

The Lattice Structure of the Subgroups of Order 16 in the Subgroup Lattices Of 3 X 3 Matrices Over Z_3

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Abstract

Let \mathcal{G} be the set of all 3 X 3 non-singular matrices $\begin{pmatrix} a & bc \\ d & ef \\ g & hi \end{pmatrix}$, where a,b,c,d,e,f,g,h,i are integers modulo p . Then \mathcal{G} is a group under matrix multiplication modulo p , of order $(p^n - 1)(p^n - p)(p^n - p^2) \dots (p^n - p^{n-1})$. Let G be the subgroup of \mathcal{G} defined by $G = \{abcdefghi \in \mathcal{G} : abcdefghi = 1\}$. Then G is of order $\frac{(p^n - 1)(p^n - p)(p^n - p^2) \dots (p^n - p^{n-1})}{p-1}$. Let $L(G)$ be the lattice formed by all subgroups G . In this paper, we give the structure of the subgroups of order 16 of $L(G)$ in the case when $P=3$.

Keywords: Matrix group, subgroups, Lagrange's theorem, Lattice, Atom.

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I. Introduction

In 1992, Karan M. Gragg and P.S Kung [12] have attempted to characterize the finite groups with a consistent lattice of subgroups. In that endeavor, they discovered that the lattice of subnormal subgroups of a finite group is consistent and dually semi modular (lower semi modular). A. Vethamanickam has cited one of their theorems and has given a counter example in his thesis [19]. Suzuki's [13] results are mainly concerned with L-isomorphic groups. That is, groups whose lattice of subgroups are isomorphic.

In 2012, R. Sulaiman [18] has given the structure of the subgroup lattice of the symmetric group S_4 and Bashir Humera and Zahid Raza [2] have given the structure of the subgroup lattice of Quasidihedral group. In 2015, Jebaraj Thiraviam. D [6], in his thesis, has given the structure of the lattice of subgroups of the group of 2 x 2 matrices over Z_p having determinant value 1, under matrix multiplication modulo p , where p is prime and studied their properties.

Let $L(G)$ be the Lattice of Subgroups of G , where G is a group of 3x3 matrices over Z_p having determinant value 1 under matrix multiplication modulo p , where p is a prime number.

Let $\mathcal{G} = \left\{ \begin{pmatrix} a & bc \\ d & ef \\ g & hi \end{pmatrix} : a, b, c, d, e, f, g, h, i \in Z_p, \begin{vmatrix} a & bc \\ d & ef \\ g & hi \end{vmatrix} \neq 0 \right\}$

Then \mathcal{G} is a group under matrix multiplication modulo p .

Let $G = \left\{ \begin{pmatrix} a & bc \\ d & ef \\ g & hi \end{pmatrix} \in \mathcal{G} : \begin{vmatrix} a & bc \\ d & ef \\ g & hi \end{vmatrix} = 1 \right\}$

Then G is a subgroup of \mathcal{G} .

we have, $o(\mathcal{G}) = (p^n - 1)(p^n - p)(p^n - p^2) \dots (p^n - p^{n-1})$

and $o(G) = \frac{(p^n - 1)(p^n - p)(p^n - p^2) \dots (p^n - p^{n-1})}{p-1}$.

In this paper, we give the structure of the subgroups of order 16 of $L(G)$ in the case when $P=3$.

II. Preliminaries

In this section we give the definition needed for the development of the paper.

Definition 2.1

A partial order on a non-empty set P is a binary relation \leq on P that is reflexive, anti-symmetric and transitive. The pair (P, \leq) is called a **partially ordered set or poset**. A poset (P, \leq) is totally ordered if every $x, y \in P$ are comparable, that is either $x \leq y$ or $y \leq x$. A non-empty subset S of P is a chain in P if S is totally ordered by \leq .

Definition 2.2

Let (P, \leq) be a poset and let $S \subseteq P$. An upper bound of S is an element $x \in P$ for which $s \leq x$ for all $s \in S$. The least upper bound of S is called the **supremum or join** of S . A lower bound for S is an element $x \in P$ for which $x \leq s$ for all $s \in S$. The greatest lower bound of S is called the **infimum or meet** of S .

Definition 2.3

Poset (P, \leq) is called a **lattice** if every pair x, y elements of P has a supremum and an infimum, which are denoted by $x \vee y$ and $x \wedge y$ respectively.

Definition 2.4

For two elements a and b in P , a is said to **cover** b or b is said to be covered by a (in notation, $a > b$ or $b < a$) if and only if $b < a$ and, for no $x \in P$, $b < x < a$.

Definition 2.5

An element $a \in P$ is called an **atom**, if $a > 0$ and it is a dual atom, if $a < 1$.

Theorem 2.6

If G is a finite group and H is a subgroup of G , then the order of H is a divisor of the order of G .

Theorem 2.7

If G is a finite group and $a \in G$, then the order of ' a ' is a divisor of the order of G .

Theorem 2.8

Let G be a finite group and let p be any prime number that divides the order of G . Then G contains an element of order p .

Theorem 2.9

If p is a prime number and $p^\alpha \mid o(G)$, $p^{\alpha+1} \nmid o(G)$, then G has a subgroup of order p^α , called a p -sylow subgroup.

Theorem 2.10

The number of p -sylow subgroups in G , for a given prime p , is of the form $1+kp$.

III. Arrangement of elements of G according to their orders

The number of elements of order 2 is 117. The number of elements of order 3 is 728. The number of elements of order 4 is 702. The number of elements of order 6 is 936. The number of elements of order 8 is 1404. The number of elements of order 13 is 1728.

IV. Subgroups of G of different orders

The number of subgroups of order 2 is 117. The number of subgroups of order 3 is 364. The number of subgroups of order 4 is 351. The number of subgroups of order 6 is 468. The number of subgroups of order 8 is 468. The number of subgroups of order 9 is 117. The number of subgroups of order 13 is 144. The number of subgroups of order 16 is 351. The number of subgroups of order 27 is 52.

V. Lattice structure of some lower intervals of subgroups of order 16 in $L(G)$ over Z_3

Let R be an arbitrary subgroup of order 16. Then the elements of U must have orders 1, 3 or 9.

We tabulate the subgroups of order 16 in $L(G)$

Table 5.1: Intervals $\{[e], R_i\}$ in $L(G)$, $i = 1, 2, \dots, 351$.

Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R_1	16	R_2	16	R_3
8	N_1	8	N_2	8	N_3
4	L_{62}, L_{285}	4	L_{18}, L_{285}	4	L_{18}, L_{266}
2	$H_{27}, H_{83}, H_{92}, H_{113}$	2	$H_{25}, H_{29}, H_{82}, H_{107}$	2	$H_{19}, H_{69}, H_{82}, H_{150}$
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R_4	16	R_5	16	R_6
8	N_4	8	N_5	8	N_6
4	L_{248}, L_{317}	4	L_{234}, L_{316}	4	L_{272}, L_{292}
2	$H_{35}, H_{81}, H_{92}, H_{111}$	2	$H_{31}, H_{50}, H_{60}, H_{104}$	2	$H_{13}, H_{46}, H_{63}, H_{92}$
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R_7	16	R_8	16	R_9
8	N_7	8	N_8	8	N_9

4	L ₂₄ , L ₁₄₂	4	L ₂₅₂ , L ₃₁₈	4	L ₅₈ , L ₁₀₄
2	H ₂₅ , H ₃₆ , H ₇₅ , H ₉₃	2	H ₆ , H ₅₀ , H ₆₅ , H ₁₁₃	2	H ₄₁ , H ₇₂ , H ₉₃ , H ₁₀₃
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₀	16	R ₁₁	16	R ₁₂
8	N ₁₀	8	N ₁₁	8	N ₁₂
4	L ₄₄ , L ₃₁₉	4	L ₁₂₅ , L ₁₅₈	4	L ₁ , L ₂₆₃
2	H ₂₄ , H ₄₀ , H ₄₇ , H ₁₁₁	2	H ₃₈ , H ₇₆ , H ₉₃ , H ₉₄	2	H ₁₉ , H ₅₀ , H ₆₇ , H ₉₈

Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₃	16	R ₁₄	16	R ₁₅
8	N ₁₃	8	N ₁₄	8	N ₁₅
4	L ₆ , L ₈₇	4	L ₁₂₃ , L ₂₅₇	4	L ₁₀₉ , L ₃₁₉
2	H ₇ , H ₃₈ , H ₇₀ , H ₉₉	2	H ₁₉ , H ₄₇ , H ₆₈ , H ₉₆	2	H ₇ , H ₁₈ , H ₂₃ , H ₅₅
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₆	16	R ₁₇	16	R ₁₈
8	N ₁₆	8	N ₁₇	8	N ₁₈
4	L ₄₇ , L ₂₆₁	4	L ₂₃ , L ₃₀₄	4	L ₃₃ , L ₂₈₀
2	H ₅₀ , H ₁₀₃ , H ₁₁₂ , H ₁₁₇	2	H ₄₀ , H ₇₅ , H ₉₈ , H ₁₀₅	2	H ₅₁ , H ₁₀₀ , H ₁₀₉ , H ₁₁₇
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₉	16	R ₂₀	16	R ₂₁
8	N ₁₉	8	N ₂₀	8	N ₂₁
4	L ₂₉ , L ₃₀₃	4	L ₂₁ , L ₂₇₅	4	L ₁₃₇ , L ₂₁₀
2	H ₄₀ , H ₇₁ , H ₁₀₂ , H ₁₀₆	2	H ₄₆ , H ₆₆ , H ₉₉ , H ₁₀₉	2	H ₂₁ , H ₂₆ , H ₃₄ , H ₅₁
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₂	16	R ₂₃	16	R ₂₄
8	N ₂₂	8	N ₂₃	8	N ₂₄
4	L ₁₄₀ , L ₃₁₃	4	L ₁₁₅ , L ₃₀₆	4	L ₁₃ , L ₂₉₇
2	H ₃₅ , H ₅₃ , H ₈₉ , H ₁₀₄	2	H ₅ , H ₂₂ , H ₇₉ , H ₁₀₄	2	H ₄₀ , H ₇₈ , H ₉₄ , H ₁₀₄
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₅	16	R ₂₆	16	R ₂₇
8	N ₂₅	8	N ₂₆	8	N ₂₇
4	L ₂₃₆ , L ₃₁₅	4	L ₂₀₉ , L ₃₀₆	4	L ₂₃₆ , L ₂₅₅
2	H ₇ , H ₁₁ , H ₇₇ , H ₁₀₄	2	H ₆ , H ₈₄ , H ₉₁ , H ₁₀₃	2	H ₁₂ , H ₁₄ , H ₆₂ , H ₈₄
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₈	16	R ₂₉	16	R ₃₀
8	N ₂₈	8	N ₂₉	8	N ₃₀
4	L ₁₃ , L ₂₇₁	4	L ₂₁₄ , L ₂₅₆	4	L ₁₉₂ , L ₂₆₈
2	H ₈ , H ₈₄ , H ₉₅ , H ₁₀₉	2	H ₁ , H ₃₃ , H ₄₆ , H ₅₉	2	H ₁₅ , H ₄₃ , H ₆₄ , H ₁₀₄
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₁	16	R ₃₂	16	R ₃₃
8	N ₃₁	8	N ₃₂	8	N ₃₃
4	L ₁₂₄ , L ₂₇₇	4	L ₁₀₀ , L ₁₂₇	4	L ₁₀₃ , L ₂₇₉
2	H ₅ , H ₂₀ , H ₃₅ , H ₄₆	2	H ₂₁ , H ₂₄ , H ₅₂ , H ₁₀₅	2	H ₁₀ , H ₁₆ , H ₄₆ , H ₆₇
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₄	16	R ₃₅	16	R ₃₆
8	N ₃₄	8	N ₃₅	8	N ₃₆
4	L ₂₉ , L ₂₉₁	4	L ₂₆ , L ₂₇₈	4	L ₂₈ , L ₃₀₀
2	H ₃₉ , H ₄₈ , H ₁₀₄ , H ₁₁₀	2	H ₈ , H ₄₆ , H ₁₀₀ , H ₁₀₈	2	H ₃₀ , H ₇₁ , H ₉₄ , H ₁₀₅
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₇	16	R ₃₈	16	R ₃₉
8	N ₃₇	8	N ₃₈	8	N ₃₉
4	L ₂₃₃ , L ₂₄₃	4	L ₉ , L ₂₇₄	4	L ₁₇₈ , L ₂₈₇
2	H ₁₄ , H ₃₂ , H ₈₆ , H ₁₀₅	2	H ₈₄ , H ₉₉ , H ₁₀₈ , H ₁₁₇	2	H ₁₈ , H ₆₈ , H ₈₃ , H ₁₀₆
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₄₀	16	R ₄₁	16	R ₄₂
8	N ₄₀	8	N ₄₁	8	N ₄₂
4	L ₂₇₃ , L ₃₁₄	4	L ₂₃₈ , L ₃₁₄	4	L ₁₆₆ , L ₁₇₈
2	H ₃₂ , H ₆₁ , H ₈₄ , H ₉₃	2	H ₃₃ , H ₆₃ , H ₈₂ , H ₁₀₆	2	H ₉ , H ₁₇ , H ₃₉ , H ₈₄
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₄₃	16	R ₄₄	16	R ₄₅
8	N ₄₃	8	N ₄₄	8	N ₄₅
4	L ₂₉ , L ₃₀₃	4	L ₂₁ , L ₂₇₅	4	L ₂₉ , L ₃₀₃
2	H ₄₀ , H ₇₁ , H ₁₀₂ , H ₁₀₆	2	H ₄₆ , H ₆₆ , H ₉₉ , H ₁₀₉	2	H ₄₀ , H ₇₁ , H ₁₀₂ , H ₁₀₆

Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₄₆	16	R ₄₇	16	R ₄₈
8	N ₄₆	8	N ₄₇	8	N ₄₈
4	L ₂₁ , L ₂₇₅	4	L ₄₈ , L ₉₀	4	L ₁₂₉ , L ₂₁₀
2	H ₄₆ , H ₆₆ , H ₉₉ , H ₁₀₉	2	H ₆₅ , H ₆₇ , H ₇₃ , H ₁₀₅	2	H ₆ , H ₁₉ , H ₅₇ , H ₁₀₆
Order	Subgroups	Order	Subgroups	Order	Subgroups

16	R ₄₉	16	R ₅₀	16	R ₅₁
8	N ₄₉	8	N ₅₀	8	N ₅₁
4	L ₁₀₀ , L ₂₇₆	4	L ₂₃₃ , L ₂₅₁	4	L ₁₉₀ , L ₃₅₁
2	H ₂₂ , H ₂₇ , H ₄₆ , H ₈₉	2	H ₁₁ , H ₁₅ , H ₅₁ , H ₉₉	2	H ₄₅ , H ₅₉ , H ₉₂ , H ₁₀₆
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₅₂	16	R ₅₃	16	R ₅₄
8	N ₅₂	8	N ₅₃	8	N ₅₄
4	L ₁₉₃ , L ₂₅₆	4	L ₈₃ , L ₃₅₁	4	L ₂₁₆ , L ₃₁₂
2	H ₁₂ , H ₄₇ , H ₆₁ , H ₁₀₅	2	H ₇ , H ₃₁ , H ₅₁ , H ₆₄	2	H ₂₉ , H ₆₉ , H ₈₄ , H ₁₁₀
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₅₅	16	R ₅₆	16	R ₅₇
8	N ₅₅	8	N ₅₆	8	N ₅₇
4	L ₁₉ , L ₆₁	4	L ₁₉ , L ₂₁₅	4	L ₁₆₃ , L ₃₁₂
2	H ₄₁ , H ₇₈ , H ₁₀₂ , H ₁₀₅	2	H ₆₆ , H ₈₄ , H ₁₀₀ , H ₁₀₇	2	H ₁₀ , H ₈₅ , H ₁₀₅ , H ₁₁₂
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16	R ₅₈	16	R ₅₉	16	R ₆₀
8	N ₅₈	8	N ₅₉	8	N ₆₀
4	L ₂₄₇ , L ₂₈₁	4	L ₁₉ , L ₂₈₁	4	L ₁₂₄ , L ₁₄₃
2	H ₂₆ , H ₈₀ , H ₉₀ , H ₁₀₅	2	H ₁₉ , H ₂₃ , H ₂₅ , H ₈₄	2	H ₂₅ , H ₇₆ , H ₁₀₃ , H ₁₀₆
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₆₁	16	R ₆₂	16	R ₆₃
8	N ₅₃	8	N ₅₄	8	N ₆₃
4	L ₂₇₀ , L ₃₁₃	4	L ₂₆ , L ₃₁₁	4	L ₂₈ , L ₆₇
2	H ₂₄ , H ₅₁ , H ₉₀ , H ₁₁₆	2	H ₄₁ , H ₇₅ , H ₉₄ , H ₁₀₆	2	H ₈ , H ₅₁ , H ₉₉ , H ₁₀₇
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8	N ₇₀	8	N ₇₁	8	N ₇₂
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2	H ₇ , H ₂₆ , H ₄₈ , H ₁₀₈	2	H ₅₂ , H ₆₅ , H ₉₂ , H ₁₀₉	2	H ₂₉ , H ₅₃ , H ₉₃ , H ₁₁₄
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2	H ₂₃ , H ₅₄ , H ₉₁ , H ₁₀₆	2	H ₃₀ , H ₃₈ , H ₅₄ , H ₅₉	2	H ₂₅ , H ₃₇ , H ₄₂ , H ₆₁
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2	H ₂₅ , H ₄₀ , H ₆₃ , H ₈₈	2	H ₃₉ , H ₄₁ , H ₅₃ , H ₆₂	2	H ₈ , H ₂₆ , H ₃₉ , H ₈₇

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8	N ₇₉	8	N ₈₀	8	N ₈₁
4	L ₁₃₈ , L ₁₆₄	4	L ₁₃₉ , L ₂₂₄	4	L ₇₅ , L ₁₄₄
2	H ₃₇ , H ₅₂ , H ₆₃ , H ₆₆	2	H ₅₂ , H ₄₀ , H ₆₁ , H ₆₉	2	H ₃₀ , H ₆₇ , H ₈₆ , H ₁₂₇
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₈₂	16	R ₈₃	16	R ₈₄
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4	L ₄₉ , L ₁₉₇	4	L ₅₅ , L ₉₉	4	L ₅₂ , L ₁₀₁
2	H ₂₅ , H ₃₁ , H ₇₂ , H ₉₄	2	H ₂₇ , H ₃₁ , H ₇₄ , H ₉₆	2	H ₂₆ , H ₃₁ , H ₇₃ , H ₉₅
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₈₅	16	R ₈₆	16	R ₈₇
8	N ₈₅	8	N ₈₆	8	N ₈₇
4	L ₅₃ , L ₁₅₃	4	L ₅₀ , L ₉₆	4	L ₅₆ , L ₉₈
2	H ₂₅ , H ₃₃ , H ₇₃ , H ₉₆	2	H ₂₇ , H ₃₃ , H ₉₂ , H ₉₅	2	H ₂₆ , H ₃₃ , H ₇₄ , H ₉₄
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₈₈	16	R ₈₉	16	R ₉₀
8	N ₈₈	8	N ₈₉	8	N ₉₀
4	L ₅₇ , L ₉₇	4	L ₅₄ , L ₁₀₂	4	L ₅₁ , L ₉₅
2	H ₂₅ , H ₃₂ , H ₇₄ , H ₉₅	2	H ₂₇ , H ₃₂ , H ₇₃ , H ₉₄	2	H ₂₆ , H ₃₂ , H ₇₂ , H ₉₆

Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₉₁	16	R ₉₂	16	R ₉₃
8	N ₉₁	8	N ₉₂	8	N ₉₃
4	L ₁₄₁ , L ₃₁₈	4	L ₁₃₆ , L ₂₈₆	4	L ₃₁ , L ₂₉₀
2	H ₇ , H ₁₀ , H ₅₄ , H ₁₁₅	2	H ₁ , H ₂₈ , H ₅₄ , H ₁₀₈	2	H ₁₄ , H ₂₇ , H ₆₅ , H ₇₁
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₉₄	16	R ₉₅	16	R ₉₆
8	N ₉₄	8	N ₉₅	8	N ₉₆
4	L ₁₂₉ , L ₁₃₇	4	L ₃ , L ₁₃₅	4	L ₄₉ , L ₁₉₇
2	H ₄ , H ₂₀ , H ₂₂ , H ₅₃	2	H ₂ , H ₂₄ , H ₂₆ , H ₇₀	2	H ₁₁ , H ₅₂ , H ₉₇ , H ₁₁₀
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₉₇	16	R ₉₈	16	R ₉₉
8	N ₉₇	8	N ₉₈	8	N ₉₉
4	L ₅₃ , L ₁₉₉	4	L ₅₇ , L ₁₉₈	4	L ₅₅ , L ₂₀₀
2	H ₁₃ , H ₅₃ , H ₉₈ , H ₁₁₀	2	H ₁₂ , H ₅₄ , H ₉₉ , H ₁₁₀	2	H ₁₃ , H ₅₄ , H ₉₇ , H ₁₁₁
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₀₀	16	R ₁₀₁	16	R ₁₀₂
8	N ₁₀₀	8	N ₁₀₁	8	N ₁₀₂
4	L ₅₀ , L ₂₀₂	4	L ₅₄ , L ₂₀₁	4	L ₅₂ , L ₂₀₃
2	H ₅₂ , H ₁₂ , H ₉₈ , H ₁₁₁	2	H ₁₁ , H ₅₃ , H ₉₉ , H ₁₁₁	2	H ₁₂ , H ₅₃ , H ₉₇ , H ₁₁₂
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₀₃	16	R ₁₀₄	16	R ₁₀₅
8	N ₁₀₃	8	N ₁₀₄	8	N ₁₀₅
4	L ₅₆ , L ₂₀₅	4	L ₅₁ , L ₂₀₄	4	L ₁₄₂ , L ₂₂₀
2	H ₁₁ , H ₉₈ , H ₁₀₃ , H ₁₁₂	2	H ₁₃ , H ₅₂ , H ₉₉ , H ₁₁₂	2	H ₉ , H ₁₄ , H ₅₂ , H ₁₁₃
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₀₆	16	R ₁₀₇	16	R ₁₀₈
8	N ₁₀₆	8	N ₁₀₇	8	N ₁₀₈
4	L ₁₄₁ , L ₂₅₂	4	L ₁₄₃ , L ₂₇₇	4	L ₁₄₀ , L ₂₇₀
2	H ₁₅ , H ₂₅ , H ₅₈ , H ₁₁₄	2	H ₂ , H ₃₄ , H ₅₂ , H ₉₀	2	H ₃ , H ₂₅ , H ₅₇ , H ₉₁
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₀₉	16	R ₁₁₀	16	R ₁₁₁
8	N ₁₀₉	8	N ₁₁₀	8	N ₁₁₁
4	L ₁₄₄ , L ₂₂₃	4	L ₆₀ , L ₁₃₉	4	L ₇₈ , L ₂₂₂
2	H ₈ , H ₅₃ , H ₆₀ , H ₆₈	2	H ₂₇ , H ₅₆ , H ₅₉ , H ₁₁₇	2	H ₅₄ , H ₆₄ , H ₆₇ , H ₁₁₇

Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₁₂	16	R ₁₁₃	16	R ₁₁₄
8	N ₁₁₂	8	N ₁₁₃	8	N ₁₁₄
4	L ₇₃ , L ₁₃₈	4	L ₃ , L ₂₂₁	4	L ₃₁ , L ₁₈₁
2	H ₂₆ , H ₅₅ , H ₆₂ , H ₆₈	2	H ₃ , H ₁₉ , H ₅₄ , H ₁₀₃	2	H ₁₅ , H ₃₆ , H ₅₃ , H ₁₀₇
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₁₅	16	R ₁₁₆	16	R ₁₁₇
8	N ₁₁₅	8	N ₁₁₆	8	N ₁₁₇
4	L ₁₃₆ , L ₁₇₄	4	L ₅₉ , L ₁₀₃	4	L ₂₇₄ , L ₂₉₈
2	H ₉ , H ₂₇ , H ₅₀ , H ₁₀₉	2	H ₃₆ , H ₃₈ , H ₇₂ , H ₁₀₆	2	H ₃ , H ₈₁ , H ₉₇ , H ₁₁₅
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₁₈	16	R ₁₁₉	16	R ₁₂₀
8	N ₁₁₈	8	N ₁₁₉	8	N ₁₂₀
4	L ₂₃₈ , L ₂₇₃	4	L ₁₆₆ , L ₂₈₇	4	L ₂₆₉ , L ₃₀₃
2	H ₃ , H ₁₅ , H ₃₁ , H ₇₇	2	H ₃ , H ₁₆ , H ₆₅ , H ₈₅	2	H ₄ , H ₄₂ , H ₉₇ , H ₁₁₄
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₂₁	16	R ₁₂₂	16	R ₁₂₃
8	N ₁₂₁	8	N ₁₂₂	8	N ₁₂₃
4	L ₄₃ , L ₁₁₀	4	L ₂₇₈ , L ₃₁₁	4	L ₅₉ , L ₂₇₉
2	H ₄ , H ₂₈ , H ₆₈ , H ₇₂	2	H ₂ , H ₅₅ , H ₉₇ , H ₁₁₃	2	H ₂ , H ₉ , H ₅₆ , H ₁₁₀
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₂₄	16	R ₁₂₅	16	R ₁₂₆
8	N ₁₂₄	8	N ₁₂₅	8	N ₁₂₆
4	L ₁₂₂ , L ₂₆₀	4	L ₆₉ , L ₈₇	4	L ₂₇ , L ₂₆₂
2	H ₂₁ , H ₄₇ , H ₆₉ , H ₉₈	2	H ₁₅ , H ₈₈ , H ₉₀ , H ₉₅	2	H ₁₅ , H ₂₉ , H ₇₅ , H ₁₁₆
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₂₇	16	R ₁₂₈	16	R ₁₂₉
8	N ₁₂₇	8	N ₁₂₈	8	N ₁₂₉
4	L ₆ , L ₆₉	4	L ₁₇₁ , L ₂₆₀	4	L ₂₅₄ , L ₂₉₆
2	H ₂₁ , H ₅₀ , H ₆₈ , H ₁₀₀	2	H ₁ , H ₅₆ , H ₈₉ , H ₉₅	2	H ₃₆ , H ₈₂ , H ₁₁₄ , H ₁₁₆
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₃₀	16	R ₁₃₁	16	R ₁₃₂
8	N ₁₃₀	8	N ₁₃₁	8	N ₁₃₂
4	L ₇₂ , L ₁₅₁	4	L ₂₅₅ , L ₃₁₅	4	L ₇₂ , L ₁₁₄

2	H ₁₄ , H ₈₅ , H ₉₁ , H ₉₅	2	H ₂ , H ₁₃ , H ₅₉ , H ₈₂	2	H ₂₁ , H ₆₇ , H ₈₂ , H ₉₆
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₃₃	16	R ₁₃₄	16	R ₁₃₅
8	N ₁₃₃	8	N ₁₃₄	8	N ₁₃₅
4	L ₁₂ , L ₂₅₄	4	L ₁₄₇ , L ₂₆₂	4	L ₁₃₃ , L ₂₄₅
2	H ₁₄ , H ₁₈ , H ₆₇ , H ₈₂	2	H ₂₆ , H ₂₈ , H ₄₇ , H ₁₁₅	2	H ₂₄ , H ₂₈ , H ₈₆ , H ₁₀₀
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₃₆	16	R ₁₃₇	16	R ₁₃₈
8	N ₁₃₆	8	N ₁₃₇	8	N ₁₃₈
4	L ₇₉ , L ₂₁₂	4	L ₁₁₂ , L ₂₈₈	4	L ₂₈₈ , L ₃₀₈
2	H ₃₅ , H ₅₈ , H ₆₃ , H ₉₅	2	H ₂₀ , H ₃₀ , H ₃₃ , H ₈₃	2	H ₃₁ , H ₆₅ , H ₈₁ , H ₈₉
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₃₉	16	R ₁₄₀	16	R ₁₄₁
8	N ₁₃₉	8	N ₁₄₀	8	N ₁₄₁
4	L ₂₃₉ , L ₃₅₀	4	L ₁₇₇ , L ₂₃₉	4	L ₈₀ , L ₁₃₀
2	H ₂₄ , H ₆₅ , H ₇₇ , H ₉₇	2	H ₃₄ , H ₆₃ , H ₈₃ , H ₉₈	2	H ₂₄ , H ₂₉ , H ₈₇ , H ₉₄
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₄₂	16	R ₁₄₃	16	R ₁₄₄
8	N ₁₄₂	8	N ₁₄₃	8	N ₁₄₄
4	L ₇₄ , L ₁₈₄	4	L ₁₂₀ , L ₂₈₃	4	L ₅ , L ₁₇₅
2	H ₄₈ , H ₆₃ , H ₁₀₁ , H ₁₀₃	2	H ₂₁ , H ₄₁ , H ₄₄ , H ₆₅	2	H ₁₁ , H ₁₉ , H ₅₈ , H ₁₁₇

Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₄₅	16	R ₁₄₆	16	R ₁₄₇
8	N ₁₄₅	8	N ₁₄₆	8	N ₁₄₇
4	L ₁₇₄ , L ₂₈₆	4	L ₁₈₆ , L ₂₈₃	4	L ₁₉₆ , L ₂₈₉
2	H ₅₈ , H ₉₃ , H ₁₀₇ , H ₁₁₆	2	H ₃₂ , H ₄₀ , H ₅₈ , H ₉₀	2	H ₁₂ , H ₂₀ , H ₂₉ , H ₄₂
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₄₈	16	R ₁₄₉	16	R ₁₅₀
8	N ₁₄₈	8	N ₁₄₉	8	N ₁₅₀
4	L ₄₆ , L ₁₈₀	4	L ₈₁ , L ₂₂₈	4	L ₁₇₂ , L ₂₄₄
2	H ₃ , H ₅₈ , H ₆₇ , H ₁₁₂	2	H ₂₃ , H ₂₈ , H ₈₇ , H ₉₇	2	H ₃₄ , H ₄₈ , H ₆₄ , H ₉₅
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₅₁	16	R ₁₅₂	16	R ₁₅₃
8	N ₁₅₁	8	N ₁₅₂	8	N ₁₅₃
4	L ₇₄ , L ₂₃₂	4	L ₈₀ , L ₁₇₉	4	L ₂₄₁ , L ₂₈₂
2	H ₂₃ , H ₆₅ , H ₈₆ , H ₉₄	2	H ₅₈ , H ₆₄ , H ₉₈ , H ₁₀₃	2	H ₂₃ , H ₂₉ , H ₇₇ , H ₁₀₀
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₅₄	16	R ₁₅₅	16	R ₁₅₆
8	N ₁₅₄	8	N ₁₅₅	8	N ₁₅₆
4	L ₁₆₃ , L ₂₁₆	4	L ₁₈₅ , L ₂₄₁	4	L ₁₈₉ , L ₂₂₅
2	H ₄ , H ₃₈ , H ₈₃ , H ₁₁₁	2	H ₃₅ , H ₆₄ , H ₈₃ , H ₁₀₁	2	H ₂₉ , H ₃₀ , H ₄₃ , H ₉₁
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₅₇	16	R ₁₅₈	16	R ₁₅₉
8	N ₁₅₇	8	N ₁₅₈	8	N ₁₅₉
4	L ₁₅₅ , L ₁₈₈	4	L ₂₄₉ , L ₂₉₂	4	L ₃₀ , L ₁₅₅
2	H ₂₃ , H ₃₉ , H ₄₃ , H ₁₁₄	2	H ₃₃ , H ₃₇ , H ₅₇ , H ₁₁₃	2	H ₂₂ , H ₃₃ , H ₅₆ , H ₁₀₇
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₆₀	16	R ₁₆₁	16	R ₁₆₂
8	N ₁₆₀	8	N ₁₆₁	8	N ₁₆₂
4	L ₂₀ , L ₂₉₂	4	L ₄ , L ₂₂₅	4	L ₁₉₁ , L ₃₁₆
2	H ₂₄ , H ₃₈ , H ₄₅ , H ₁₀₉	2	H ₁₉ , H ₂₈ , H ₃₂ , H ₅₅	2	H ₄₄ , H ₆₂ , H ₉₃ , H ₁₀₅
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₆₃	16	R ₁₆₄	16	R ₁₆₅
8	N ₁₆₃	8	N ₁₆₄	8	N ₁₆₅
4	L ₉₄ , L ₁₉₇	4	L ₁₀₁ , L ₂₀₃	4	L ₉₉ , L ₂₀₀
2	H ₅ , H ₁₆ , H ₄₃ , H ₁₀₀	2	H ₆ , H ₁₆ , H ₄₅ , H ₁₀₂	2	H ₁₆ , H ₄₄ , H ₁₀₁ , H ₁₁₆
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₆₆	16	R ₁₆₇	16	R ₁₆₈
8	N ₁₆₆	8	N ₁₆₇	8	N ₁₆₈
4	L ₁₅₃ , L ₁₉₉	4	L ₉₈ , L ₂₀₅	4	L ₉₆ , L ₂₀₂
2	H ₁₈ , H ₄₅ , H ₁₀₀ , H ₁₁₆	2	H ₅ , H ₁₈ , H ₄₄ , H ₁₀₂	2	H ₆ , H ₁₈ , H ₄₃ , H ₁₀₁
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₆₉	16	R ₁₇₀	16	R ₁₇₁
8	N ₁₆₉	8	N ₁₇₀	8	N ₁₇₁
4	L ₉₇ , L ₁₉₈	4	L ₉₅ , L ₂₀₄	4	L ₁₀₂ , L ₂₀₁
2	H ₆ , H ₁₇ , H ₄₄ , H ₁₀₀	2	H ₁₇ , H ₄₃ , H ₁₀₂ , H ₁₁₆	2	H ₅ , H ₁₇ , H ₄₅ , H ₁₀₁
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₇₂	16	R ₁₇₃	16	R ₁₇₄
8	N ₁₇₂	8	N ₁₇₃	8	N ₁₇₄

4	L ₂₅ , L ₂₀₇	4	L ₆₈ , L ₂₄₆	4	L ₁₂₆ , L ₂₀₆
2	H ₃₅ , H ₄₃ , H ₆₇ , H ₁₀₈	2	H ₃₁ , H ₄₉ , H ₆₉ , H ₁₀₉	2	H ₈ , H ₁₀ , H ₂₀ , H ₄₃
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₇₅	16	R ₁₇₆	16	R ₁₇₇
8	N ₁₇₅	8	N ₁₇₆	8	N ₁₇₇
4	L ₁₈₂ , L ₂₈₉	4	L ₂₄₃ , L ₂₅₁	4	L ₂₆₈ , L ₂₇₂
2	H ₂₁ , H ₃₁ , H ₄₈ , H ₆₆	2	H ₁ , H ₂ , H ₄₅ , H ₆₃	2	H ₄ , H ₃₂ , H ₄₇ , H ₆₂

Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₇₈	16	R ₁₇₉	16	R ₁₈₀
8	N ₁₇₈	8	N ₁₇₉	8	N ₁₈₀
4	L ₂₂₇ , L ₂₅₀	4	L ₁₇₅ , L ₃₀₉	4	L ₆₆ , L ₁₇₃
2	H ₄ , H ₁₄ , H ₄₄ , H ₆₁	2	H ₃₆ , H ₄₄ , H ₆₆ , H ₈₉	2	H ₁₀ , H ₃₃ , H ₄₂ , H ₉₁
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₈₁	16	R ₁₈₂	16	R ₁₈₃
8	N ₁₈₁	8	N ₁₈₂	8	N ₁₈₃
4	L ₂₁₁ , L ₃₀₂	4	L ₁₇₀ , L ₃₂₁	4	L ₁₉₃ , L ₂₁₄
2	H ₄₅ , H ₆₉ , H ₁₀₃ , H ₁₁₅	2	H ₃₂ , H ₃₅ , H ₈₈ , H ₁₁₄	2	H ₃ , H ₇ , H ₄₃ , H ₆₀
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₈₄	16	R ₁₈₅	16	R ₁₈₆
8	N ₁₈₄	8	N ₁₈₅	8	N ₁₈₆
4	L ₈₂ , L ₂₂₇	4	L ₁₂₆ , L ₃₁₀	4	L ₁₈₂ , L ₁₉₆
2	H ₂ , H ₃₃ , H ₈₇ , H ₉₂	2	H ₃₁ , H ₃₆ , H ₄₁ , H ₇₆	2	H ₉ , H ₁₉ , H ₄₀ , H ₄₅
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₈₇	16	R ₁₈₈	16	R ₁₈₉
8	N ₁₈₇	8	N ₁₈₈	8	N ₁₈₉
4	L ₂₅ , L ₁₅₉	4	L ₁₅₆ , L ₂₄₆	4	L ₁₉₅ , L ₃₂₁
2	H ₃₁ , H ₃₈ , H ₇₅ , H ₁₀₃	2	H ₃₄ , H ₃₇ , H ₄₄ , H ₁₀₇	2	H ₂₂ , H ₄₄ , H ₆₈ , H ₁₁₃
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₉₀	16	R ₁₉₁	16	R ₁₉₂
8	N ₁₉₀	8	N ₁₉₁	8	N ₁₉₂
4	L ₃₂ , L ₃₀₂	4	L ₆₆ , L ₁₉₄	4	L ₅ , L ₃₀₉
2	H ₂₃ , H ₃₂ , H ₆₇ , H ₇₁	2	H ₂₈ , H ₄₅ , H ₉₀ , H ₁₁₇	2	H ₈ , H ₂₉ , H ₃₃ , H ₇₀
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₉₃	16	R ₁₉₄	16	R ₁₉₅
8	N ₁₉₃	8	N ₁₉₄	8	N ₁₉₅
4	L ₁₂₃ , L ₁₅₀	4	L ₂ , L ₁₅₄	4	L ₁₂ , L ₂₉₆
2	H ₃₇ , H ₇₆ , H ₉₂ , H ₉₉	2	H ₃₇ , H ₇₀ , H ₉₃ , H ₁₀₁	2	H ₅ , H ₇ , H ₂₈ , H ₇₈
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₉₆	16	R ₁₉₇	16	R ₁₉₈
8	N ₁₉₆	8	N ₁₉₇	8	N ₁₉₈
4	L ₁₁₄ , L ₁₅₁	4	L ₂₄ , L ₂₂₀	4	L ₁₂₅ , L ₂₆₅
2	H ₇ , H ₃₇ , H ₇₉ , H ₉₄	2	H ₅ , H ₁₀ , H ₄₇ , H ₁₀₈	2	H ₂₀ , H ₄₇ , H ₆₇ , H ₁₀₀
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₁₉₉	16	R ₂₀₀	16	R ₂₀₁
8	N ₁₉₉	8	N ₂₀₀	8	N ₂₀₁
4	L ₁ , L ₁₆₈	4	L ₁₀₄ , L ₂₆₄	4	L ₄₀ , L ₁₃₄
2	H ₃₉ , H ₇₀ , H ₉₂ , H ₉₄	2	H ₈ , H ₁₆ , H ₃₅ , H ₄₇	2	H ₇ , H ₃₀ , H ₄₉ , H ₁₁₀
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₀₂	16	R ₂₀₃	16	R ₂₀₄
8	N ₂₀₂	8	N ₂₀₃	8	N ₂₀₄
4	L ₁₁₁ , L ₂₅₉	4	L ₁₃₁ , L ₂₁₃	4	L ₂₁₃ , L ₂₅₈
2	H ₂₀ , H ₆₈ , H ₈₂ , H ₉₈	2	H ₁₇ , H ₉₃ , H ₈₀ , H ₁₁₇	2	H ₁₈ , H ₃₀ , H ₃₄ , H ₈₂
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₀₅	16	R ₂₀₆	16	R ₂₀₇
8	N ₂₀₅	8	N ₂₀₆	8	N ₂₀₇
4	L ₁₁₁ , L ₁₆₇	4	L ₂ , L ₂₅₃	4	L ₁₂₂ , L ₁₇₁
2	H ₃₉ , H ₇₉ , H ₉₃ , H ₉₉	2	H ₂₀ , H ₅₀ , H ₆₉ , H ₉₆	2	H ₇ , H ₃₉ , H ₇₆ , H ₁₀₁
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₀₈	16	R ₂₀₉	16	R ₂₁₀
8	N ₂₀₈	8	N ₂₀₉	8	N ₂₁₀
4	L ₄₇ , L ₉₁	4	L ₁₀₇ , L ₁₃₄	4	L ₂₅₀ , L ₈₂
2	H ₈ , H ₂₄ , H ₇₄ , H ₉₂	2	H ₁₇ , H ₄₁ , H ₂₂ , H ₅₀	2	H ₅₀ , H ₁₁ , H ₃ , H ₆₄

Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₁₁	16	R ₂₁₂	16	R ₂₁₃
8	N ₂₁₁	8	N ₂₁₂	8	N ₂₁₃
4	L ₁₁₈ , L ₁₆₁	4	L ₂₆₇ , L ₃₁₇	4	L ₁₇₂ , L ₂₃₁
2	H ₃₈ , H ₇₉ , H ₉₂ , H ₁₀₁	2	H ₂₃ , H ₈₂ , H ₆₆ , H ₁₁₀	2	H ₉ , H ₅₇ , H ₅₉ , H ₁₀₂
Order	Subgroups	Order	Subgroups	Order	Subgroups

16	R ₂₁₄	16	R ₂₁₅	16	R ₂₁₆
8	N ₂₁₄	8	N ₂₁₅	8	N ₂₁₆
4	L ₁₃₃ , L ₁₇₆	4	L ₇₉ , L ₂₂₉	4	L ₁₈₄ , L ₂₃₂
2	H ₁₀ , H ₅₇ , H ₆₁ , H ₉₆	2	H ₂₂ , H ₆₅ , H ₈₇ , H ₁₀₀	2	H ₃₆ , H ₅₇ , H ₆₀ , H ₉₉
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₁₇	16	R ₂₁₈	16	R ₂₁₉
8	N ₂₁₇	8	N ₂₁₈	8	N ₂₁₉
4	L ₁₉₁ , L ₂₃₄	4	L ₇₆ , L ₂₁₇	4	L ₃₂₇ , L ₃₃₀
2	H ₁ , H ₁₃ , H ₈₇ , H ₁₀₆	2	H ₆ , H ₄₁ , H ₆₈ , H ₈₇	2	H ₇₀ , H ₇₁ , H ₇₂ , H ₇₇
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₂₀	16	R ₂₂₁	16	R ₂₂₂
8	N ₂₂₀	8	N ₂₂₁	8	N ₂₂₂
4	L ₁₆₅ , L ₂₂₂	4	L ₂₁₂ , L ₂₂₉	4	L ₂₃₁ , L ₂₄₄
2	H ₅ , H ₆₁ , H ₆₆ , H ₈₈	2	H ₁₀ , H ₄₉ , H ₅₉ , H ₉₉	2	H ₂₂ , H ₂₉ , H ₈₆ , H ₉₇
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₂₃	16	R ₂₂₄	16	R ₂₂₅
8	N ₂₂₃	8	N ₂₂₄	8	N ₂₂₅
4	L ₁₃₀ , L ₁₇₉	4	L ₁₈₃ , L ₂₂₈	4	L ₁₆ , L ₃₄
2	H ₃₆ , H ₄₉ , H ₆₁ , H ₁₀₂	2	H ₉ , H ₄₉ , H ₆₀ , H ₉₆	2	H ₂₁ , H ₆₄ , H ₇₃ , H ₁₀₈
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₂₆	16	R ₂₂₇	16	R ₂₂₈
8	N ₂₂₆	8	N ₂₂₇	8	N ₂₂₈
4	L ₁₀ , L ₃₈	4	L ₃₅ , L ₈₆	4	L ₁₅ , L ₆₃
2	H ₆₃ , H ₇₆ , H ₁₀₉ , H ₁₁₂	2	H ₂₀ , H ₆₀ , H ₇₄ , H ₁₀₉	2	H ₂₁ , H ₆₁ , H ₇₅ , H ₁₁₁
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₂₉	16	R ₂₃₀	16	R ₂₃₁
8	N ₂₂₉	8	N ₂₃₀	8	N ₂₃₁
4	L ₃₆ , L ₈₅	4	L ₁₄ , L ₃₇	4	L ₃₆ , L ₁₁₆
2	H ₆₃ , H ₇₄ , H ₉₀ , H ₁₁₄	2	H ₆₂ , H ₇₁ , H ₈₉ , H ₁₁₀	2	H ₅₉ , H ₇₀ , H ₁₁₀ , H ₁₁₅
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₃₂	16	R ₂₃₃	16	R ₂₃₄
8	N ₂₃₂	8	N ₂₃₃	8	N ₂₃₄
4	L ₃₇ , L ₈₄	4	L ₃₅ , L ₂₄₂	4	L ₁₅ , L ₈₉
2	H ₆₁ , H ₇₃ , H ₉₁ , H ₁₁₃	2	H ₈₇ , H ₉₁ , H ₁₀₈ , H ₁₁₁	2	H ₅₉ , H ₈₉ , H ₁₀₇ , H ₁₇₂
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₃₅	16	R ₂₃₆	16	R ₂₃₇
8	N ₂₃₅	8	N ₂₃₆	8	N ₂₃₇
4	L ₃₈ , L ₈₈	4	L ₃₄ , L ₂₂₆	4	L ₃₉ , L ₁₀₈
2	H ₁₉ , H ₆₂ , H ₇₂ , H ₁₁₅	2	H ₂₀ , H ₈₆ , H ₁₁₂ , H ₁₁₄	2	H ₃ , H ₁₀ , H ₃₇ , H ₇₃
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₃₈	16	R ₂₃₉	16	R ₂₄₀
8	N ₂₃₈	8	N ₂₃₉	8	N ₂₄₀
4	L ₅₈ , L ₂₆₄	4	L ₄₀ , L ₁₀₇	4	L ₄₁ , L ₁₀₆
2	H ₁₄ , H ₃₄ , H ₅₅ , H ₁₁₀	2	H ₁₅ , H ₃₅ , H ₄₀ , H ₇₄	2	H ₉ , H ₆₉ , H ₇₄ , H ₁₀₄
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₄₁	16	R ₂₄₂	16	R ₂₄₃
8	N ₂₄₁	8	N ₂₄₂	8	N ₂₄₃
4	L ₄₂ , L ₁₀₅	4	L ₄₄ , L ₁₀₉	4	L ₄₅ , L ₆₅
2	H ₇ , H ₃₄ , H ₆₆ , H ₇₃	2	H ₁ , H ₂₂ , H ₇₂ , H ₁₁₇	2	H ₈ , H ₁₅ , H ₅₇ , H ₁₁₁

Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₄₄	16	R ₂₄₅	16	R ₂₄₆
8	N ₂₄₄	8	N ₂₄₅	8	N ₂₄₆
4	L ₄₅ , L ₉₃	4	L ₄₆ , L ₉₂	4	L ₄₂ , L ₁₂₈
2	H ₁₄ , H ₂₃ , H ₃₀ , H ₇₃	2	H ₂ , H ₂₉ , H ₃₉ , H ₇₄	2	H ₂₂ , H ₄₂ , H ₉₃ , H ₁₁₂
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₄₇	16	R ₂₄₈	16	R ₂₄₉
8	N ₂₄₇	8	N ₂₄₈	8	N ₂₄₉
4	L ₄₁ , L ₁₆₉	4	L ₆₀ , L ₁₁₇	4	L ₁₇ , L ₆₀
2	H ₂₈ , H ₈₈ , H ₁₀₆ , H ₁₁₁	2	H ₆₄ , H ₁₁₁ , H ₁₁₃ , H ₁₇₉	2	H ₆₀ , H ₇₈ , H ₉₀ , H ₁₁₂
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₅₀	16	R ₂₅₁	16	R ₂₅₂
8	N ₂₅₀	8	N ₂₅₁	8	N ₂₅₂
4	L ₁₁₉ , L ₂₄₇	4	L ₁₆₁ , L ₂₆₆	4	L ₆₃ , L ₈₉
2	H ₄ , H ₂₇ , H ₃₅ , H ₇₉	2	H ₁ , H ₈₅ , H ₉₀ , H ₁₀₂	2	H ₁₈ , H ₆₀ , H ₇₆ , H ₁₁₄
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₅₃	16	R ₂₅₂	16	R ₂₅₅
8	N ₂₅₃	8	N ₂₅₂	8	N ₂₅₅
4	L ₂₃₇ , L ₃₀₇	4	L ₆₃ , L ₈₉	4	L ₁₁₂ , L ₃₀₈
2	H ₁₆ , H ₇₇ , H ₈₉ , H ₁₁₅	2	H ₁₈ , H ₆₀ , H ₇₆ , H ₁₁₄	2	H ₉ , H ₃₂ , H ₇₉ , H ₁₁₇

Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₅₆	16	R ₂₅₇	16	R ₂₅₈
8	N ₂₅₆	8	N ₂₅₇	8	N ₂₅₈
4	L ₁₆₇ , L ₂₅₉	4	L ₂₈₄ , L ₂₉₅	4	L ₁₁₃ , L ₂₈₄
2	H ₁₅ , H ₈₅ , H ₈₉ , H ₉₇	2	H ₈ , H ₁₂ , H ₈₃ , H ₉₁	2	H ₁₁ , H ₂₈ , H ₄₀ , H ₇₉
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₅₉	16	R ₂₆₀	16	R ₂₆₁
8	N ₂₅₉	8	N ₂₆₀	8	N ₂₆₁
4	L ₁₀ , L ₈₈	4	L ₈₆ , L ₂₄₂	4	L ₁₂₇ , L ₂₇₆
2	H ₁₇ , H ₆₄ , H ₇₅ , H ₉₁	2	H ₁₆ , H ₆₂ , H ₇₀ , H ₁₀₇	2	H ₃ , H ₆ , H ₂₃ , H ₇₆
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₆₂	16	R ₂₆₃	16	R ₂₆₄
8	N ₂₆₂	8	N ₂₆₃	8	N ₂₆₄
4	L ₁₅₈ , L ₂₆₅	4	L ₂₀₆ , L ₃₁₀	4	L ₄ , L ₁₈₉
2	H ₁₄ , H ₅₆ , H ₉₀ , H ₁₉₇	2	H ₉ , H ₁₁ , H ₅₅ , H ₉₀	2	H ₁₃ , H ₆₅ , H ₆₆ , H ₇₆
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₆₅	16	R ₂₆₆	16	R ₂₆₇
8	N ₂₆₅	8	N ₂₆₆	8	N ₂₆₇
4	L ₁₅₀ , L ₂₅₇	4	L ₁₂₀ , L ₁₈₆	4	L ₁₂₁ , L ₂₁₈
2	H ₁₅ , H ₅₆ , H ₉₁ , H ₁₀₂	2	H ₁₀ , H ₁₂ , H ₃₀ , H ₇₀	2	H ₃₄ , H ₇₀ , H ₁₀₅ , H ₁₁₆
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₆₈	16	R ₂₆₉	16	R ₂₇₀
8	N ₂₆₈	8	N ₂₆₉	8	N ₂₇₀
4	L ₁₆₈ , L ₂₆₃	4	L ₃₂₂ , L ₃₂₇	4	L ₁₅₄ , L ₂₅₃
2	H ₁₄ , H ₈₈ , H ₈₉ , H ₁₀₂	2	H ₇₄ , H ₇₆ , H ₇₈ , H ₈₆	2	H ₁ , H ₉₁ , H ₉₇ , H ₈₈
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₇₁	16	R ₂₇₂	16	R ₂₇₃
8	N ₂₇₁	8	N ₂₇₂	8	N ₂₇₃
4	L ₁₃₅ , L ₂₂₁	4	L ₂₀ , L ₂₄₉	4	L ₇₀ , L ₂₉₄
2	H ₄ , H ₅ , H ₄₉ , H ₈₉	2	H ₁₃ , H ₃₅ , H ₃₉ , H ₇₁	2	H ₁₃ , H ₆₇ , H ₈₀ , H ₁₁₄
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₇₄	16	R ₂₇₅	16	R ₂₇₆
8	N ₂₇₄	8	N ₂₇₅	8	N ₂₇₆
4	L ₂₀₈ , L ₂₉₃	4	L ₂₀₈ , L ₂₃₅	4	L ₁₁₅ , L ₂₀₉
2	H ₃₆ , H ₅₉ , H ₈₀ , H ₉₆	2	H ₆₂ , H ₈₃ , H ₉₅ , H ₁₀₃	2	H ₂ , H ₂₁ , H ₈₀ , H ₁₁₆

Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₇₇	16	R ₂₇₈	16	R ₂₇₉
8	N ₂₇₇	8	N ₂₇₈	8	N ₂₇₉
4	L ₁₁ , L ₇₀	4	L ₃₂ , L ₂₁₁	4	L ₆₅ , L ₉₃
2	H ₁₂ , H ₈₅ , H ₁₀₃ , H ₁₀₉	2	H ₁₁ , H ₅₇ , H ₆₈ , H ₁₀₇	2	H ₁ , H ₁₇ , H ₄₂ , H ₁₀₃
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₈₀	16	R ₂₈₁	16	R ₂₈₂
8	N ₂₈₀	8	N ₂₈₁	8	N ₂₈₂
4	L ₈₁ , L ₁₈₃	4	L ₂₄₈ , L ₂₆₇	4	L ₁₈₅ , L ₂₈₂
2	H ₃₄ , H ₅₈ , H ₆₂ , H ₁₀₁	2	H ₁ , H ₄₁ , H ₈₀ , H ₁₁₂	2	H ₁₀ , H ₆₀ , H ₈₀ , H ₁₀₂
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₈₃	16	R ₂₈₄	16	R ₂₈₅
8	N ₂₈₃	8	N ₂₈₄	8	N ₂₈₅
4	L ₁₇₀ , L ₁₉₅	4	L ₉₁ , L ₂₆₁	4	L ₁₀₅ , L ₁₂₈
2	H ₁₂ , H ₃₈ , H ₄₉ , H ₁₁₅	2	H ₁₄ , H ₁₈ , H ₄₉ , H ₆₆	2	H ₁₆ , H ₄₀ , H ₅₇ , H ₉₂
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₈₆	16	R ₂₈₇	16	R ₂₈₈
8	N ₂₈₆	8	N ₂₈₇	8	N ₂₈₈
4	L ₈ , L ₁₃₂	4	L ₈ , L ₂₉₉	4	L ₃₅₀ , L ₁₇₇
2	H ₃₂ , H ₃₉ , H ₈₀ , H ₁₀₈	2	H ₃₃ , H ₃₄ , H ₆₈ , H ₇₈	2	H ₈₀ , H ₆₁ , H ₉₉ , H ₉
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₈₉	16	R ₂₉₀	16	R ₂₉₁
8	N ₂₈₉	8	N ₂₉₀	8	N ₂₉₁
4	L ₁₇₆ , L ₂₄₅	4	L ₇₃ , L ₁₆₄	4	L ₁₆₀ , L ₂₂₄
2	H ₃₅ , H ₄₈ , H ₆₂ , H ₉₈	2	H ₃₀ , H ₅₆ , H ₆₄ , H ₁₁₆	2	H ₆ , H ₃₉ , H ₅₅ , H ₆₀
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₉₂	16	R ₂₉₃	16	R ₂₉₄
8	N ₂₉₂	8	N ₂₉₃	8	N ₂₉₄
4	L ₇₅ , L ₂₂₃	4	L ₁₈₁ , L ₂₉₀	4	L ₂₂ , L ₂₁₉
2	H ₅ , H ₄₂ , H ₆₃ , H ₆₉	2	H ₁ , H ₅ , H ₄₈ , H ₁₁₅	2	H ₆ , H ₉ , H ₇₁ , H ₉₂
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₉₅	16	R ₂₉₆	16	R ₂₉₇
8	N ₂₉₅	8	N ₂₉₆	8	N ₂₉₇
4	L ₁₁₃ , L ₂₉₅	4	L ₉₂ , L ₁₈₀	4	L ₁₈ , L ₆₂

2	H ₁₃ , H ₂₁ , H ₃₆ , H ₈₁	2	H ₄ , H ₁₈ , H ₃₆ , H ₈₈	2	H ₁ , H ₁₀ , H ₂₆ , H ₇₈
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₂₉₈	16	R ₂₉₉	16	R ₃₀₀
8	N ₂₉₈	8	N ₂₉₉	8	N ₃₀₀
4	L ₁₃₁ , L ₂₅₈	4	L ₂₇₅ , L ₃₀₅	4	L ₆₈ , L ₁₅₆
2	H ₁₅ , H ₁₆ , H ₂₄ , H ₈₁	2	H ₄ , H ₅₅ , H ₉₆ , H ₁₁₅	2	H ₁₃ , H ₂₃ , H ₈₈ , H ₁₀₈
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₀₁	16	R ₃₀₂	16	R ₃₀₃
8	N ₃₀₁	8	N ₃₀₂	8	N ₃₀₃
4	L ₁₇₃ , L ₁₉₄	4	L ₈₃ , L ₁₉₀	4	L ₃₀ , L ₁₈₈
2	H ₁₃ , H ₄₁ , H ₄₈ , H ₈₉	2	H ₄ , H ₁₂ , H ₈₆ , H ₉₃	2	H ₁₂ , H ₂₄ , H ₆₉ , H ₇₅
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₀₄	16	R ₃₀₅	16	R ₃₀₆
8	N ₃₀₄	8	N ₃₀₅	8	N ₃₀₆
4	L ₁₃₂ , L ₂₉₉	4	L ₂₃₅ , L ₂₉₃	4	L ₁₀₆ , L ₁₆₉
2	H ₂₄ , H ₃₁ , H ₈₅ , H ₁₁₅	2	H ₂₂ , H ₂₈ , H ₇₇ , H ₉₄	2	H ₁₆ , H ₃₇ , H ₅₈ , H ₁₀₅
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₀₇	16	R ₃₀₈	16	R ₃₀₉
8	N ₃₀₇	8	N ₃₀₈	8	N ₃₀₉
4	L ₉₀ , L ₁₈₇	4	L ₇₇ , L ₂₃₀	4	L ₇₁ , L ₁₅₂
2	H ₂ , H ₁₇ , H ₄₈ , H ₆₉	2	H ₃₈ , H ₈₆ , H ₁₁₆ , H ₁₁₇	2	H ₆ , H ₈ , H ₆₂ , H ₈₅

Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₁₀	16	R ₃₁₁	16	R ₃₁₂
8	N ₃₁₀	8	N ₃₁₁	8	N ₃₁₂
4	L ₇₁ , L ₃₄₉	4	L ₃₂₂ , L ₃₃₀	4	L ₃₃₂ , L ₃₄₆
2	H ₅₉ , H ₆₇ , H ₈₁ , H ₁₁₆	2	H ₇₃ , H ₇₅ , H ₇₉ , H ₈₇	2	H ₅₃ , H ₅₅ , H ₈₅ , H ₈₇
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₁₃	16	R ₃₁₄	16	R ₃₁₅
8	N ₃₁₃	8	N ₃₁₄	8	N ₃₁₅
4	L ₂₄₀ , L ₃₂₀	4	L ₁₆₂ , L ₃₂₀	4	L ₁₅₉ , L ₂₀₇
2	H ₂₅ , H ₆₆ , H ₆₉ , H ₇₇	2	H ₂₆ , H ₄₁ , H ₆₀ , H ₈₅	2	H ₁₁ , H ₃₄ , H ₅₆ , H ₁₁₃
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₁₆	16	R ₃₁₇	16	R ₃₁₈
8	N ₃₁₆	8	N ₃₁₇	8	N ₃₁₈
4	L ₆₄ , L ₃₀₄	4	L ₂₇₁ , L ₂₉₇	4	L ₆₁ , L ₂₁₅
2	H ₃ , H ₅₅ , H ₁₀₁ , H ₁₁₄	2	H ₂ , H ₈₁ , H ₉₆ , H ₁₁₄	2	H ₄ , H ₈₁ , H ₁₀₁ , H ₁₁₃
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₁₉	16	R ₃₂₀	16	R ₃₂₁
8	N ₃₁₉	8	N ₃₂₀	8	N ₃₂₁
4	L ₁₆₂ , L ₂₄₀	4	L ₂₈₀ , L ₃₀₁	4	L ₆₇ , L ₃₀₀
2	H ₂₇ , H ₃₈ , H ₆₄ , H ₈₁	2	H ₃ , H ₄₂ , H ₉₆ , H ₁₁₃	2	H ₂ , H ₄₂ , H ₁₀₁ , H ₁₁₅
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₂₂	16	R ₃₂₃	16	R ₃₂₄
8	N ₃₂₂	8	N ₃₂₃	8	N ₃₂₄
4	L ₃₃₉ , L ₃₄₃	4	L ₃₃₉ , L ₃₄₅	4	L ₃₂₅ , L ₃₃₄
2	H ₄₉ , H ₅₅ , H ₇₃ , H ₈₂	2	H ₄₇ , H ₅₇ , H ₇₄ , H ₈₁	2	H ₄₅ , H ₅₅ , H ₅₈ , H ₇₉
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₂₅	16	R ₃₂₆	16	R ₃₂₇
8	N ₃₂₅	8	N ₃₂₆	8	N ₃₂₇
4	L ₃₂₈ , L ₃₄₆	4	L ₃₂₆ , L ₃₄₇	4	L ₃₃₃ , L ₃₄₇
2	H ₅₄ , H ₅₆ , H ₈₁ , H ₈₆	2	H ₅₄ , H ₅₇ , H ₇₆ , H ₈₄	2	H ₅₁ , H ₅₂ , H ₇₀ , H ₈₀
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₂₈	16	R ₃₂₉	16	R ₃₃₀
8	N ₃₂₈	8	N ₃₂₉	8	N ₃₃₀
4	L ₃₃₈ , L ₃₄₁	4	L ₃₂₉ , L ₃₃₁	4	L ₃₂₉ , L ₃₃₇
2	H ₄₈ , H ₅₆ , H ₇₄ , H ₈₄	2	H ₄₃ , H ₅₀ , H ₅₁ , H ₇₇	2	H ₄₄ , H ₄₇ , H ₈₄ , H ₈₆
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₃₁	16	R ₃₃₂	16	R ₃₃₃
8	N ₃₃₁	8	N ₃₃₂	8	N ₃₃₃
4	L ₃₄₁ , L ₃₄₄	4	L ₃₃₆ , L ₃₄₈	4	L ₃₂₃ , L ₃₄₈
2	H ₅₁ , H ₇₂ , H ₈₃ , H ₈₈	2	H ₅₀ , H ₅₂ , H ₇₁ , H ₈₃	2	H ₅₃ , H ₅₈ , H ₇₅ , H ₈₂
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₃₄	16	R ₃₃₅	16	R ₃₃₆
8	N ₃₃₄	8	N ₃₃₅	8	N ₃₃₆
4	L ₃₄₃ , L ₃₄₅	4	L ₃₃₁ , L ₃₃₇	4	L ₃₃₈ , L ₃₄₄
2	H ₄₂ , H ₅₀ , H ₇₀ , H ₈₀	2	H ₄₅ , H ₄₆ , H ₈₂ , H ₈₇	2	H ₄₆ , H ₅₈ , H ₇₃ , H ₈₅
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₃₇	16	R ₃₃₈	16	R ₃₃₉
8	N ₃₃₇	8	N ₃₃₈	8	N ₃₃₉

4	L ₃₃₄ , L ₃₄₀	4	L ₃₂₄ , L ₃₃₅	4	L ₃₃₅ , L ₃₄₂
2	H ₄₂ , H ₄₃ , H ₇₀ , H ₈₃	2	H ₄₄ , H ₅₆ , H ₅₇ , H ₇₈	2	H ₄₃ , H ₇₁ , H ₈₀ , H ₈₈
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₄₀	16	R ₃₄₁	16	R ₃₄₂
8	N ₃₄₀	8	N ₃₄₁	8	N ₃₄₂
4	L ₃₂₃ , L ₃₃₆	4	L ₃₂₆ , L ₃₃₃	4	L ₃₂₅ , L ₃₄₀
2	H ₄₇ , H ₄₈ , H ₅₄ , H ₇₈	2	H ₄₆ , H ₄₉ , H ₅₃ , H ₇₉	2	H ₄₄ , H ₄₈ , H ₇₆ , H ₈₁

Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₄₃	16	R ₃₄₄	16	R ₃₄₅
8	N ₃₄₃	8	N ₃₄₄	8	N ₃₄₅
4	L ₃₂₈ , L ₃₃₂	4	L ₃₂₄ , L ₃₄₂	4	L ₁₄ , L ₈₄
2	H ₄₂ , H ₅₂ , H ₇₇ , H ₈₈	2	H ₄₅ , H ₄₉ , H ₇₅ , H ₈₅	2	H ₁₈ , H ₈₆ , H ₉₀ , H ₁₀₉
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₄₆	16	R ₃₄₇	16	R ₃₄₈
8	N ₃₄₆	8	N ₃₄₇	8	N ₃₄₈
4	L ₁₁ , L ₂₉₄	4	L ₈₅ , L ₁₁₆	4	L ₁₅₂ , L ₃₄₉
2	H ₁₁ , H ₂₂ , H ₃₇ , H ₇₈	2	H ₁₇ , H ₂₁ , H ₈₇ , H ₁₁₃	2	H ₅ , H ₃₇ , H ₄₀ , H ₇₇
Order	Subgroups	Order	Subgroups	Order	Subgroups
16	R ₃₄₉	16	R ₃₅₀	16	R ₃₅₁
8	N ₃₄₉	8	N ₃₅₀	8	N ₃₅₁
4	L ₁₇ , L ₁₁₇	4	L ₇ , L ₂₃₇	4	L ₁₆ , L ₂₂₆
2	H ₁₉ , H ₇₇ , H ₁₀₇ , H ₁₁₀	2	H ₁₈ , H ₂₀ , H ₆₃ , H ₇₈	2	H ₁₆ , H ₁₉ , H ₅₉ , H ₇₁

We display one typical interval $[\{e\}, R_1]$ of $L(G)$ in the following diagram.

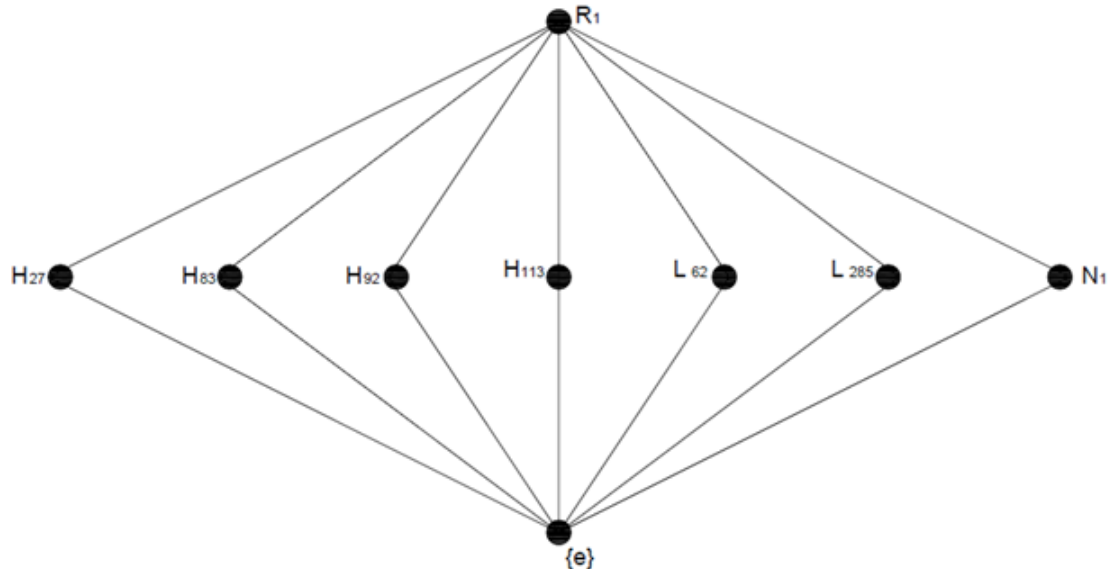


Fig. 5.1: The Interval $[\{e\}, R_1]$

VI. Conclusion:

In this paper, we produced the lattice structure of subgroups of order 16 in the subgroup lattices of 3x3 matrices over Z_3 .

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