁ , L ₂₇₅ | 4 | L ₄₈ , L ₉₀ | 4 | L ₁₂₉ , L ₂₁₀ |
| 2 | H ₄₆ , H ₆₆ , H ₉₉ , H ₁₀₉ | 2 | H ₆₅ , H ₆₇ , H ₇₃ , H ₁₀₅ | 2 | H ₆ , H ₁₉ , H ₅₇ , H ₁₀₆ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |

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| 16 | R ₄₉ | 16 | R ₅₀ | 16 | R ₅₁ |
|-------|---|-------|---|-------|---|
| 8 | N ₄₉ | 8 | N ₅₀ | 8 | N ₅₁ |
| 4 | L ₁₀₀ , L ₂₇₆ | 4 | L ₂₃₃ , L ₂₅₁ | 4 | L ₁₉₀ , L ₃₅₁ |
| 2 | H ₂₂ , H ₂₇ , H ₄₆ , H ₈₉ | 2 | H ₁₁ , H ₁₅ , H ₅₁ , H ₉₉ | 2 | H ₄₅ , H ₅₉ , H ₉₂ , H ₁₀₆ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₅₂ | 16 | R ₅₃ | 16 | R ₅₄ |
| 8 | N ₅₂ | 8 | N ₅₃ | 8 | N ₅₄ |
| 4 | L ₁₉₃ , L ₂₅₆ | 4 | L ₈₃ , L ₃₅₁ | 4 | L ₂₁₆ , L ₃₁₂ |
| 2 | H ₁₂ , H ₄₇ , H ₆₁ , H ₁₀₅ | 2 | H ₇ , H ₃₁ , H ₅₁ , H ₆₄ | 2 | H ₂₉ , H ₆₉ , H ₈₄ , H ₁₁₀ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₅₅ | 16 | R ₅₆ | 16 | R ₅₇ |
| 8 | N ₅₅ | 8 | N ₅₆ | 8 | N ₅₇ |
| 4 | L ₁₉ , L ₆₁ | 4 | L ₁₉ , L ₂₁₅ | 4 | L ₁₆₃ , L ₃₁₂ |
| 2 | H ₄₁ , H ₇₈ , H ₁₀₂ , H ₁₀₅ | 2 | H ₆₆ , H ₈₄ , H ₁₀₀ , H ₁₀₇ | 2 | H ₁₀ , H ₈₅ , H ₁₀₅ , H ₁₁₂ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₅₈ | 16 | R ₅₉ | 16 | R ₆₀ |
| 8 | N ₅₈ | 8 | N ₅₉ | 8 | N ₆₀ |
| 4 | L ₂₄₇ , L ₂₈₁ | 4 | L ₁₉ , L ₂₈₁ | 4 | L ₁₂₄ , L ₁₄₃ |
| 2 | H ₂₆ , H ₈₀ , H ₉₀ , H ₁₀₅ | 2 | H ₁₉ , H ₂₃ , H ₂₅ , H ₈₄ | 2 | H ₂₅ , H ₇₆ , H ₁₀₃ , H ₁₀₆ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₆₁ | 16 | R ₆₂ | 16 | R ₆₃ |
| 8 | N ₅₃ | 8 | N ₅₄ | 8 | N ₆₃ |
| 4 | L ₂₇₀ , L ₃₁₃ | 4 | L ₂₆ , L ₃₁₁ | 4 | L ₂₈ , L ₆₇ |
| 2 | H ₂₄ , H ₅₁ , H ₉₀ , H ₁₁₆ | 2 | H ₄₁ , H ₇₅ , H ₉₄ , H ₁₀₆ | 2 | H ₈ , H ₅₁ , H ₉₉ , H ₁₀₇ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₆₄ | 16 | R ₆₅ | 16 | R ₆₆ |
| 8 | N ₆₄ | 8 | N ₆₅ | 8 | N ₆₆ |
| 4 | L ₄₃ , L ₁₅₇ | 4 | L ₃₃ , L ₃₀₁ | 4 | L ₂₃ , L ₃₀₄ |
| 2 | H ₃₇ , H ₄₆ , H ₆₅ , H ₁₁₂ | 2 | H ₄₁ , H ₇₁ , H ₉₈ , H ₁₀₄ | 2 | H ₄₆ , H ₆₄ , H ₁₀₇ , H ₁₁₇ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₆₇ | 16 | R ₆₈ | 16 | R ₆₉ |
| 8 | N ₆₇ | 8 | N ₆₈ | 8 | N ₆₉ |
| 4 | L ₄₈ , L ₁₈₇ | 4 | L ₁₁₀ , L ₁₅₇ | 4 | L ₁₂₁ , L ₁₄₉ |
| 2 | H ₃₆ , H ₅₁ , H ₆₈ , H ₁₁₁ | 2 | H ₁₇ , H ₂₉ , H ₅₆ , H ₁₀₄ | 2 | H ₂₀ , H ₂₇ , H ₄₉ , H ₁₀₄ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₇₀ | 16 | R ₇₁ | 16 | R ₇₂ |
| 8 | N ₇₀ | 8 | N ₇₁ | 8 | N ₇₂ |
| 4 | L ₂₂ , L ₁₄₈ | 4 | L ₂₇ , L ₁₄₇ | 4 | L ₁₄₈ , L ₂₁₉ |
| 2 | H ₇ , H ₂₆ , H ₄₈ , H ₁₀₈ | 2 | H ₅₂ , H ₆₅ , H ₉₂ , H ₁₀₉ | 2 | H ₂₉ , H ₅₃ , H ₉₃ , H ₁₁₄ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₇₃ | 16 | R ₇₄ | 16 | R ₇₅ |
| 8 | N ₇₃ | 8 | N ₇₄ | 8 | N ₇₅ |
| 4 | L ₁₄₉ , L ₂₁₈ | 4 | L ₁₄₅ , L ₂₁₇ | 4 | L ₁₄₆ , L ₂₃₀ |
| 2 | H ₂₃ , H ₅₄ , H ₉₁ , H ₁₀₆ | 2 | H ₃₀ , H ₃₈ , H ₅₄ , H ₅₉ | 2 | H ₂₅ , H ₃₇ , H ₄₂ , H ₆₁ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₇₆ | 16 | R ₇₇ | 16 | R ₇₈ |
| 8 | N ₇₆ | 8 | N ₇₇ | 8 | N ₇₈ |
| 4 | L ₇₆ , L ₁₄₅ | 4 | L ₇₇ , L ₁₄₆ | 4 | L ₇₈ , L ₁₆₅ |
| 2 | H ₂₅ , H ₄₀ , H ₆₃ , H ₈₈ | 2 | H ₃₉ , H ₄₁ , H ₅₃ , H ₆₂ | 2 | H ₈ , H ₂₆ , H ₃₉ , H ₈₇ |

| Order | Subgroups | Order | Subgroups | Order | Subgroups |
|-------|---|-------|---|-------|--|
| 16 | R ₇₉ | 16 | R ₈₀ | 16 | R ₈₁ |
| 8 | N ₇₉ | 8 | N ₈₀ | 8 | N ₈₁ |
| 4 | L ₁₃₈ , L ₁₆₄ | 4 | L ₁₃₉ , L ₂₂₄ | 4 | L ₇₅ , L ₁₄₄ |
| 2 | H ₃₇ , H ₅₂ , H ₆₃ , H ₆₆ | 2 | H ₅₂ , H ₄₀ , H ₆₁ , H ₆₉ | 2 | H ₃₀ , H ₆₇ , H ₈₆ , H ₁₂₇ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₈₂ | 16 | R ₈₃ | 16 | R ₈₄ |
| 8 | N ₈₂ | 8 | N ₈₃ | 8 | N ₈₄ |
| 4 | L ₄₉ , L ₁₉₇ | 4 | L ₅₅ , L ₉₉ | 4 | L ₅₂ , L ₁₀₁ |
| 2 | H ₂₅ , H ₃₁ , H ₇₂ , H ₉₄ | 2 | H ₂₇ , H ₃₁ , H ₇₄ , H ₉₆ | 2 | H ₂₆ , H ₃₁ , H ₇₃ , H ₉₅ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₈₅ | 16 | R ₈₆ | 16 | R ₈₇ |
| 8 | N ₈₅ | 8 | N ₈₆ | 8 | N ₈₇ |
| 4 | L ₅₃ , L ₁₅₃ | 4 | L ₅₀ , L ₉₆ | 4 | L ₅₆ , L ₉₈ |
| 2 | H ₂₅ , H ₃₃ , H ₇₃ , H ₉₆ | 2 | H ₂₇ , H ₃₃ , H ₉₂ , H ₉₅ | 2 | H ₂₆ , H ₃₃ , H ₇₄ , H ₉₄ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₈₈ | 16 | R ₈₉ | 16 | R ₉₀ |
| 8 | N ₈₈ | 8 | N ₈₉ | 8 | N ₉₀ |
| 4 | L ₅₇ , L ₉₇ | 4 | L ₅₄ , L ₁₀₂ | 4 | L ₅₁ , L ₉₅ |
| 2 | H ₂₅ , H ₃₂ , H ₇₄ , H ₉₅ | 2 | H ₂₇ , H ₃₂ , H ₇₃ , H ₉₄ | 2 | H ₂₆ , H ₃₂ , H ₇₂ , H ₉₆ |

The Lattice Structure of the Subgroups of Order 16in the Subgroup Lattices Of 3 X 3 Matrices ..

| Order | Subgroups | Order | Subgroups | Order | Subgroups |
|--------------|---|--------------|--|--------------|--|
| 16 | R ₉₁ | 16 | R ₉₂ | 16 | R ₉₃ |
| 8 | N ₉₁ | 8 | N ₉₂ | 8 | N ₉₃ |
| 4 | L ₁₄₁ , L ₃₁₈ | 4 | L ₁₃₆ , L ₂₈₆ | 4 | L ₃₁ , L ₂₉₀ |
| 2 | H ₇ , H ₁₀ , H ₅₄ , H ₁₁₅ | 2 | H ₁ , H ₂₈ , H ₅₄ , H ₁₀₈ | 2 | H ₁₄ , H ₂₇ , H ₆₅ , H ₇₁ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₉₄ | 16 | R ₉₅ | 16 | R ₉₆ |
| 8 | N ₉₄ | 8 | N ₉₅ | 8 | N ₉₆ |
| 4 | L ₁₂₉ , L ₁₃₇ | 4 | L ₃ , L ₁₃₅ | 4 | L ₄₉ , L ₁₉₇ |
| 2 | H ₄ , H ₂₀ , H ₂₂ , H ₅₃ | 2 | H ₂ , H ₂₄ , H ₂₆ , H ₇₀ | 2 | H ₁₁ , H ₅₂ , H ₉₇ , H ₁₁₀ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₉₇ | 16 | R ₉₈ | 16 | R ₉₉ |
| 8 | N ₉₇ | 8 | N ₉₈ | 8 | N ₉₉ |
| 4 | L ₅₃ , L ₁₉₉ | 4 | L ₅₇ , L ₁₉₈ | 4 | L ₅₅ , L ₂₀₀ |
| 2 | H ₁₃ , H ₅₃ , H ₉₈ , H ₁₁₀ | 2 | H ₁₂ , H ₅₄ , H ₉₉ , H ₁₁₀ | 2 | H ₁₃ , H ₅₄ , H ₉₇ , H ₁₁₁ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₀₀ | 16 | R ₁₀₁ | 16 | R ₁₀₂ |
| 8 | N ₁₀₀ | 8 | N ₁₀₁ | 8 | N ₁₀₂ |
| 4 | L ₅₀ , L ₂₀₂ | 4 | L ₅₄ , L ₂₀₁ | 4 | L ₅₂ , L ₂₀₃ |
| 2 | H ₅₂ , H ₁₂ , H ₉₈ , H ₁₁₁ | 2 | H ₁₁ , H ₅₃ , H ₉₉ , H ₁₁₁ | 2 | H ₁₂ , H ₅₃ , H ₉₇ , H ₁₁₂ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₀₃ | 16 | R ₁₀₄ | 16 | R ₁₀₅ |
| 8 | N ₁₀₃ | 8 | N ₁₀₄ | 8 | N ₁₀₅ |
| 4 | L ₅₆ , L ₂₀₅ | 4 | L ₅₁ , L ₂₀₄ | 4 | L ₁₄₂ , L ₂₂₀ |
| 2 | H ₁₁ , H ₉₈ , H ₁₀₃ , H ₁₁₂ | 2 | H ₁₃ , H ₅₂ , H ₉₉ , H ₁₁₂ | 2 | H ₉ , H ₁₄ , H ₅₂ , H ₁₁₃ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₀₆ | 16 | R ₁₀₇ | 16 | R ₁₀₈ |
| 8 | N ₁₀₆ | 8 | N ₁₀₇ | 8 | N ₁₀₈ |
| 4 | L ₁₄₁ , L ₂₅₂ | 4 | L ₁₄₃ , L ₂₇₇ | 4 | L ₁₄₀ , L ₂₇₀ |
| 2 | H ₁₅ , H ₂₅ , H ₅₈ , H ₁₁₄ | 2 | H ₂ , H ₃₄ , H ₅₂ , H ₉₀ | 2 | H ₃ , H ₂₅ , H ₅₇ , H ₉₁ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₀₉ | 16 | R ₁₁₀ | 16 | R ₁₁₁ |
| 8 | N ₁₀₉ | 8 | N ₁₁₀ | 8 | N ₁₁₁ |
| 4 | L ₁₄₄ , L ₂₂₃ | 4 | L ₆₀ , L ₁₃₉ | 4 | L ₇₈ , L ₂₂₂ |
| 2 | H ₈ , H ₅₃ , H ₆₀ , H ₆₈ | 2 | H ₂₇ , H ₅₆ , H ₅₉ , H ₁₁₇ | 2 | H ₅₄ , H ₆₄ , H ₆₇ , H ₁₁₇ |

| Order | Subgroups | Order | Subgroups | Order | Subgroups |
|--------------|--|--------------|--|--------------|---|
| 16 | R ₁₁₂ | 16 | R ₁₁₃ | 16 | R ₁₁₄ |
| 8 | N ₁₁₂ | 8 | N ₁₁₃ | 8 | N ₁₁₄ |
| 4 | L ₇₃ , L ₁₃₈ | 4 | L ₃ , L ₂₂₁ | 4 | L ₃₁ , L ₁₈₁ |
| 2 | H ₂₆ , H ₅₅ , H ₆₂ , H ₆₈ | 2 | H ₃ , H ₁₉ , H ₅₄ , H ₁₀₃ | 2 | H ₁₅ , H ₃₆ , H ₅₃ , H ₁₀₇ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₁₅ | 16 | R ₁₁₆ | 16 | R ₁₁₇ |
| 8 | N ₁₁₅ | 8 | N ₁₁₆ | 8 | N ₁₁₇ |
| 4 | L ₁₃₆ , L ₁₇₄ | 4 | L ₅₉ , L ₁₀₃ | 4 | L ₂₇₄ , L ₂₉₈ |
| 2 | H ₉ , H ₂₇ , H ₅₀ , H ₁₀₉ | 2 | H ₃₆ , H ₃₈ , H ₇₂ , H ₁₀₆ | 2 | H ₃ , H ₈₁ , H ₉₇ , H ₁₁₅ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₁₈ | 16 | R ₁₁₉ | 16 | R ₁₂₀ |
| 8 | N ₁₁₈ | 8 | N ₁₁₉ | 8 | N ₁₂₀ |
| 4 | L ₂₃₈ , L ₂₇₃ | 4 | L ₁₆₆ , L ₂₈₇ | 4 | L ₂₆₉ , L ₃₀₃ |
| 2 | H ₃ , H ₁₅ , H ₃₁ , H ₇₇ | 2 | H ₃ , H ₁₆ , H ₆₅ , H ₈₅ | 2 | H ₄ , H ₄₂ , H ₉₇ , H ₁₁₄ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₂₁ | 16 | R ₁₂₂ | 16 | R ₁₂₃ |
| 8 | N ₁₂₁ | 8 | N ₁₂₂ | 8 | N ₁₂₃ |
| 4 | L ₄₃ , L ₁₁₀ | 4 | L ₂₇₈ , L ₃₁₁ | 4 | L ₅₉ , L ₂₇₉ |
| 2 | H ₄ , H ₂₈ , H ₆₈ , H ₇₂ | 2 | H ₂ , H ₅₅ , H ₉₇ , H ₁₁₃ | 2 | H ₂ , H ₉ , H ₅₆ , H ₁₁₀ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₂₄ | 16 | R ₁₂₅ | 16 | R ₁₂₆ |
| 8 | N ₁₂₄ | 8 | N ₁₂₅ | 8 | N ₁₂₆ |
| 4 | L ₁₂₂ , L ₂₆₀ | 4 | L ₆₉ , L ₈₇ | 4 | L ₂₇ , L ₂₆₂ |
| 2 | H ₂₁ , H ₄₇ , H ₆₉ , H ₉₈ | 2 | H ₁₅ , H ₈₈ , H ₉₀ , H ₉₅ | 2 | H ₁₅ , H ₂₉ , H ₇₅ , H ₁₁₆ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₂₇ | 16 | R ₁₂₈ | 16 | R ₁₂₉ |
| 8 | N ₁₂₇ | 8 | N ₁₂₈ | 8 | N ₁₂₉ |
| 4 | L ₆ , L ₆₉ | 4 | L ₁₇₁ , L ₂₆₀ | 4 | L ₂₅₄ , L ₂₉₆ |
| 2 | H ₂₁ , H ₅₀ , H ₆₈ , H ₁₀₀ | 2 | H ₁ , H ₅₆ , H ₈₉ , H ₉₅ | 2 | H ₃₆ , H ₈₂ , H ₁₁₄ , H ₁₁₆ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₃₀ | 16 | R ₁₃₁ | 16 | R ₁₃₂ |
| 8 | N ₁₃₀ | 8 | N ₁₃₁ | 8 | N ₁₃₂ |
| 4 | L ₇₂ , L ₁₅₁ | 4 | L ₂₅₅ , L ₃₁₅ | 4 | L ₇₂ , L ₁₁₄ |

The Lattice Structure of the Subgroups of Order 16in the Subgroup Lattices Of 3 X 3 Matrices ..

| | | | | | |
|--------------|---|--------------|--|--------------|--|
| 2 | H ₁₄ , H ₈₅ , H ₉₁ , H ₉₅ | 2 | H ₂ , H ₁₃ , H ₅₉ , H ₈₂ | 2 | H ₂₁ , H ₆₇ , H ₈₂ , H ₉₆ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₃₃ | 16 | R ₁₃₄ | 16 | R ₁₃₅ |
| 8 | N ₁₃₃ | 8 | N ₁₃₄ | 8 | N ₁₃₅ |
| 4 | L ₁₂ , L ₂₅₄ | 4 | L ₁₄₇ , L ₂₆₂ | 4 | L ₁₃₃ , L ₂₄₅ |
| 2 | H ₁₄ , H ₁₈ , H ₆₇ , H ₈₂ | 2 | H ₂₆ , H ₂₈ , H ₄₇ , H ₁₁₅ | 2 | H ₂₄ , H ₂₈ , H ₈₆ , H ₁₀₀ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₃₆ | 16 | R ₁₃₇ | 16 | R ₁₃₈ |
| 8 | N ₁₃₆ | 8 | N ₁₃₇ | 8 | N ₁₃₈ |
| 4 | L ₇₉ , L ₂₁₂ | 4 | L ₁₁₂ , L ₂₈₈ | 4 | L ₂₈₈ , L ₃₀₈ |
| 2 | H ₃₅ , H ₅₈ , H ₆₃ , H ₉₅ | 2 | H ₂₀ , H ₃₀ , H ₃₃ , H ₈₃ | 2 | H ₃₁ , H ₆₅ , H ₈₁ , H ₈₉ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₃₉ | 16 | R ₁₄₀ | 16 | R ₁₄₁ |
| 8 | N ₁₃₉ | 8 | N ₁₄₀ | 8 | N ₁₄₁ |
| 4 | L ₂₃₉ , L ₃₅₀ | 4 | L ₁₇₇ , L ₂₃₉ | 4 | L ₈₀ , L ₁₃₀ |
| 2 | H ₂₄ , H ₆₅ , H ₇₇ , H ₉₇ | 2 | H ₃₄ , H ₆₃ , H ₈₃ , H ₉₈ | 2 | H ₂₄ , H ₂₉ , H ₈₇ , H ₉₄ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₄₂ | 16 | R ₁₄₃ | 16 | R ₁₄₄ |
| 8 | N ₁₄₂ | 8 | N ₁₄₃ | 8 | N ₁₄₄ |
| 4 | L ₇₄ , L ₁₈₄ | 4 | L ₁₂₀ , L ₂₈₃ | 4 | L ₅ , L ₁₇₅ |
| 2 | H ₄₈ , H ₆₃ , H ₁₀₁ , H ₁₀₃ | 2 | H ₂₁ , H ₄₁ , H ₄₄ , H ₆₅ | 2 | H ₁₁ , H ₁₉ , H ₅₈ , H ₁₁₇ |

| | | | | | |
|--------------|---|--------------|---|--------------|---|
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₄₅ | 16 | R ₁₄₆ | 16 | R ₁₄₇ |
| 8 | N ₁₄₅ | 8 | N ₁₄₆ | 8 | N ₁₄₇ |
| 4 | L ₁₇₄ , L ₂₈₆ | 4 | L ₁₈₆ , L ₂₈₃ | 4 | L ₁₉₆ , L ₂₈₉ |
| 2 | H ₅₈ , H ₉₃ , H ₁₀₇ , H ₁₁₆ | 2 | H ₃₂ , H ₄₀ , H ₅₈ , H ₉₀ | 2 | H ₁₂ , H ₂₀ , H ₂₉ , H ₄₂ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₄₈ | 16 | R ₁₄₉ | 16 | R ₁₅₀ |
| 8 | N ₁₄₈ | 8 | N ₁₄₉ | 8 | N ₁₅₀ |
| 4 | L ₄₆ , L ₁₈₀ | 4 | L ₈₁ , L ₂₂₈ | 4 | L ₁₇₂ , L ₂₄₄ |
| 2 | H ₃ , H ₅₈ , H ₆₇ , H ₁₁₂ | 2 | H ₂₃ , H ₂₈ , H ₈₇ , H ₉₇ | 2 | H ₃₄ , H ₄₈ , H ₆₄ , H ₉₅ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₅₁ | 16 | R ₁₅₂ | 16 | R ₁₅₃ |
| 8 | N ₁₅₁ | 8 | N ₁₅₂ | 8 | N ₁₅₃ |
| 4 | L ₇₄ , L ₂₃₂ | 4 | L ₈₀ , L ₁₇₉ | 4 | L ₂₄₁ , L ₂₈₂ |
| 2 | H ₂₃ , H ₆₅ , H ₈₆ , H ₉₄ | 2 | H ₅₈ , H ₆₄ , H ₉₈ , H ₁₀₃ | 2 | H ₂₃ , H ₂₉ , H ₇₇ , H ₁₀₀ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₅₄ | 16 | R ₁₅₅ | 16 | R ₁₅₆ |
| 8 | N ₁₅₄ | 8 | N ₁₅₅ | 8 | N ₁₅₆ |
| 4 | L ₁₆₃ , L ₂₁₆ | 4 | L ₁₈₅ , L ₂₄₁ | 4 | L ₁₈₉ , L ₂₂₅ |
| 2 | H ₄ , H ₃₈ , H ₈₃ , H ₁₁₁ | 2 | H ₃₅ , H ₆₄ , H ₈₃ , H ₁₀₁ | 2 | H ₂₉ , H ₃₀ , H ₄₃ , H ₉₁ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₅₇ | 16 | R ₁₅₈ | 16 | R ₁₅₉ |
| 8 | N ₁₅₇ | 8 | N ₁₅₈ | 8 | N ₁₅₉ |
| 4 | L ₁₅₅ , L ₁₈₈ | 4 | L ₂₄₉ , L ₂₉₂ | 4 | L ₃₀ , L ₁₅₅ |
| 2 | H ₂₃ , H ₃₉ , H ₄₃ , H ₁₁₄ | 2 | H ₃₃ , H ₃₇ , H ₅₇ , H ₁₁₃ | 2 | H ₂₂ , H ₃₃ , H ₅₆ , H ₁₀₇ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₆₀ | 16 | R ₁₆₁ | 16 | R ₁₆₂ |
| 8 | N ₁₆₀ | 8 | N ₁₆₁ | 8 | N ₁₆₂ |
| 4 | L ₂₀ , L ₂₉₂ | 4 | L ₄ , L ₂₂₅ | 4 | L ₁₉₁ , L ₃₁₆ |
| 2 | H ₂₄ , H ₃₈ , H ₄₅ , H ₁₀₉ | 2 | H ₁₉ , H ₂₈ , H ₃₂ , H ₅₅ | 2 | H ₄₄ , H ₆₂ , H ₉₃ , H ₁₀₅ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₆₃ | 16 | R ₁₆₄ | 16 | R ₁₆₅ |
| 8 | N ₁₆₃ | 8 | N ₁₆₄ | 8 | N ₁₆₅ |
| 4 | L ₉₄ , L ₁₉₇ | 4 | L ₁₀₁ , L ₂₀₃ | 4 | L ₉₉ , L ₂₀₀ |
| 2 | H ₅ , H ₁₆ , H ₄₃ , H ₁₀₀ | 2 | H ₆ , H ₁₆ , H ₄₅ , H ₁₀₂ | 2 | H ₁₆ , H ₄₄ , H ₁₀₁ , H ₁₁₆ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₆₆ | 16 | R ₁₆₇ | 16 | R ₁₆₈ |
| 8 | N ₁₆₆ | 8 | N ₁₆₇ | 8 | N ₁₆₈ |
| 4 | L ₁₅₃ , L ₁₉₉ | 4 | L ₉₈ , L ₂₀₅ | 4 | L ₉₆ , L ₂₀₂ |
| 2 | H ₁₈ , H ₄₅ , H ₁₀₀ , H ₁₁₆ | 2 | H ₅ , H ₁₈ , H ₄₄ , H ₁₀₂ | 2 | H ₆ , H ₁₈ , H ₄₃ , H ₁₀₁ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₆₉ | 16 | R ₁₇₀ | 16 | R ₁₇₁ |
| 8 | N ₁₆₉ | 8 | N ₁₇₀ | 8 | N ₁₇₁ |
| 4 | L ₉₇ , L ₁₉₈ | 4 | L ₉₅ , L ₂₀₄ | 4 | L ₁₀₂ , L ₂₀₁ |
| 2 | H ₆ , H ₁₇ , H ₄₄ , H ₁₀₀ | 2 | H ₁₇ , H ₄₃ , H ₁₀₂ , H ₁₁₆ | 2 | H ₅ , H ₁₇ , H ₄₅ , H ₁₀₁ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₇₂ | 16 | R ₁₇₃ | 16 | R ₁₇₄ |
| 8 | N ₁₇₂ | 8 | N ₁₇₃ | 8 | N ₁₇₄ |

The Lattice Structure of the Subgroups of Order 16in the Subgroup Lattices Of 3 X 3 Matrices ..

| | | | | | |
|--------------|--|--------------|--|--------------|--|
| 4 | L ₂₅ , L ₂₀₇ | 4 | L ₆₈ , L ₂₄₆ | 4 | L ₁₂₆ , L ₂₀₆ |
| 2 | H ₃₅ , H ₄₃ , H ₆₇ , H ₁₀₈ | 2 | H ₃₁ , H ₄₉ , H ₆₉ , H ₁₀₉ | 2 | H ₈ , H ₁₀ , H ₂₀ , H ₄₃ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₇₅ | 16 | R ₁₇₆ | 16 | R ₁₇₇ |
| 8 | N ₁₇₅ | 8 | N ₁₇₆ | 8 | N ₁₇₇ |
| 4 | L ₁₈₂ , L ₂₈₉ | 4 | L ₂₄₃ , L ₂₅₁ | 4 | L ₂₆₈ , L ₂₇₂ |
| 2 | H ₂₁ , H ₃₁ , H ₄₈ , H ₆₆ | 2 | H ₁ , H ₂ , H ₄₅ , H ₆₃ | 2 | H ₄ , H ₃₂ , H ₄₇ , H ₆₂ |

| Order | Subgroups | Order | Subgroups | Order | Subgroups |
|-------|--|-------|--|-------|--|
| 16 | R ₁₇₈ | 16 | R ₁₇₉ | 16 | R ₁₈₀ |
| 8 | N ₁₇₈ | 8 | N ₁₇ | 8 | N ₁₈₀ |
| 4 | L ₂₂₇ , L ₂₅₀ | 4 | L ₁₇₅ , L ₃₀₉ | 4 | L ₆₆ , L ₁₇₃ |
| 2 | H ₄ , H ₁₄ , H ₄₄ , H ₆₁ | 2 | H ₃₆ , H ₄₄ , H ₆₆ , H ₈₉ | 2 | H ₁₀ , H ₃₃ , H ₄₂ , H ₉₁ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₈₁ | 16 | R ₁₈₂ | 16 | R ₁₈₃ |
| 8 | N ₁₈₁ | 8 | N ₁₈₂ | 8 | N ₁₈₃ |
| 4 | L ₂₁₁ , L ₃₀₂ | 4 | L ₁₇₀ , L ₃₂₁ | 4 | L ₁₉₃ , L ₂₁₄ |
| 2 | H ₄₅ , H ₆₉ , H ₁₀₃ , H ₁₅ | 2 | H ₃₂ , H ₃₅ , H ₈₈ , H ₁₁₄ | 2 | H ₃ , H ₇ , H ₄₃ , H ₆₀ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₈₄ | 16 | R ₁₈₅ | 16 | R ₁₈₆ |
| 8 | N ₁₈₄ | 8 | N ₁₈₅ | 8 | N ₁₈₆ |
| 4 | L ₈₂ , L ₂₂₇ | 4 | L ₁₂₆ , L ₃₁₀ | 4 | L ₁₈₂ , L ₁₉₆ |
| 2 | H ₂ , H ₃₃ , H ₈₇ , H ₉₂ | 2 | H ₃₁ , H ₃₆ , H ₄₁ , H ₇₆ | 2 | H ₉ , H ₁₉ , H ₄₀ , H ₄₅ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₈₇ | 16 | R ₁₈₈ | 16 | R ₁₈₉ |
| 8 | N ₁₈₇ | 8 | N ₁₈₈ | 8 | N ₁₈₉ |
| 4 | L ₂₅ , L ₁₅₉ | 4 | L ₁₅₆ , L ₂₄₆ | 4 | L ₁₉₅ , L ₃₂₁ |
| 2 | H ₃₁ , H ₃₈ , H ₇₅ , H ₁₀₃ | 2 | H ₃₄ , H ₃₇ , H ₄₄ , H ₁₀₇ | 2 | H ₂₂ , H ₄₄ , H ₆₈ , H ₁₁₃ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₉₀ | 16 | R ₁₉₁ | 16 | R ₁₉₂ |
| 8 | N ₁₉₀ | 8 | N ₁₉₁ | 8 | N ₁₉₂ |
| 4 | L ₃₂ , L ₃₀₂ | 4 | L ₆₆ , L ₁₉₄ | 4 | L ₅ , L ₃₀₉ |
| 2 | H ₂₃ , H ₃₂ , H ₆₇ , H ₇₁ | 2 | H ₂₈ , H ₄₅ , H ₉₀ , H ₁₁₇ | 2 | H ₈ , H ₂₉ , H ₃₃ , H ₇₀ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₉₃ | 16 | R ₁₉₄ | 16 | R ₁₉₅ |
| 8 | N ₁₉₃ | 8 | N ₁₉₄ | 8 | N ₁₉₅ |
| 4 | L ₁₂₃ , L ₁₅₀ | 4 | L ₂ , L ₁₅₄ | 4 | L ₁₂ , L ₂₉₆ |
| 2 | H ₃₇ , H ₇₆ , H ₉₂ , H ₉₉ | 2 | H ₃₇ , H ₇₀ , H ₉₃ , H ₁₀₁ | 2 | H ₅ , H ₇ , H ₂₈ , H ₇₈ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₉₆ | 16 | R ₁₉₇ | 16 | R ₁₉₈ |
| 8 | N ₁₉₆ | 8 | N ₁₉₇ | 8 | N ₁₉₈ |
| 4 | L ₁₁₄ , L ₁₅₁ | 4 | L ₂₄ , L ₂₂₀ | 4 | L ₁₂₅ , L ₂₆₅ |
| 2 | H ₇ , H ₃₇ , H ₇₉ , H ₉₄ | 2 | H ₅ , H ₁₀ , H ₄₇ , H ₁₀₈ | 2 | H ₂₀ , H ₄₇ , H ₆₇ , H ₁₀₀ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₁₉₉ | 16 | R ₂₀₀ | 16 | R ₂₀₁ |
| 8 | N ₁₉₉ | 8 | N ₂₀₀ | 8 | N ₂₀₁ |
| 4 | L ₁ , L ₁₆₈ | 4 | L ₁₀₄ , L ₂₆₄ | 4 | L ₄₀ , L ₁₃₄ |
| 2 | H ₃₉ , H ₇₀ , H ₉₂ , H ₉₄ | 2 | H ₈ , H ₁₆ , H ₃₅ , H ₄₇ | 2 | H ₇ , H ₃₀ , H ₄₉ , H ₁₁₀ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₀₂ | 16 | R ₂₀₃ | 16 | R ₂₀₄ |
| 8 | N ₂₀₂ | 8 | N ₂₀₃ | 8 | N ₂₀₄ |
| 4 | L ₁₁₁ , L ₂₅₉ | 4 | L ₁₃₁ , L ₂₁₃ | 4 | L ₂₁₃ , L ₂₅₈ |
| 2 | H ₂₀ , H ₆₈ , H ₈₂ , H ₉₈ | 2 | H ₁₇ , H ₉₃ , H ₈₀ , H ₁₁₇ | 2 | H ₁₈ , H ₃₀ , H ₃₄ , H ₈₂ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₀₅ | 16 | R ₂₀₆ | 16 | R ₂₀₇ |
| 8 | N ₂₀₅ | 8 | N ₂₀₆ | 8 | N ₂₀₇ |
| 4 | L ₁₁₁ , L ₁₆₇ | 4 | L ₂ , L ₂₅₃ | 4 | L ₁₂₂ , L ₁₇₁ |
| 2 | H ₃₉ , H ₇₉ , H ₉₃ , H ₉₉ | 2 | H ₂₀ , H ₅₀ , H ₆₉ , H ₉₆ | 2 | H ₇ , H ₃₉ , H ₇₆ , H ₁₀₁ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₀₈ | 16 | R ₂₀₉ | 16 | R ₂₁₀ |
| 8 | N ₂₀₈ | 8 | N ₂₀₉ | 8 | N ₂₁₀ |
| 4 | L ₄₇ , L ₉₁ | 4 | L ₁₀₇ , L ₁₃₄ | 4 | L ₂₅₀ , L ₈₂ |
| 2 | H ₈ , H ₂₄ , H ₇₄ , H ₉₂ | 2 | H ₁₇ , H ₄₁ , H ₂₂ , H ₅₀ | 2 | H ₅₀ , H ₁₁ , H ₃ , H ₆₄ |

| Order | Subgroups | Order | Subgroups | Order | Subgroups |
|-------|--|-------|--|-------|---|
| 16 | R ₂₁₁ | 16 | R ₂₁₂ | 16 | R ₂₁₃ |
| 8 | N ₂₁₁ | 8 | N ₂₁₂ | 8 | N ₂₁₃ |
| 4 | L ₁₁₈ , L ₁₆₁ | 4 | L ₂₆₇ , L ₃₁₇ | 4 | L ₁₇₂ , L ₂₃₁ |
| 2 | H ₃₈ , H ₇₉ , H ₉₂ , H ₁₀₁ | 2 | H ₂₃ , H ₈₂ , H ₆₆ , H ₁₁₀ | 2 | H ₉ , H ₅₇ , H ₅₉ , H ₁₀₂ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |

The Lattice Structure of the Subgroups of Order 16 in the Subgroup Lattices Of 3 X 3 Matrices ..

| 16 | R ₂₁₄ | 16 | R ₂₁₅ | 16 | R ₂₁₆ |
|-------|---|-------|---|-------|---|
| 8 | N ₂₁₄ | 8 | N ₂₁₅ | 8 | N ₂₁₆ |
| 4 | L ₁₃₃ , L ₁₇₆ | 4 | L ₇₉ , L ₂₂₉ | 4 | L ₁₈₄ , L ₂₃₂ |
| 2 | H ₁₀ , H ₅₇ , H ₆₁ , H ₉₆ | 2 | H ₂₂ , H ₆₅ , H ₈₇ , H ₁₀₀ | 2 | H ₃₆ , H ₅₇ , H ₆₀ , H ₉₉ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₁₇ | 16 | R ₂₁₈ | 16 | R ₂₁₉ |
| 8 | N ₂₁₇ | 8 | N ₂₁₈ | 8 | N ₂₁₉ |
| 4 | L ₁₉₁ , L ₂₃₄ | 4 | L ₇₆ , L ₂₁₇ | 4 | L ₃₂₇ , L ₃₃₀ |
| 2 | H ₁ , H ₁₃ , H ₈₇ , H ₁₀₆ | 2 | H ₆ , H ₄₁ , H ₆₈ , H ₈₇ | 2 | H ₇₀ , H ₇₁ , H ₇₂ , H ₇₇ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₂₀ | 16 | R ₂₂₁ | 16 | R ₂₂₂ |
| 8 | N ₂₂₀ | 8 | N ₂₂₁ | 8 | N ₂₂₂ |
| 4 | L ₁₆₅ , L ₂₂₂ | 4 | L ₂₁₂ , L ₂₂₉ | 4 | L ₂₃₁ , L ₂₄₄ |
| 2 | H ₅ , H ₆₁ , H ₆₆ , H ₈₈ | 2 | H ₁₀ , H ₄₉ , H ₅₉ , H ₉₉ | 2 | H ₂₂ , H ₂₉ , H ₈₆ , H ₉₇ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₂₃ | 16 | R ₂₂₄ | 16 | R ₂₂₅ |
| 8 | N ₂₂₃ | 8 | N ₂₂₄ | 8 | N ₂₂₅ |
| 4 | L ₁₃₀ , L ₁₇₉ | 4 | L ₁₈₃ , L ₂₂₈ | 4 | L ₁₆ , L ₃₄ |
| 2 | H ₃₆ , H ₄₉ , H ₆₁ , H ₁₀₂ | 2 | H ₉ , H ₄₉ , H ₆₀ , H ₉₆ | 2 | H ₂₁ , H ₆₄ , H ₇₃ , H ₁₀₈ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₂₆ | 16 | R ₂₂₇ | 16 | R ₂₂₈ |
| 8 | N ₂₂₆ | 8 | N ₂₂₇ | 8 | N ₂₂₈ |
| 4 | L ₁₀ , L ₃₈ | 4 | L ₃₅ , L ₈₆ | 4 | L ₁₅ , L ₆₃ |
| 2 | H ₆₃ , H ₇₆ , H ₁₀₉ , H ₁₁₂ | 2 | H ₂₀ , H ₆₀ , H ₇₄ , H ₁₀₉ | 2 | H ₂₁ , H ₆₁ , H ₇₅ , H ₁₁₁ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₂₉ | 16 | R ₂₃₀ | 16 | R ₂₃₁ |
| 8 | N ₂₂₉ | 8 | N ₂₃₀ | 8 | N ₂₃₁ |
| 4 | L ₃₆ , L ₈₅ | 4 | L ₁₄ , L ₃₇ | 4 | L ₃₆ , L ₁₁₆ |
| 2 | H ₆₃ , H ₇₄ , H ₉₀ , H ₁₁₄ | 2 | H ₆₂ , H ₇₁ , H ₈₉ , H ₁₁₀ | 2 | H ₃₉ , H ₇₀ , H ₁₁₀ , H ₁₁₅ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₃₂ | 16 | R ₂₃₃ | 16 | R ₂₃₄ |
| 8 | N ₂₃₂ | 8 | N ₂₃₃ | 8 | N ₂₃₄ |
| 4 | L ₃₇ , L ₈₄ | 4 | L ₃₅ , L ₂₄₂ | 4 | L ₁₅ , L ₈₉ |
| 2 | H ₆₁ , H ₇₃ , H ₉₁ , H ₁₁₃ | 2 | H ₈₇ , H ₉₁ , H ₁₀₈ , H ₁₁₁ | 2 | H ₅₉ , H ₈₉ , H ₁₀₇ , H ₁₇₂ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₃₅ | 16 | R ₂₃₆ | 16 | R ₂₃₇ |
| 8 | N ₂₃₅ | 8 | N ₂₃₆ | 8 | N ₂₃₇ |
| 4 | L ₃₈ , L ₈₈ | 4 | L ₃₄ , L ₂₂₆ | 4 | L ₃₉ , L ₁₀₈ |
| 2 | H ₁₉ , H ₆₂ , H ₇₂ , H ₁₁₅ | 2 | H ₂₀ , H ₈₆ , H ₁₁₂ , H ₁₁₄ | 2 | H ₃ , H ₁₀ , H ₃₇ , H ₇₃ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₃₈ | 16 | R ₂₃₉ | 16 | R ₂₄₀ |
| 8 | N ₂₃₈ | 8 | N ₂₃₉ | 8 | N ₂₄₀ |
| 4 | L ₅₈ , L ₂₆₄ | 4 | L ₄₀ , L ₁₀₇ | 4 | L ₄₁ , L ₁₀₆ |
| 2 | H ₁₄ , H ₃₄ , H ₅₅ , H ₁₁₀ | 2 | H ₁₅ , H ₃₅ , H ₄₀ , H ₇₄ | 2 | H ₉ , H ₆₉ , H ₇₄ , H ₁₀₄ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₄₁ | 16 | R ₂₄₂ | 16 | R ₂₄₃ |
| 8 | N ₂₄₁ | 8 | N ₂₄₂ | 8 | N ₂₄₃ |
| 4 | L ₄₂ , L ₁₀₅ | 4 | L ₄₄ , L ₁₀₉ | 4 | L ₄₅ , L ₆₅ |
| 2 | H ₇ , H ₃₄ , H ₆₆ , H ₇₃ | 2 | H ₁ , H ₂₂ , H ₇₂ , H ₁₁₇ | 2 | H ₈ , H ₁₅ , H ₅₇ , H ₁₁₁ |

| Order | Subgroups | Order | Subgroups | Order | Subgroups |
|-------|---|-------|--|-------|--|
| 16 | R ₂₄₄ | 16 | R ₂₄₅ | 16 | R ₂₄₆ |
| 8 | N ₂₄₄ | 8 | N ₂₄₅ | 8 | N ₂₄₆ |
| 4 | L ₄₅ , L ₉₃ | 4 | L ₄₆ , L ₉₂ | 4 | L ₄₂ , L ₁₂₈ |
| 2 | H ₁₄ , H ₂₃ , H ₃₀ , H ₇₃ | 2 | H ₂ , H ₂₉ , H ₃₉ , H ₇₄ | 2 | H ₂₂ , H ₄₂ , H ₉₃ , H ₁₁₂ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₄₇ | 16 | R ₂₄₈ | 16 | R ₂₄₉ |
| 8 | N ₂₄₇ | 8 | N ₂₄₈ | 8 | N ₂₄₉ |
| 4 | L ₄₁ , L ₁₆₉ | 4 | L ₆₀ , L ₁₁₇ | 4 | L ₁₇ , L ₆₀ |
| 2 | H ₂₈ , H ₈₈ , H ₁₀₆ , H ₁₁₁ | 2 | H ₆₄ , H ₁₁₁ , H ₁₁₃ , H ₁₇₉ | 2 | H ₆₀ , H ₇₈ , H ₉₀ , H ₁₁₂ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₅₀ | 16 | R ₂₅₁ | 16 | R ₂₅₂ |
| 8 | N ₂₅₀ | 8 | N ₂₅₁ | 8 | N ₂₅₂ |
| 4 | L ₁₁₉ , L ₂₄₇ | 4 | L ₁₆₁ , L ₂₆₆ | 4 | L ₆₃ , L ₈₉ |
| 2 | H ₄ , H ₂₇ , H ₃₅ , H ₇₉ | 2 | H ₁ , H ₈₅ , H ₉₀ , H ₁₀₂ | 2 | H ₁₈ , H ₆₀ , H ₇₆ , H ₁₁₄ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₅₃ | 16 | R ₂₅₂ | 16 | R ₂₅₅ |
| 8 | N ₂₅₃ | 8 | N ₂₅₂ | 8 | N ₂₅₅ |
| 4 | L ₂₃₇ , L ₃₀₇ | 4 | L ₆₃ , L ₈₉ | 4 | L ₁₁₂ , L ₃₀₈ |
| 2 | H ₁₆ , H ₇₇ , H ₈₉ , H ₁₁₅ | 2 | H ₁₈ , H ₆₀ , H ₇₆ , H ₁₁₄ | 2 | H ₉ , H ₃₂ , H ₇₉ , H ₁₁₇ |

| Order | Subgroups | Order | Subgroups | Order | Subgroups |
|-------|--|-------|--|-------|---|
| 16 | R ₂₅₆ | 16 | R ₂₅₇ | 16 | R ₂₅₈ |
| 8 | N ₂₅₆ | 8 | N ₂₅₇ | 8 | N ₂₅₈ |
| 4 | L ₁₆₇ , L ₂₅₉ | 4 | L ₂₈₄ , L ₂₉₅ | 4 | L ₁₁₃ , L ₂₈₄ |
| 2 | H ₁₅ , H ₈₅ , H ₈₉ , H ₉₇ | 2 | H ₈ , H ₁₂ , H ₈₃ , H ₉₁ | 2 | H ₁₁ , H ₂₈ , H ₄₀ , H ₇₉ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₅₉ | 16 | R ₂₆₀ | 16 | R ₂₆₁ |
| 8 | N ₂₅₉ | 8 | N ₂₆₀ | 8 | N ₂₆₁ |
| 4 | L ₁₀ , L ₈₈ | 4 | L ₈₆ , L ₂₄₂ | 4 | L ₁₂₇ , L ₂₇₆ |
| 2 | H ₁₇ , H ₆₄ , H ₇₅ , H ₉₁ | 2 | H ₁₆ , H ₆₂ , H ₇₀ , H ₁₀₇ | 2 | H ₃ , H ₆ , H ₂₃ , H ₇₆ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₆₂ | 16 | R ₂₆₃ | 16 | R ₂₆₄ |
| 8 | N ₂₆₂ | 8 | N ₂₆₃ | 8 | N ₂₆₄ |
| 4 | L ₁₅₈ , L ₂₆₅ | 4 | L ₂₀₆ , L ₃₁₀ | 4 | L ₄ , L ₁₈₉ |
| 2 | H ₁₄ , H ₅₆ , H ₉₀ , H ₁₉₇ | 2 | H ₉ , H ₁₁ , H ₅₅ , H ₉₀ | 2 | H ₁₃ , H ₆₅ , H ₆₆ , H ₇₆ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₆₅ | 16 | R ₂₆₆ | 16 | R ₂₆₇ |
| 8 | N ₂₆₅ | 8 | N ₂₆₆ | 8 | N ₂₆₇ |
| 4 | L ₁₅₀ , L ₂₅₇ | 4 | L ₁₂₀ , L ₁₈₆ | 4 | L ₁₂₁ , L ₂₁₈ |
| 2 | H ₁₅ , H ₅₆ , H ₉₁ , H ₁₀₂ | 2 | H ₁₀ , H ₁₂ , H ₃₀ , H ₇₀ | 2 | H ₃₄ , H ₇₀ , H ₁₀₅ , H ₁₁₆ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₆₈ | 16 | R ₂₆₉ | 16 | R ₂₇₀ |
| 8 | N ₂₆₈ | 8 | N ₂₆₉ | 8 | N ₂₇₀ |
| 4 | L ₁₆₈ , L ₂₆₃ | 4 | L ₃₂₂ , L ₃₂₇ | 4 | L ₁₅₄ , L ₂₅₃ |
| 2 | H ₁₄ , H ₈₈ , H ₈₉ , H ₁₀₂ | 2 | H ₇₄ , H ₇₆ , H ₇₈ , H ₈₆ | 2 | H ₁ , H ₉₁ , H ₉₇ , H ₈₈ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₇₁ | 16 | R ₂₇₂ | 16 | R ₂₇₃ |
| 8 | N ₂₇₁ | 8 | N ₂₇₂ | 8 | N ₂₇₃ |
| 4 | L ₁₃₅ , L ₂₂₁ | 4 | L ₂₀ , L ₂₄₉ | 4 | L ₇₀ , L ₂₉₄ |
| 2 | H ₄ , H ₅ , H ₄₉ , H ₈₉ | 2 | H ₁₃ , H ₃₅ , H ₃₉ , H ₇₁ | 2 | H ₁₃ , H ₆₇ , H ₈₀ , H ₁₁₄ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₇₄ | 16 | R ₂₇₅ | 16 | R ₂₇₆ |
| 8 | N ₂₇₄ | 8 | N ₂₇₅ | 8 | N ₂₇₆ |
| 4 | L ₂₀₈ , L ₂₉₃ | 4 | L ₂₀₈ , L ₂₃₅ | 4 | L ₁₁₅ , L ₂₀₉ |
| 2 | H ₃₆ , H ₅₉ , H ₈₀ , H ₉₆ | 2 | H ₆₂ , H ₈₃ , H ₉₅ , H ₁₀₃ | 2 | H ₂ , H ₂₁ , H ₈₀ , H ₁₁₆ |

| Order | Subgroups | Order | Subgroups | Order | Subgroups |
|-------|---|-------|--|-------|--|
| 16 | R ₂₇₇ | 16 | R ₂₇₈ | 16 | R ₂₇₉ |
| 8 | N ₂₇₇ | 8 | N ₂₇₈ | 8 | N ₂₇₉ |
| 4 | L ₁₁ , L ₇₀ | 4 | L ₃₂ , L ₂₁₁ | 4 | L ₆₅ , L ₉₃ |
| 2 | H ₁₂ , H ₈₅ , H ₁₀₃ , H ₁₀₉ | 2 | H ₁₁ , H ₅₇ , H ₆₈ , H ₁₀₇ | 2 | H ₁ , H ₁₇ , H ₄₂ , H ₁₀₃ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₈₀ | 16 | R ₂₈₁ | 16 | R ₂₈₂ |
| 8 | N ₂₈₀ | 8 | N ₂₈₁ | 8 | N ₂₈₂ |
| 4 | L ₈₁ , L ₁₈₃ | 4 | L ₂₄₈ , L ₂₆₇ | 4 | L ₁₈₅ , L ₂₈₂ |
| 2 | H ₃₄ , H ₅₈ , H ₆₂ , H ₁₀₁ | 2 | H ₁ , H ₄₁ , H ₈₀ , H ₁₁₂ | 2 | H ₁₀ , H ₆₀ , H ₈₀ , H ₁₀₂ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₈₃ | 16 | R ₂₈₄ | 16 | R ₂₈₅ |
| 8 | N ₂₈₃ | 8 | N ₂₈₄ | 8 | N ₂₈₅ |
| 4 | L ₁₇₀ , L ₁₉₅ | 4 | L ₉₁ , L ₂₆₁ | 4 | L ₁₀₅ , L ₁₂₈ |
| 2 | H ₁₂ , H ₃₈ , H ₄₉ , H ₁₁₅ | 2 | H ₁₄ , H ₁₈ , H ₄₉ , H ₆₆ | 2 | H ₁₆ , H ₄₀ , H ₅₇ , H ₉₂ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₈₆ | 16 | R ₂₈₇ | 16 | R ₂₈₈ |
| 8 | N ₂₈₆ | 8 | N ₂₈₇ | 8 | N ₂₈₈ |
| 4 | L ₈ , L ₁₃₂ | 4 | L ₈ , L ₂₉₉ | 4 | L ₃₅₀ , L ₁₇₇ |
| 2 | H ₃₂ , H ₃₉ , H ₈₀ , H ₁₀₈ | 2 | H ₃₃ , H ₃₄ , H ₆₈ , H ₇₈ | 2 | H ₈₀ , H ₆₁ , H ₉₉ , H ₉ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₈₉ | 16 | R ₂₉₀ | 16 | R ₂₉₁ |
| 8 | N ₂₈₉ | 8 | N ₂₉₀ | 8 | N ₂₉₁ |
| 4 | L ₁₇₆ , L ₂₄₅ | 4 | L ₇₃ , L ₁₆₄ | 4 | L ₁₆₀ , L ₂₂₄ |
| 2 | H ₃₅ , H ₄₈ , H ₆₂ , H ₉₈ | 2 | H ₃₀ , H ₅₆ , H ₆₄ , H ₁₁₆ | 2 | H ₆ , H ₃₉ , H ₅₅ , H ₆₀ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₉₂ | 16 | R ₂₉₃ | 16 | R ₂₉₄ |
| 8 | N ₂₉₂ | 8 | N ₂₉₃ | 8 | N ₂₉₄ |
| 4 | L ₇₅ , L ₂₂₃ | 4 | L ₁₈₁ , L ₂₉₀ | 4 | L ₂₂ , L ₂₁₉ |
| 2 | H ₅ , H ₄₂ , H ₆₃ , H ₆₉ | 2 | H ₁ , H ₅ , H ₄₈ , H ₁₁₅ | 2 | H ₆ , H ₉ , H ₇₁ , H ₉₂ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₉₅ | 16 | R ₂₉₆ | 16 | R ₂₉₇ |
| 8 | N ₂₉₅ | 8 | N ₂₉₆ | 8 | N ₂₉₇ |
| 4 | L ₁₁₃ , L ₂₉₅ | 4 | L ₉₂ , L ₁₈₀ | 4 | L ₁₈ , L ₆₂ |

The Lattice Structure of the Subgroups of Order 16in the Subgroup Lattices Of 3 X 3 Matrices ..

| | | | | | |
|--------------|--|--------------|---|--------------|--|
| 2 | H ₁₃ , H ₂₁ , H ₃₆ , H ₈₁ | 2 | H ₄ , H ₁₈ , H ₃₆ , H ₈₈ | 2 | H ₁ , H ₁₀ , H ₂₆ , H ₇₈ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₂₉₈ | 16 | R ₂₉₉ | 16 | R ₃₀₀ |
| 8 | N ₂₉₈ | 8 | N ₂₉₉ | 8 | N ₃₀₀ |
| 4 | L ₁₃₁ , L ₂₅₈ | 4 | L ₂₇₅ , L ₃₀₅ | 4 | L ₆₈ , L ₁₅₆ |
| 2 | H ₁₅ , H ₁₆ , H ₂₄ , H ₈₁ | 2 | H ₄ , H ₅₅ , H ₉₆ , H ₁₁₅ | 2 | H ₁₃ , H ₂₃ , H ₈₈ , H ₁₀₈ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₃₀₁ | 16 | R ₃₀₂ | 16 | R ₃₀₃ |
| 8 | N ₃₀₁ | 8 | N ₃₀₂ | 8 | N ₃₀₃ |
| 4 | L ₁₇₃ , L ₁₉₄ | 4 | L ₈₃ , L ₁₉₀ | 4 | L ₃₀ , L ₁₈₈ |
| 2 | H ₁₃ , H ₄₁ , H ₄₈ , H ₈₉ | 2 | H ₄ , H ₁₂ , H ₈₆ , H ₉₃ | 2 | H ₁₂ , H ₂₄ , H ₆₉ , H ₇₅ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₃₀₄ | 16 | R ₃₀₅ | 16 | R ₃₀₆ |
| 8 | N ₃₀₄ | 8 | N ₃₀₅ | 8 | N ₃₀₆ |
| 4 | L ₁₃₂ , L ₂₉₉ | 4 | L ₂₃₅ , L ₂₉₃ | 4 | L ₁₀₆ , L ₁₆₉ |
| 2 | H ₂₄ , H ₃₁ , H ₈₅ , H ₁₁₅ | 2 | H ₂₂ , H ₂₈ , H ₇₇ , H ₉₄ | 2 | H ₁₆ , H ₃₇ , H ₅₈ , H ₁₀₅ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₃₀₇ | 16 | R ₃₀₈ | 16 | R ₃₀₉ |
| 8 | N ₃₀₇ | 8 | N ₃₀₈ | 8 | N ₃₀₉ |
| 4 | L ₉₀ , L ₁₈₇ | 4 | L ₇₇ , L ₂₃₀ | 4 | L ₇₁ , L ₁₅₂ |
| 2 | H ₂ , H ₁₇ , H ₄₈ , H ₆₉ | 2 | H ₃₈ , H ₈₆ , H ₁₁₆ , H ₁₁₇ | 2 | H ₆ , H ₈ , H ₆₂ , H ₈₅ |

| | | | | | |
|--------------|--|--------------|---|--------------|--|
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₃₁₀ | 16 | R ₃₁₁ | 16 | R ₃₁₂ |
| 8 | N ₃₁₀ | 8 | N ₃₁₁ | 8 | N ₃₁₂ |
| 4 | L ₇₁ , L ₃₄₉ | 4 | L ₃₂₂ , L ₃₃₀ | 4 | L ₃₃₂ , L ₃₄₆ |
| 2 | H ₅₉ , H ₆₇ , H ₈₁ , H ₁₁₆ | 2 | H ₇₃ , H ₇₅ , H ₇₉ , H ₈₇ | 2 | H ₅₃ , H ₅₅ , H ₈₅ , H ₈₇ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₃₁₃ | 16 | R ₃₁₄ | 16 | R ₃₁₅ |
| 8 | N ₃₁₃ | 8 | N ₃₁₄ | 8 | N ₃₁₅ |
| 4 | L ₂₄₀ , L ₃₂₀ | 4 | L ₁₆₂ , L ₃₂₀ | 4 | L ₁₅₉ , L ₂₀₇ |
| 2 | H ₂₅ , H ₆₆ , H ₆₉ , H ₇₇ | 2 | H ₂₆ , H ₄₁ , H ₆₀ , H ₈₅ | 2 | H ₁₁ , H ₃₄ , H ₅₆ , H ₁₁₃ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₃₁₆ | 16 | R ₃₁₇ | 16 | R ₃₁₈ |
| 8 | N ₃₁₆ | 8 | N ₃₁₇ | 8 | N ₃₁₈ |
| 4 | L ₆₄ , L ₃₀₄ | 4 | L ₂₇₁ , L ₂₉₇ | 4 | L ₆₁ , L ₂₁₅ |
| 2 | H ₃ , H ₅₅ , H ₁₀₁ , H ₁₁₄ | 2 | H ₂ , H ₈₁ , H ₉₆ , H ₁₁₄ | 2 | H ₄ , H ₈₁ , H ₁₀₁ , H ₁₁₃ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₃₁₉ | 16 | R ₃₂₀ | 16 | R ₃₂₁ |
| 8 | N ₃₁₉ | 8 | N ₃₂₀ | 8 | N ₃₂₁ |
| 4 | L ₁₆₂ , L ₂₄₀ | 4 | L ₂₈₀ , L ₃₀₁ | 4 | L ₆₇ , L ₃₀₀ |
| 2 | H ₂₇ , H ₃₈ , H ₆₄ , H ₈₁ | 2 | H ₃ , H ₄₂ , H ₉₆ , H ₁₁₃ | 2 | H ₂ , H ₄₂ , H ₁₀₁ , H ₁₁₅ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₃₂₂ | 16 | R ₃₂₃ | 16 | R ₃₂₄ |
| 8 | N ₃₂₂ | 8 | N ₃₂₃ | 8 | N ₃₂₄ |
| 4 | L ₃₃₉ , L ₃₄₃ | 4 | L ₃₃₉ , L ₃₄₅ | 4 | L ₃₂₅ , L ₃₃₄ |
| 2 | H ₄₉ , H ₅₅ , H ₇₃ , H ₈₂ | 2 | H ₄₇ , H ₅₇ , H ₇₄ , H ₈₁ | 2 | H ₄₅ , H ₅₅ , H ₅₈ , H ₇₉ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₃₂₅ | 16 | R ₃₂₆ | 16 | R ₃₂₇ |
| 8 | N ₃₂₅ | 8 | N ₃₂₆ | 8 | N ₃₂₇ |
| 4 | L ₃₂₈ , L ₃₄₆ | 4 | L ₃₂₆ , L ₃₄₇ | 4 | L ₃₃₃ , L ₃₄₇ |
| 2 | H ₅₄ , H ₅₆ , H ₈₁ , H ₈₆ | 2 | H ₅₄ , H ₅₇ , H ₇₆ , H ₈₄ | 2 | H ₅₁ , H ₅₂ , H ₇₀ , H ₈₀ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₃₂₈ | 16 | R ₃₂₉ | 16 | R ₃₃₀ |
| 8 | N ₃₂₈ | 8 | N ₃₂₉ | 8 | N ₃₃₀ |
| 4 | L ₃₃₈ , L ₃₄₁ | 4 | L ₃₂₉ , L ₃₃₁ | 4 | L ₃₂₉ , L ₃₃₇ |
| 2 | H ₄₈ , H ₅₆ , H ₇₄ , H ₈₄ | 2 | H ₄₃ , H ₅₀ , H ₅₁ , H ₇₇ | 2 | H ₄₄ , H ₄₇ , H ₈₄ , H ₈₆ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₃₃₁ | 16 | R ₃₃₂ | 16 | R ₃₃₃ |
| 8 | N ₃₃₁ | 8 | N ₃₃₂ | 8 | N ₃₃₃ |
| 4 | L ₃₄₁ , L ₃₄₄ | 4 | L ₃₃₆ , L ₃₄₈ | 4 | L ₃₂₃ , L ₃₄₈ |
| 2 | H ₅₁ , H ₇₂ , H ₈₃ , H ₈₈ | 2 | H ₅₀ , H ₅₂ , H ₇₁ , H ₈₃ | 2 | H ₅₃ , H ₅₈ , H ₇₅ , H ₈₂ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₃₃₄ | 16 | R ₃₃₅ | 16 | R ₃₃₆ |
| 8 | N ₃₃₄ | 8 | N ₃₃₅ | 8 | N ₃₃₆ |
| 4 | L ₃₄₃ , L ₃₄₅ | 4 | L ₃₃₁ , L ₃₃₇ | 4 | L ₃₃₈ , L ₃₄₄ |
| 2 | H ₄₂ , H ₅₀ , H ₇₀ , H ₈₀ | 2 | H ₄₅ , H ₄₆ , H ₈₂ , H ₈₇ | 2 | H ₄₆ , H ₅₈ , H ₇₃ , H ₈₅ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R ₃₃₇ | 16 | R ₃₃₈ | 16 | R ₃₃₉ |
| 8 | N ₃₃₇ | 8 | N ₃₃₈ | 8 | N ₃₃₉ |

| | | | | | |
|--------------|----------------------------------|--------------|----------------------------------|--------------|----------------------------------|
| 4 | L_{334}, L_{340} | 4 | L_{324}, L_{335} | 4 | L_{335}, L_{342} |
| 2 | $H_{42}, H_{43}, H_{70}, H_{83}$ | 2 | $H_{44}, H_{56}, H_{57}, H_{78}$ | 2 | $H_{43}, H_{71}, H_{80}, H_{88}$ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R_{340} | 16 | R_{341} | 16 | R_{342} |
| 8 | N_{340} | 8 | N_{341} | 8 | N_{342} |
| 4 | L_{323}, L_{336} | 4 | L_{326}, L_{333} | 4 | L_{325}, L_{340} |
| 2 | $H_{47}, H_{48}, H_{54}, H_{78}$ | 2 | $H_{46}, H_{49}, H_{53}, H_{79}$ | 2 | $H_{44}, H_{48}, H_{76}, H_{81}$ |

| Order | Subgroups | Order | Subgroups | Order | Subgroups |
|--------------|------------------------------------|--------------|-----------------------------------|--------------|-----------------------------------|
| 16 | R_{343} | 16 | R_{344} | 16 | R_{345} |
| 8 | N_{343} | 8 | N_{344} | 8 | N_{345} |
| 4 | L_{328}, L_{332} | 4 | L_{324}, L_{342} | 4 | L_{14}, L_{84} |
| 2 | $H_{42}, H_{52}, H_{77}, H_{88}$ | 2 | $H_{45}, H_{49}, H_{75}, H_{85}$ | 2 | $H_{18}, H_{86}, H_{90}, H_{109}$ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R_{346} | 16 | R_{347} | 16 | R_{348} |
| 8 | N_{346} | 8 | N_{347} | 8 | N_{348} |
| 4 | L_{11}, L_{294} | 4 | L_{85}, L_{116} | 4 | L_{152}, L_{349} |
| 2 | $H_{11}, H_{22}, H_{37}, H_{78}$ | 2 | $H_{17}, H_{21}, H_{87}, H_{113}$ | 2 | $H_5, H_{37}, H_{40}, H_{77}$ |
| Order | Subgroups | Order | Subgroups | Order | Subgroups |
| 16 | R_{349} | 16 | R_{350} | 16 | R_{351} |
| 8 | N_{349} | 8 | N_{350} | 8 | N_{351} |
| 4 | L_{17}, L_{117} | 4 | L_7, L_{237} | 4 | L_{16}, L_{226} |
| 2 | $H_{19}, H_{77}, H_{107}, H_{110}$ | 2 | $H_{18}, H_{20}, H_{63}, H_{78}$ | 2 | $H_{16}, H_{19}, H_{59}, H_{71}$ |

We display one typical interval $[\{e\}, R_1]$ of $L(G)$ in the following diagram.

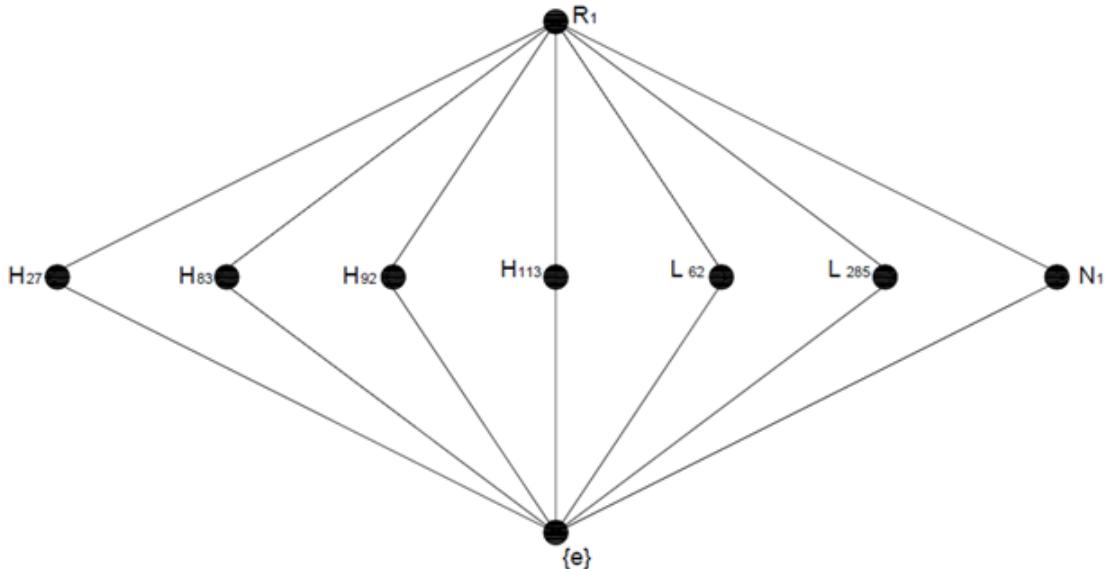


Fig. 5.1:The Interval $[\{e\}, R_1]$

VI. Conclusion:

In this paper, we produced the lattice structure of subgroups of order 16 in the subgroup lattices of 3×3 matrices over \mathbb{Z}_3 .

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