IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-0661,p-ISSN: 2278-8727, Volume 20, Issue 3, Ser. 1 (May. – June. 2024), PP 14-27
www.iosrjournals.org

Analysis Of Vedic Mathematics Ekadhikena Purvena Sutra In Decimal Representation Of A Fraction

Rohit Ranjan Lal[1] & Dharmendra Kumar Yadav[2]
[1] *University Department Of Mathematics, Lalit Narayan Mithila University, Darbhanga, Bihar, India*
[2] *University Department Of Mathematics, Lalit Narayan Mithila University, Darbhanga, Bihar, India*

Abstract:
In the paper we have reviewed, analyzed and generalized the two applications of Vedic Mathematics Ekadhikena Purvena Sutra: the multiplication method and the division method, in finding the decimal representation of vulgar fraction having the standard numerator 1 and the denominator ending with the digits 1, 3, 7 and 9. The unit and non-unit fractions with denominators ending with the digits 2, 4, 5, 6, and 8 have also been discussed as exceptions. The short cut method available for both the multiplication and division methods, which reduces half of the procedures and thus time as well as space, has been found not suitable for all types of fractions i.e., it is not applicable for all the fractions. An example has been given to prove it. We have also discussed the logic hidden in the procedures of Binomial theorem for negative integral index by using the decimal representation of all terms of the Binomial expansion of fraction. Thereafter we have extended this sutra for general form of vulgar fraction also with some examples and have suggested the general working rule for the two methods. The paper ends with a short note on limitations of the sutra and the future scope of research in this field.

Key-words: *Ekadhikena Purvena Sutra, Fraction, Decimal Representation, Recurring, Non-recurring, Binomial theorem for negative integral index.*

Date of Submission: 13-05-2024 Date of Acceptance: 23-05-2024

I. Introduction

In India the four Vedas are known as the source of all knowledge. They are not only treated as the oldest but also the best source of information about human beings and its surroundings. The Vedas have guided millions of aspirants on the path of knowledge (Bose, 2021; Ganesh, 2018; Maharaja, 2015). With the passage of time most of the Vedic knowledge has been scattered and have not been discussed properly. Vedic Mathematics is no exception to it. Now it is inspiring to observe that the deep interest is being taken in reviving the Vedic Sciences (Gupta, 2015, 2018). In the Vedic mathematics system, the very first step is to recognize the pattern of the problem and pick up the most efficient Vedic sutras (Thakur, 2013; Halai, 2018). Further, at each subsequent step, we recognize the pattern and complete the task by using the superfast mental working procedure, because in Vedic mathematics we have multiple choices available at each stage of working (Khatru, 2022). In a nutshell we can say that Vedic mathematics provides the cosmic software for the human mind to save time and space. It originated from the Vedas, which manifest divine knowledge (Murthy, 2009). Any knowledge derived from the Vedas is bound to have a touch of the divine bliss. Therefore, the very natural, easy and superfast algorithms of Vedic Mathematics bring an upsurge of joy and bliss (Khare, 2006; Parajuli, 2020, 2021).

It is a system of mathematical techniques that originated in ancient India. These techniques were completed in the early 20th century by Swami Bharti Krishna Tirtha Ji Maharaja, whose aim was to simplify and expedite mathematical arithmetical calculations (Pethkar, 2020; Maharaja, 2015; Priya et al., 2021). Some key principles of Vedic mathematics include various sutras and sub-sutras that cover addition, subtraction, multiplication, division, square roots, cube roots, integration, differentiation, decimal representation and many more (Maharaja, 2015; Selvaraj, 2021). The system is known for its simplicity and efficiency making mental calculations quicker (Sharma et al., 2022; Shashtri, 2011).

Vedic Mathematics is a collective name given to a set of sixteen mathematical formulae and each formula deals with a different branch of Mathematics. These sixteen formulas can be used to solve problems ranging from arithmetic to algebra to geometry to conics to calculus and so on (Maharaja, 2015; Shembalkar et al., 2017). The formulae are completed themselves and applicable to virtually and all kind of mathematical problems. Complex mathematical questions that otherwise take numerous steps to solve can be solved with the help of a few steps and in some cases without any intermediate steps at all and these systems are so simple that even people with an average knowledge of Mathematics can easily understand them (Maharaja, 2015; Tekriwal, 2017; Tripathi, 2022). It is the study of mathematical relationships in keeping with the Vedic tradition of intuitive thinking. In other words, it is an approach that exploits both halves of the brain by using the pattern recognition capabilities of one and the analytical capabilities of the other (Gupta, 2015; Maharaja, 2015). Each sixteen sutra also covers a wide range of applications (Joshi, 2017; Srivastava, 2011).

As far as the current research on Vedic Mathematics is concerned, Yadav (2007) has propounded the Aanuruppen Binomial Method to find the nth power of integers and rational numbers in terminating decimal form by extending the Aanuruppen Vedic sutra followed by the computer algorithm in C/C++ of Aanuruppen Binomial Method by Knnojiya & Yadav (2008). Yadav & Lal (2023) have analysed the Vedic Mathematics Ekadhikena Purvena Sutra in squaring and multiplication. Thereafter Lal & Yadav (2023) discussed the relevance of Vedic Mathematics Ekanyunena Purvena and Ekadhikena Purvena Sutras in Calculus.

In this paper we have analysed application of Ekadhikena Purvena Sutra in the decimal representation of a fraction. The factors in the denominator of a fraction play an important role in its equivalent decimal forms. Each of 2, 5, and 10 contribute one significant digit to the decimal point. Denominator having mixed factors partly 2, 5 and partly 3, 7, 9, etc. gives us a mixed decimal representation i.e., partly recurring decimal and partly non-recurring decimal. Each 2, 5 or 10 contribute one non-recurring digit to the decimal. Each 3 or 9 contributes only one recurring digit, 11 contributes 2, 7 contributes 6 and other numbers contribute their own individual digits (Maharaja, 2015).

The multiplication of the last digit of the denominator and the last digit of the decimal representation of that fraction also play an important role. In every non-recurring decimal representation with the standard numerator 1, the last digit of the denominator and the last digit of the equivalent decimal, multiplied together, will always yield a product ending in 0. In every recurring decimal representation with the standard numerator 1, it is observed that 9 is the last digit of the product of the last digit of the denominator and the last digit of the recurring decimal representation. It is known that when the last digit of the denominator ends with 9, the equivalent recurring decimal representation ends in 1. The product is actually a continuous series of nines. The above concept enables us to determine the last digit of the recurring decimal equivalent of a given vulgar fraction. Thus, $\frac{1}{17}$ in its decimal shape end with 7, $\frac{1}{19}$ with 1, $\frac{1}{21}$ with 9, $\frac{1}{23}$ with 3, and so on. For the fractions of the form $\frac{1}{7}, \frac{1}{13}$ etc., the Vedic sutra Sesanyankena Caramena which means the remainders by the last digit is used to write their decimal representation, which is entirely different from the Ekadhikena Purvena sutra (Maharaja, 2015). In this paper, our study is bound to the Ekadhikena Purvena sutra.

II. Preliminary Ideas

Ekadhikena Purvena is a technique in Vedic Mathematics, which means 'by one more than the previous one'. It is used in many calculations like integration, squaring, multiplication, division, addition, etc. (Bose, 2021; Gupta, 2018; Thakur, 2013; Halai, 2018; Maharaja, 2015). It is also used in writing the decimal representation of a fraction of specific forms, which will be explained after revising some basic concepts of fractions.

Vulgar Fraction: A vulgar fraction is a fraction whose numerator and denominator are both integers. If the numerator of a vulgar fraction is 1, it is called a unit fraction (James & James, 2001) and such numerator is known as a standard numerator (Maharaja, 2015).

Decimal Fraction: In it a decimal point '.' is used to denote a number, which is called a decimal point. The number in decimal notation has no digits other than zeros to the left of the decimal point. A mixed decimal is a decimal plus an integer. For example, 23.342 is a mixed decimal (James & James, 2001).

A finite or terminating decimal is a decimal that contains a finite number of digits. An infinite or non-terminating decimal uses an unending string of digits to the right of the decimal point. A repeating or periodic decimal is a decimal that either is finite or is infinite and has a finite block of digits that eventually repeats indefinitely. A fraction has two forms of representations: vulgar fraction in the form $\frac{a}{b}$, where a and b (≠0) are integers and the decimal fraction like 0.230246 (James & James, 2001).

Vedic mathematics sutra uses some shortcut calculations to express the vulgar fraction of some particular type in which the denominator is an integer ending with the digit 9 and the numerator is 1 in decimal form by using both multiplication and division. Although it can be applied to those fractions, in which the numerator is other than 1 and the denominator ends with the digit 9 after some modification (Bose, 2021; Halai, 2018).

Conversion of Vulgar Fraction into Decimal Representation: This can be done by two different ways: multiplication method and division method, followed by a short-cut method, which reduces half of the work and process. The working rules for these methods are as follows:

Multiplication Method: The Ekadhikena Purvena sutra can be used to convert those vulgar unit fractions into decimal forms whose numerator is 1 and the denominator ends with 9 such as $\frac{1}{19}$, $\frac{1}{49}$, $\frac{1}{299}$, etc. using multiplication process. The working rules to find the decimal representation of the unit fraction $\frac{1}{a9}$, where $1 \leq a < \infty$ (although no such explanation on 'a' is available in the references), are as follows:

1. Find the multiplier b = (a + 1) by adding one in the previous number 'a' written left of 9 in the denominator of the unit fraction, which is the 'Ekadhika Purva' of a.
2. Write 1 as the last digit in the decimal representation of $\frac{1}{a9}$ i.e., we put 1 as the right hand most digit.
3. Multiply 1 by the multiplier b and write unit digit of the answer (let it be dc), towards left side of 1 as c1 and carry forward d to add in next step.
4. Multiply c by b and add d to the answer (let it be bc + d = fe). Write e towards left side of c in the decimal representation and carry forward f to add it in next step.
5. Repeat step 4 till we get the numbers repeated from right to left.
6. Put up the usual recurring marks (dots) on the first and the last digit of the answer or just write one complete block or repeated digits to denote that it is a recurring decimal.

The reason behind writing 1 as the right hand most digit in the decimal representation is that the multiplication of the last digit of the denominator and the last digit of the decimal equivalent of the fraction must end with 9 (Bose, 2021; Maharaja, 2015).

Example 1: Find the decimal representation of the unit fraction $\frac{1}{19}$ by using multiplication method of Ekadhikena Purvena Sutra (Bose, 2021; Maharaja, 2015).

Here we have $\frac{1}{a9} = \frac{1}{19}$, where the multiplier is M (= b) = a +1 = 1 + 1 = 2. Let us denote P = Product with multiplier, U = Unit Digit, R = Carry Forwarded Number, and Q = Product + Carry Forwarded Number. Then the calculations can be carried out as follows shown in the table-1:

Table-1

Steps	M	P	Q	U	R	Decimal Representation
1	--	--	--	1*	0	1
2	2	2	2 + 0 = 2	2	0	21
3	2	4	4 + 0 = 4	4	0	421
4	2	8	8 + 0 = 8	8	0	8421
5	2	16	16 + 0 = 16	6	1	68421
6	2	12	12 + 1 = 13	3	1	368421
7	2	6	6 + 1 = 7	7	0	7368421
8	2	14	14 + 0 = 14	4	1	47368421
9	2	8	8 + 1 = 9	9	0	947368421
10	2	**18**	18 + 0 = **18****	8	1	8947368421
11	2	16	16 + 1 = 17	7	1	78947368421
12	2	14	14 + 1 = 15	5	1	578947368421
13	2	10	10 + 1 = 11	1	1	1578947368421
14	2	2	2 + 1 = 3	3	0	31578947368421
15	2	6	6 + 0 = 6	6	0	631578947368421
16	2	12	12 + 0 = 12	2	1	2631578947368421
17	2	4	4 + 1 = 5	5	0	52631578947368421
18	2	10	10 + 0 = 10	0	1	052631578947368421
19	2	0	0 + 1 = 1	1	0	Repeated step-1, Stop

In the above table-1, in step-1 the mark 1* denotes that we have taken U = 1 as we have to start the process with taking 1 to get the decimal representation. Step 19 is the repetition of the first step 1, so we stopped the process. Our decimal representation has been obtained in the last column in step 18 and it is given by

$$\frac{1}{19} = 0.\dot{0}5263157894736842\dot{1} = 0.\overline{052631578947368421}$$

The result can be verified using Computer software Mathematica using the inbuilt technique as

$$\mathrm{In}[1] \coloneqq \mathrm{N}[\frac{1}{19}, 17]$$

Out[1]:=0.052631578947368421

where Mathematica code N[F, n] gives the result with n-digit precision of the fraction F. Since the value of the above unit fraction is a periodic fraction and the above decimal representation makes a complete block of the value, as can be seen from following decimal representation with 53-digit precision

$$\mathrm{In}[2] \coloneqq \mathrm{N}[\frac{1}{19}, 53]$$

Out[2]:=0.**052631578947368421**052631578947368421**052631578947368421**

In the above result three blocks of the repeated values have been shown with different fonts. There is an observation in this sutra that 'for any fraction of the form $\frac{1}{a9}$, the value of the fraction is always written in recurring decimal form and the repeating block's right most digit is always 1'.

This sutra can also be used for conversion of vulgar fractions, whose denominators are ending with 1, 3, 7 such as $\frac{1}{11}, \frac{1}{21}, \frac{1}{31}, \frac{1}{13}, \frac{1}{23}, \frac{1}{17}, \ldots$ etc. by expressing them in standard forms like $\frac{9}{99}, \frac{9}{189}, \frac{9}{279}, \frac{3}{39}, \frac{3}{69}, \frac{7}{119}, \ldots$ etc. After finding the decimal representations of the fractions $\frac{1}{99}, \frac{1}{189}, \frac{1}{279}, \frac{1}{39}, \frac{1}{69}, \frac{1}{119}, \ldots$ etc. and thereafter multiplying by 9, 9, 9, 3, 3, 7, … etc. respectively, we can find their decimal representations using the above method. Although a new procedure will be discussed to find the value of the fractions having denominators made with the digit 9 only like $\frac{1}{9}, \frac{1}{99}, \frac{1}{999}, \frac{1}{9999}, \frac{1}{99999}, \frac{1}{999999}, \ldots$ etc. later in discussion section.

Division Method: Since the division is the inverse operation to multiplication and the multiplication method writes the decimal representation of the fraction from right hand most side to left hand most side till the decimal point, the division method will write it from left hand most side after decimal point to right hand most side till the repetition of the digits (one block) starts. As stated earlier the Ekadhikena Purvena sutra is used to convert the vulgar unit fractions into decimal forms whose numerator is 1 and the denominator ends with 9 using the multiplication process, so the same can be evaluated using the division process. In this case the working rules to find the decimal representation of the unit fraction of the form $\frac{1}{a9}$, where $1 \leq a < \infty$, are as follows:

1. Find the divisor b = a + 1.
2. Find the dividend, the numerator of the unit fraction, here it is 1.
3. Divide the dividend 1 by the divisor b. If the numerator is greater than 1 (let it be p), first find for 1 and then multiply the result by p.
4. If we divide the dividend 1 by divisor b > 1, we first take 0 as the quotient after decimal point and 1 as the remainder. Modern mathematics technique says that the number of zeros should be equal to the number of digits in the divisor.
5. We, therefore, set down 0 as the first digit of the quotient and prefix the remainder 1 to that very digit of the quotient 0 and thus obtain 10 as our next dividend in case of $\frac{1}{19}$.
6. Divide 10 by b (2 for $\frac{1}{19}$) and repeat the step 5.
7. Repeat steps 5 and 6 by carrying the process of straight, continuous division by b (2 for $\frac{1}{19}$) till we get 1 as the quotient and 0 as the remainder i.e., till the repetition of the digits starts for the next block continuously.
8. We stop the division process and put down the usual recurring symbols (dots on the first and last digits or bar on the block) to show that the whole of it is a circulating decimal (Bose, 2021; Maharaja, 2015).

Example 2: Find the decimal representation of the unit fraction $\frac{1}{19}$ using division method of Ekadhikena Purvena Sutra (Bose, 2021; Maharaja, 2015).

Here we have $\frac{1}{a9} = \frac{1}{19}$, where the divisor is D (= b) = a +1 = 1 + 1 = 2. Let us denote P = Dividend, Q = Quotient and R = Remainder as the Carry Forwarded Number towards left of Q in P from step-2 and U = Unit Digit. Then the calculations can be carried out as follows shown in the table-2:

Table-2

Steps	D	P ≈ RQ	Q	U	R	Decimal Representation
1	2	1#	0	0	1	0.0
2	2	10	5	5	0	0.05
3	2	05	2	2	1	0.052
4	2	12	6	6	0	0.0526
5	2	06	3	3	0	0.05263
6	2	03	1	1	1	0.052631
7	2	11	5	5	1	0.0526315
8	2	15	7	7	1	0.05263157
9	2	17	8	8	1	0.052631578
10	2	**18****	9	9	0	0.0526315789
11	2	09	4	4	1	0.05263157894
12	2	14	7	7	0	0.052631578947
13	2	07	3	3	1	0.0526315789473
14	2	13	6	6	1	0.05263157894736
15	2	16	8	8	0	0.052631578947368

16	2	08	4	4	0	0.0526315789473684
17	2	04	2	2	0	0.05263157894736842
18	2	02	1	1	0	0.052631578947368421
19	2	01	0	0	1	Repeats Step 1, Stop

In table-2 in step-1, the number 1# denotes that we have to take 1 as the first dividend for this example. Here P ≈ RQ denotes that RQ is just the placement of two numbers R and Q i.e., it denotes a two digit numbers, a new dividend, if both R and Q are of one digit numbers. There doesn't exist any mathematical operation between them.

From above table-2 we have

$$\frac{1}{19} = 0.\dot{0}5263157894736842\dot{1} = 0.\overline{052631578947368421}$$

In the multiplication method each surplus digit is added in the next step, whereas in division method each remainder is prefixed to the left i.e., just immediately to the left of the next dividend digit to make it a complete number which finally acts as a dividend (Bose, 2021; Maharaja, 2015).

Short-Cut Method: A short cut method is available to write the decimal representation of the unit fraction, which reduces half of the work of multiplication and division methods. When we apply this technique, we continue the process of either method till we reach at the number, which is equal to the difference between denominator and numerator (i.e., denominator – numerator). Thereafter we take the complements of these digits from 9 and write the remaining digits of the complete block (Bose, 2021; Maharaja, 2015). For example in the previous decimal representation

$$\frac{1}{19} = 0.052631578947368421$$

we put down the first 9 digits of the decimal representation in one horizontal row above and the other 9 digits in another horizontal row just below and observe their sum column-wise as follows:

$$\begin{array}{rccccccccc} \text{Row 1:} & 0 & 5 & 2 & 6 & 3 & 1 & 5 & 7 & 8 \\ \text{Row 2:} & 9 & 4 & 7 & 3 & 6 & 8 & 4 & 2 & 1 \\ \text{Sum:} & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 \end{array}$$

We observe that each set of digits in vertical positions totals 9, which indicates that if half of the work has been completed by either of the methods, the other half need not be obtained by the same process. The remaining can be obtained by subtracting from 9 each of the digits already obtained. Therefore the work is automatically reduced by 50%.

To decide that whether the work is half finished or not in either method, we have to take care that if and as soon as we reach the difference between the denominator and numerator of the unit fraction (19 – 1 = 18 in case of above examples shown by 18** in both table 1 and 2 in step-10), we have completed exactly half of the work and with this knowledge, we can know exactly when and where we may stop the process of both methods and when and where we can start reeling off the complements from 9 to write the remaining digits of the answer (Bose, 2021; Maharaja, 2015).

This method can be applied on many fractions. We can easily find the decimal representation of the fractions

$$\frac{1}{29} = 0.0344827586206896551724137931$$

which satisfies the short-cut method property that two halves are all complements of each other as follows:

$$\begin{array}{rcccccccccccccc} Row\,1 & 0 & 3 & 4 & 4 & 8 & 2 & 7 & 5 & 8 & 6 & 2 & 0 & 6 & 8 \\ Row\,2 & 9 & 6 & 5 & 5 & 1 & 7 & 2 & 4 & 1 & 3 & 7 & 9 & 3 & 1 \\ Sum & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 \end{array}$$

and

$$\frac{1}{49} = 0.020408163265306122448979591836734693877551$$

which satisfies the short-cut method property that two halves are all complements of each other as follows:

$$\begin{array}{rccccccccccccccccccccc} Row\,1 & 0 & 2 & 0 & 4 & 0 & 8 & 1 & 6 & 3 & 2 & 6 & 5 & 3 & 0 & 6 & 1 & 2 & 2 & 4 & 4 & 8 \\ Row\,2 & 9 & 7 & 9 & 5 & 9 & 1 & 8 & 3 & 6 & 7 & 3 & 4 & 6 & 9 & 3 & 8 & 7 & 7 & 5 & 5 & 1 \\ Sum & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 \end{array}$$

This method fails for some fraction, which will be discussed in next section.

III. Discussion

Out of the two methods and one short-cut method discussed in previous section, it has been found that there are some fractions, on which the short-cut method fails. For this let us consider the following example:

Example 3: Find the decimal representation of $\frac{1}{39}$ using both multiplication and division methods and verify the short-cut method.

For multiplication method, we have $\frac{1}{a9} = \frac{1}{39}$, where the multiplier is M (= b) = a +1 = 3 + 1 = 4, P = Product, Q = Product + Carry Forwarded Number, R = Carry Forwarded Number, and U = Unit Digit. Then the calculations can be carried out as follows in the table-3:

Table-3

Steps	M	P	Q	U	R	Decimal Representation
1	--	--	--	1	0	1
2	4	4	4 + 0 = 4	4	0	41
3	4	16	16 + 0 = 16	6	1	641
4	4	24	24 + 1 = 25	5	2	5641
5	4	20	20 + 2 = 22	2	2	25641
6	4	8	8 + 2 = 10	0	1	025641
7	4	0	0 + 1 = 1	1	0	Repeated Step-1, Stop

The step 7 hints us to stop our calculation and further steps show the repetition of the value of the fraction in decimal representation, as it is a periodic fraction and its one block is 025641. Its decimal value is given by

$$\frac{1}{39} = 0.\overline{025641}$$

Now let us find the same using division method. Here we have $\frac{1}{a9} = \frac{1}{39}$, where the divisor is D (= b) = a +1 = 3 + 1 = 4. Let us denote P = Dividend, Q = Quotient and R = Remainder as the Carry Forwarded Number towards left of Q in P from step-2 and U = Unit Digit. Then the calculations can be carried out as follows shown in the table-4:

Table-4

Steps	D	P ≈ RQ	Q	U	R	Decimal Representation
1	4	$1^{\#}$	0	0	1	0.0
2	4	10	2	2	2	0.02
3	4	22	5	5	2	0.025
4	4	25	6	6	1	0.0256
5	4	16	4	4	0	0.02564
6	4	04	1	1	0	0.025641
7	4	01	0	0	1	Repeated Step-1, Stop

From the table-4 we found one block of recurring decimal as 0.025641. Therefore the decimal representation of the unit vulgar fraction is

$$\frac{1}{39} = 0.\overline{025641}$$

Using Mathematica code, we can verify the value of the fraction $\frac{1}{39}$ as follows:

$$\text{In}[3] := \text{N}[\frac{1}{39}, 5]$$

Out[3]:=0.025641

$$\text{In}[4] := \text{N}[\frac{1}{39}, 11]$$

Out[4]:=0.025641**025641**

$$\text{In}[5] := \text{N}[\frac{1}{39}, 17]$$

Out[5]:=0.**025641**025641**025641**

Mathematica codes show that the decimal representation of the above vulgar fraction obtained using two methods of Ekadhikena Purvena sutra is true. But in both tables 3 and 4, we haven't found the difference of denominator and numerator i.e., 39 – 1 = 38 in the column under P. Let us arrange the digits of the decimal representation in two rows as before, we see that

$$\begin{array}{cccc} Row\,1 & 0 & 2 & 5 \\ Row\,2 & 6 & 4 & 1 \\ Sum & 6 & 6 & 6 \end{array}$$

We find that each set of digits in vertical positions totals 6 and not 9. So if half of the work has been found by either method, the other need not be found by the same process. The remaining can be obtained by subtracting from 6 each of the digits already obtained i.e., we find complements from 6 and not from 9.

Maharaja (2015) gives its reason regarding the failure of the short-cut method. But the explanation given by him doesn't help to propound a general rule for all such fractions. He states one of the reasons that 39 is not a prime number and so it doesn't satisfy the conditions of the short-cut method. But for the fraction $\frac{1}{49}$, where the denominator 49 is also not a prime number but the condition of the short-cut method is satisfied by its decimal representation as has been mentioned in the preliminary ideas section. Let us do another example as follows:

Example 4: Find the decimal representation of the unit fraction $\frac{1}{129}$ using multiplication method of Ekadhikena Purvena Sutra.

Here we have $\frac{1}{a9} = \frac{1}{129}$, where the multiplier is M (= b) = a +1 = 12 + 1 = 13. Let us denote P = Product, Q = Product + Carry Forwarded Number, R = Carry Forwarded Number, and U = Unit Digit, then the calculations can be carried out as follows in the table-5:

Table-5

Steps	M	P	Q	U	R	Decimal Representation
1	--	--	--	1	0	1
2	13	13	13 + 0 = 13	3	1	31
3	13	39	39 + 1 = 40	0	4	031
4	13	0	0 + 4 = 4	4	0	4031
5	13	52	52 + 0 = 52	2	5	24031
6	13	26	26 + 5 = 31	1	3	124031
7	13	13	13 + 3 = 16	6	1	6124031
8	13	78	78 + 1 = 79	9	7	96124031
9	13	117	117 + 7 = 124	4	12	496124031
10	13	52	52 + 12 = 64	4	6	4496124031
11	13	52	52 + 6 = 58	8	5	84496124031
12	13	104	104 + 5 = 109	9	10	984496124031
13	13	117	117 + 10 = 127	7	12	7984496124031
14	13	91	91 + 12 = 103	3	10	37984496124031
15	13	39	39 + 10 = 49	9	4	937984496124031
16	13	117	117 + 4 = 121	1	12	1937984496124031
17	13	13	13 + 12 = 25	5	2	51937984496124031
18	13	65	65 + 2 = 67	7	6	751937984496124031
19	13	91	91 + 6 = 97	7	9	7751937984496124031
20	13	91	91 + 9 = 100	0	10	07751937984496124031
21	13	0	0 + 10 = 10	0	1	007751937984496124031
22	13	0	0 + 1 = 1	1	0	1007751937984496124031

The step 22 hints us to stop our calculation and step 23 onwards will show the repetition of the value of the fraction, as it is a periodic fraction and its one block is 007751937984496124031. Therefore the decimal representation of the above unit vulgar fraction is

$$\frac{1}{129} = 0.\overline{007751937984496124031}$$

Using Mathematica code, we can verify the value of the fraction $\frac{1}{129}$ as follows:

$$\mathrm{In}[6] \coloneqq \mathrm{N}[\frac{1}{129}, 19]$$

Out[6]:= 0.007751937984496124031

$$\mathrm{In}[7] \coloneqq \mathrm{N}[\frac{1}{129}, 40]$$

Out[7]:= 0.007751937984496124031**007751937984496124031**

$$\mathrm{In}[8] \coloneqq \mathrm{N}[\frac{1}{129}, 61]$$

Out[8]:= 0.**007751937984496124031**007751937984496124031**007751937984496124031**

Which show that the decimal representation of the above vulgar fraction obtained using Ekadhikena Purvena sutra is true. In the above decimal representation, we see that the difference of denominator and numerator i.e., 129 – 1 = 128 doesn't appear in the column under P and there are 21 digits in the decimal form, which cannot be written in two rows to verify the validity of the short-cut method. So in this case also, the short-cut method fails.

If we increase the number of digits in the denominator, the decimal form may contain more terms and using the above two methods will require more space and time to write the final decimal representation. Therefore we can use Mathematica software to find the equivalent decimal representation and may verify the validity of the short-cut method. But we have got two examples which disprove the validity of short-cut method for some fractions. So there is no need to verify it for another example now. Thus we can conclude that the short-cut method cannot be generalized and is not true for all fractions.

To verify the relevance of the two methods for the other fractions having more digits in the denominator, let us consider the following example:

Example 5: Find the decimal representation of the unit fraction $\frac{1}{3129}$.

Using Mathematica we find that in its decimal representation, we need 444 digits in a block and we get it as

$$In[9] \coloneqq N[\frac{1}{3129}, 441]$$

Out[9]:=0.$\dot{0}$0031959092361776925535314797059763502716522850751038670501757750079897730
904442313838286992649408756791307126877596676254394375199744327261105784595717481623521891
978267817193991690635985937999360818152764461489293704058804729945669542984979226589964844
998402045381911153723234260147011824864173857462448066474912112496005113454777884308085650
367529562160434643656120166187280281240012783636944710770214125918823905401086609140300415
468200703$\dot{1}$

$$In[10] \coloneqq N[\frac{1}{3129}, 885]$$

Out[10]:=0.0003195909236177692553531479705976350271652285075103867050175775007989773
090444231383828699264940875679130712687759667625439437519974432726110578459571748162352189
197826781719399169063598593799936081815276446148929370405880472994566954298497922658996484
499840204538191115372323426014701182486417385746244806647491211249600511345477788430808565
036752956216043464365612016618728028124001278363694471077021412591882390540108660914030041
54682007031***0003195909236177692553531479705976350271652285075103867050175775007989773090444***
231383828699264940875679130712687759667625439437519974432726110578459571748162352189197826
781719399169063598593799936081815276446148929370405880472994566954298497922658996484499840
204538191115372323426014701182486417385746244806647491211249600511345477788430808565036752
956216043464365612016618728028124001278363694471077021412591882390540108660914030041546820
07031

In the decimal representation of the above fraction, each block, considering zeros as digits also, contains 444 digits after decimal point. We can verify that the two methods work well for this example also.

But as the number of digits increases in the denominator, generally the number of digits in each block in recurring decimals contains more digits and so to find it from right hand most side using multiplication methods will take too much time and space whereas in the era of approximation, we generally need value of fraction to three to four decimal places and so the division method will play a major role in the decimal representation of fractions. But division method also works easy if the number ‘a’ in $\frac{1}{a9}$ is of one to two digits maximum. If ‘a’ has more than two digits, we have to face a problem of division with no fast method. So in either method, it would be useful to apply the two methods it if the denominator of the fraction has two to three digits only.

Failure of Two Methods: In the fractions discussed earlier, we observe that each denominator ends with 9 and when we use the multiplication method, we start with 1 from right hand most side and when we use the division method, the result ends with 1 i.e., the multiplication of the denominator of the fraction and the last digit of the decimal representation always equals 1 x 9 = 9. For example, in the decimal representation

$$\frac{1}{129} = 0.007751937984496124031$$

we have, last digit of the answer multiplied by the last digit of the fraction is, 1 x 9 $=$ 9.

The question arises, can we think the same pattern of the working rules of the two methods, for the fraction having denominators ending with the digits 1, 3, 7 by starting the rule with the right hand most digit 9, 3, and 7 respectively, because

$$1 \times 9 = 9, \qquad 3 \times 3 = 9, \qquad 7 \times 7 = 49$$

i.e., the multiplication of the last digit of the denominator of such fraction and the last digit of the answer obtained by the same procedure will always end with 9 at the right hand most side digit. Let us verify it by some examples:

When we apply the above procedures, we get the values of such fraction as follows: For the unit fraction $\frac{1}{11}$, we start with 9 from the right hand most side and apply the procedures of multiplication method, we get here $\frac{1}{a1} = \frac{1}{11}$, where the multiplier is M (= b) = a +1 = 1 + 1 = 2. Let us denote P = Product, Q = Product + Carry Forwarded Number, R = Carry Forwarded Number, and U = Unit Digit, then the calculations can be carried out as follows in the table-6:

Table-6

Steps	M	P	Q	U	R	Decimal Representation
1	--	--	--	9	0	9
2	2	18	18 + 0 = 18	8	1	89
3	2	16	16 + 1 = 17	7	1	789
4	2	14	14 + 1 = 15	5	1	5789
5	2	10	10 + 1 = 11	1	1	15789
6	2	2	2 + 1 = 3	3	0	315789
7	2	6	6 + 0 = 6	6	0	6315789
8	2	12	12 + 0 = 12	2	1	26315789
9	2	4	4 + 1 = 5	5	0	526315789
10	2	10	10 + 0 = 10	0	1	0526315789
11	2	2	0 + 1 = 1	1	0	10526315789
12	2	2	2 + 0 = 2	2	0	210526315789
13	2	4	4 + 0 = 4	4	0	4210526315789
14	2	8	8 + 0 = 8	8	0	84210526315789
15	2	16	16 + 0 = 16	6	1	684210526315789
16	2	12	12 + 1 = 13	3	1	3684210526315789
17	2	6	6 + 1 = 7	7	0	73684210526315789
18	2	14	14 + 0 = 14	4	1	473684210526315789
19	2	8	8 + 1 = 9	9	0	9473684210526315789

In the above table-6, we see that values started repetition from step-19, so we stop the process and write the answer as

$$\frac{1}{11} = 0.\overline{473684210526315789}$$

whereas, using Mathematica software, we find its value as

$$In[11] := N\left[\frac{1}{11}, 22\right]$$

Out[11]:=0.09090909090909090909091=0.$\overline{09}$

which shows that the procedure of multiplication method fails for this fraction. Similarly we can show that this procedure fail for the fractions $\frac{1}{31}, \frac{1}{13}, \frac{1}{17}$ also if we start the procedure with the digit 9, 3 and 7 respectively at the right hand most side digit. Thus the above two methods will not be applicable directly for those fractions whose denominator ends with 1 or 3 or 7, and when we start the multiplication method with the digit 9 or 3 or 7 respectively at the right hand most side digit in decimal representation. But as discussed earlier, we can apply the two methods after making the last digit of the denominator 9 by multiplying by a suitable factor.

Maharaja (2015) mentioned the method to find the decimal value of the fraction having standard numerator 1 and denominator ending with 1, 3 or 7 with some modification. To conclude to a general procedure, let us analysis the following examples discussed by Maharaja (2015) as follows:

For the fraction $\frac{1}{7}$, he wrote it as $\frac{1}{7} = 7.\frac{1}{49}$ and took the multiplier P (the Ekadhika Purva) = 4 + 1 = 5 and the last digit of the decimal value 7. He then found its value as

$$\frac{1}{7} = 0.\overline{142857}$$

By this process, two methods with their shot-cut method hold true. In applying the short-cut method, the process is stopped, when we reach at new denominator – new numerator = 49 – 7 = 42.

For the fraction $\frac{1}{13}$, he wrote it as $\frac{1}{13} = 3.\frac{1}{39}$ and took the multiplier P (the Ekadhika Purva) = 3 + 1 = 4 and the last digit of the decimal value 3. He then found its value as

$$\frac{1}{13} = 0.\overline{076923}$$

By this process, two methods with their shot-cut method hold true. In applying the short-cut method, the process is stopped, when we reach at new denominator – new numerator = 39 – 3 = 36.

For the fraction $\frac{1}{11}$, he wrote it as $\frac{1}{11} = 9.\frac{1}{99}$ and took the multiplier P (the Ekadhika Purva) = 9 + 1 = 10 and the last digit of the decimal value 9. He then found its value as

$$\frac{1}{11} = 0.\overline{09}$$

By this process, two methods with their shot-cut method hold true. In applying the short-cut method, the process is stopped, when we reach at new denominator – new numerator = 99 – 9 = 90, which we get after one step only.

For the fraction $\frac{1}{23}$, he wrote it as $\frac{1}{23} = 3.\frac{1}{69}$ and took the multiplier P (the Ekadhika Purva) = 6 + 1 = 7 and the last digit of the decimal value 3. He then found its value as

$$\frac{1}{23} = 0.\overline{0434782608695652173913}$$

By this process, two methods with their shot-cut method hold true. In applying the short-cut method, the process is stopped, when we reach at new denominator – new numerator = 69 – 3 = 66, which we get after one step only.

Similarly, for the fraction $\frac{1}{17}$, he wrote it as $\frac{1}{17} = 7.\frac{1}{119}$ and took the multiplier P (the Ekadhika Purva) = 11 + 1 = 12 and the last digit of the decimal value 7. He then found its value as

$$\frac{1}{17} = 0.\overline{0588235294117647}$$

We see some differences between the two methods previously discussed in preliminary ideas section and the method applied in the above five examples. The basic difference is that the last digit of the decimal value is decided from the last digit of the denominator of the given fraction, whereas the multiplier and divisor are decided on the basis of the new modified fraction. The difference between the denominator and denominator is also decided from the new modified fraction. Therefore we can conclude that the working rules for decimal fraction cannot be generalized in a single method for all types of fractions in finding its equivalent decimal representation. Yet we can present it in a single umbrella of working rules with some special cases as follows:

Generalized Working Rules: For the given fraction of the form $\frac{1}{\mathrm{BA}}$, where $0 \leq \mathrm{B} < \infty$ and $\mathrm{A} = 1, 3, 7$ or 9. BA denotes a number of any number of digits (let it be r) in which A is a number of only one digit (1, 3, 7, or 9) and B is a number of any number of digits (r – 1 digits, if BA is a number of r digits) from 0 to less than ∞. It should be kept in mind that no mathematical operation exists between A and B. The working rules to find the equivalent decimal representation of such fraction can be expressed using the two methods discussed earlier with a little modification as follows:

Case-I: If A = 9, we apply the same working rules as discussed in preliminary ideas section. In this case, we take multiplier = B + 1 = divisor, dividend = 1 and the last digit of the decimal representation 1. Here we may suppose that we have multiplied numerator and denominator by 1 to make the procedures similar to other cases.

Case-II: If A ≠ 9 and it is 1, 3, or 7. We make it 9 by multiplying both the numerator and denominator of the given fraction by the least number 9, 3, or 7 respectively. So there arise three more sub-cases:

(a) If $\frac{1}{\mathrm{BA}} = \frac{1}{\mathrm{B1}}$, we multiply it by 9 and get $\frac{1}{\mathrm{BA}} = 9.\frac{1}{9 \times \mathrm{B1}} = 9.\frac{1}{\mathrm{C9}}$, where $\mathrm{C9} = \mathrm{B1} \times 9$. Now we take the multiplier = C + 1 = divisor, dividend = 9 and the last digit of the decimal representation 9. In this case for short-cut method, we look for the difference, to stop the process if it follows the complements rule from 9, equal to $\mathrm{C9} - 9$.

(b) If $\frac{1}{\mathrm{BA}} = \frac{1}{\mathrm{B3}}$, we multiply it by 3 and get $\frac{1}{\mathrm{BA}} = 3.\frac{1}{3 \times \mathrm{B3}} = 3.\frac{1}{\mathrm{C9}}$, where $\mathrm{C9} = \mathrm{B3} \times 3$. Now we take the multiplier = C + 1 = divisor, dividend = 3 and the last digit of the decimal representation 3. In this case for short-cut method, we look for the difference, to stop the process if it follows the complements rule from 9, equal to $\mathrm{C9} - 3$.

(c) If $\frac{1}{\mathrm{BA}} = \frac{1}{\mathrm{B7}}$, we multiply it by 7 and get $\frac{1}{\mathrm{BA}} = 7.\frac{1}{7 \times \mathrm{B7}} = 7.\frac{1}{\mathrm{C9}}$, where $\mathrm{C9} = \mathrm{B7} \times 7$. Now we take the multiplier = C + 1 = divisor, dividend = 7 and the last digit of the decimal representation 7. In this case for short-cut method, we look for the difference, to stop the process if it follows the complements rule from 9, equal to $\mathrm{C9} - 7$.

From above three sub-cases, we observe that the last digit of the equivalent decimal value of the fraction is the least number, which is multiplied to numerator and denominator of the fraction to make the last digit of the denominator 9. This rule can be verified by applying on the fractions discussed earlier or any other fractions. But the biggest demerit of this rule is that it is applied to only those fractions, which has 1, 3, 7, or 9 as the last digit in its denominator.

Although we can apply the two methods to unit and non-unit fractions, whose denominator ends with the digits 0, 2, 4, 5, 6, 8 after some modification but no general rule can be developed for these types of fractions. For example, for the following fractions and their modified forms:

$$\frac{1}{190} = \frac{1}{10.19} = \frac{1}{10}.\frac{1}{19}; \ \frac{1}{120} = \frac{1}{40.3} = \frac{3}{40}.\frac{1}{9},$$

$$\frac{1}{112} = \frac{1}{2.8.7} = \frac{7}{16}.\frac{1}{49}; \ \frac{1}{132} = \frac{1}{4.33} = \frac{3}{4}.\frac{1}{99}; \ \frac{1}{42} = \frac{1}{2.21} = \frac{9}{2}.\frac{1}{189},$$

$$\frac{1}{14} = \frac{1}{2.7} = \frac{7}{2}.\frac{1}{49}; \ \frac{1}{24} = \frac{3}{8}.\frac{1}{9}; \ \frac{1}{114} = \frac{1}{2.57} = \frac{7}{2}.\frac{1}{379},$$

$$\frac{1}{15}=\frac{1}{5.3}=\frac{3}{5}.\frac{1}{9};\ \frac{1}{115}=\frac{1}{5.23}=\frac{3}{5}.\frac{1}{69},$$
$$\frac{1}{6}=\frac{3}{2}.\frac{1}{9};\ \frac{1}{26}=\frac{3}{2}.\frac{1}{39};\ \frac{1}{116}=\frac{1}{4}.\frac{1}{29},$$
$$\frac{1}{18}=\frac{1}{2}.\frac{1}{9};\ \frac{1}{28}=\frac{7}{4}.\frac{1}{49};\ \frac{1}{58}=\frac{1}{2}.\frac{1}{29},$$

we can apply the two methods. But these methods will make the calculations more complex than the general method available for calculating their decimal representations. Still there are many fractions, whose denominators cannot be expressed in terms of numbers having 9 as a unit digit like

$$\frac{1}{8},\frac{1}{25},\frac{1}{125},\frac{1}{10},\text{etc.}$$

We can forcefully multiply both numerator and denominator by 9 and then can bring one 9 in the denominator but it will be too much for the subject. At last but not the least, a special type of fraction having denominators made by the digit 9's only exist, which has a very special decimal representation, as has been shown below:

Denominators Made with Digit 9: Such type of fractions occurred in many situations like

$$\frac{1}{9},\frac{1}{99},\frac{1}{999},\ldots\text{etc.}$$

For these types of fractions, we can develop some short-cut technique to write their decimal representations without any calculations. Using Mathematica, we find that

$$In[12]:=N\left[\frac{1}{9},20\right]$$

Out[12]:=0.11111111111111111111=$0.\bar{1}$

We observe that number of digit 9 in the denominator n = 1, number of zeros after decimal point and before non-zero digit is z = 0, and the number of one after the decimal point is p = many times repeated as it is a non-terminating recurring decimal number. Let us observe the changes in the pattern after increasing the value of n, we have

$$In[13]:=N[\frac{1}{99},20]$$

Out[13]:=0.01010101010101010101010=$0.\overline{01}$

In this fraction we have n = 2, z = 1, and p = 1. One block contains 01. For n = 3, we have

$$In[14]:=N[\frac{1}{999},20]$$

Out[14]:=0.0010010010010010010010=$0.\overline{001}$

Here we have n = 3, = 2, and p = 1. One block contains 001. For n = 4, we have

$$In[15]:=N[\frac{1}{9999},20]$$

Out[15]:=0.0001000100010001000100=$0.\overline{0001}$

Here we have n = 4, z = 3, and p = 1. One block contains 0001. For n = 5, we have

$$In[16]:=N[\frac{1}{99999},20]$$

Out[16]:=0.00001000010000100000100000=$0.\overline{00001}$

For this we have n = 5, z = 4, and p = 1. One block contains 00001.

Proceeding in this way, we can generalize this as a rule that if a unit fraction of the form

$$\frac{1}{\underbrace{999\ldots\ldots99}_{n\ times}}$$

contains 'n' number of digit 9's only in the denominator, then its decimal representation will be a non-terminating recurring decimal number and it will contain (n-1) zeros after decimal point followed by 1 and its one block will contain (n-1) zeros and one 1 at the end at right hand most side. We can directly write the unit fraction of this type as a decimal representation like

$$\frac{1}{\underbrace{999\ldots\ldots99}_{n\ times}}=0.\underbrace{0000\ldots0}_{\text{(n−1) zeros}}1=0.\overline{\underbrace{0000\ldots0}_{\text{(n−1) zeros}}1}$$

where after decimal point in right hand side, there are (n-1) zeros before 1. This is one block of the periodic decimal representation.

As far as the logics behind the two methods for decimal representation of unit fractions are concerned, it has been found that it is hidden in the Binomial theorem expansion for negative integral index '-1', which can be seen as follows:

Justification of Methods: From Binomial theorem for negative integral index '-1', we know that

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots; \quad |x| < 1$$

for integers n = 0, 1, 2, 3, …

We know that any vulgar fraction of the form $\frac{1}{a9}$ can be written as

$$\frac{1}{a9} = \frac{1}{(a+1)z - 1}, \text{where } z = 10^n \text{ for } n = 1, 2, 3, \ldots$$

For example,

$$\frac{1}{19} = \frac{1}{(1+1)10 - 1}; \ \frac{1}{129} = \frac{1}{(12+1)10 - 1}, \text{etc.}$$

Applying the above expansion, we find that

$$\frac{1}{a9} = \frac{1}{[(a+1)z - 1]} = \frac{1}{(a+1)z\left[1 - \frac{1}{(a+1)z}\right]}$$

$$= \frac{1}{(a+1)z}\left[1 - \frac{1}{(a+1)z}\right]^{-1}; \quad \left|\frac{1}{(a+1)z}\right| < 1$$

$$= \frac{1}{(a+1)z}\left[1 + \frac{1}{(a+1)z} + \frac{1}{(a+1)^2z^2} + \frac{1}{(a+1)^3z^3} + \cdots\right]$$

$$= \frac{1}{(a+1)z} + \frac{1}{(a+1)^2z^2} + \frac{1}{(a+1)^3z^3} + \frac{1}{(a+1)^4z^4} + \cdots$$

$$= \frac{1}{(a+1)}z^{-1} + \frac{1}{(a+1)^2}z^{-2} + \frac{1}{(a+1)^3}z^{-3} + \frac{1}{(a+1)^4}z^{-4} + \cdots$$

$$= \frac{1}{(a+1)}10^{-1} + \frac{1}{(a+1)^2}10^{-2} + \frac{1}{(a+1)^3}10^{-3} + \frac{1}{(a+1)^4}10^{-4} + \cdots, \text{for } z = 10$$

That is we can directly calculate the decimal representation of a unit fraction having denominator of two digits ending with 9 only, using the formula

$$\frac{1}{a9} = \frac{1}{(a+1)}10^{-1} + \frac{1}{(a+1)^2}10^{-2} + \frac{1}{(a+1)^3}10^{-3} + \frac{1}{(a+1)^4}10^{-4} + \cdots$$

Using this expansion, we get the decimal representation of a unit vulgar fraction discussed in example-3 as:

$$\frac{1}{39} = \frac{1}{(3+1)}10^{-1} + \frac{1}{(3+1)^2}10^{-2} + \frac{1}{(3+1)^3}10^{-3} + \frac{1}{(3+1)^4}10^{-4} + \cdots$$

$$= \frac{1}{(4)}10^{-1} + \frac{1}{(4)^2}10^{-2} + \frac{1}{(4)^3}10^{-3} + \frac{1}{(4)^4}10^{-4} + \cdots \quad (i)$$

$$= (0.25)10^{-1} + (0.0625)10^{-2} + (0.015625)10^{-3} + (0.00390625)10^{-4} + \cdots$$

$$= 0.025 + 0.000625 + 0.000015625 + 0.000000390625 + \cdots$$

$= 0.\mathbf{025641}0156\ldots = 0.\overline{025641}$

To which decimal precision the decimal representation is true depends on the number of terms we have taken into consideration, as can be verified from the table-7:

Table-7

Number of Terms Taken in Series (i)	**Sum of Terms**	**Value Correct to Decimal Places**
1	0.025	3 Decimal Places 0.025
2	0.025625	4 Decimal Places 0.0256
3	0.0256406625	5 Decimal Places 0.02564
4	0.025641053125	7 Decimal Places 0.0256410

We know from example-3 that the value of the unit fraction $\frac{1}{39}$ is a non-terminating recurring decimal number and its one block is 0.025641. From the table-7 we find that after taking four terms of the series (i), we get the correct value of one complete block and one more correct value after the block. When we increase the number of terms in the expansion, we get second block of the decimal representation.

Although to find the required number of terms to get the desired accuracy, we can use Taylor's series method of expansion and error in a series approximation but it is out of the scope of the present paper and needs a special work on it as approximation, which is not needed for the present paper (Grewal, 2014).

We can apply the above series technique to find the decimal representation of the unit and non-unit fractions also but the above two methods of Ekadhikena Purvena Sutra suit only for those fraction whose denominator ends with the digit 9 or can be converted into 9 after some modification. We can also verify the examples-1 and 2 as follows:

We have

$$\frac{1}{29}=\frac{1}{30-1}=\frac{1}{(2+1)10-1}=\frac{1}{3.10}\left[1-\frac{1}{3.10}\right]^{-1}$$
$$=\frac{1}{3.10}\left[1+\frac{1}{3.10}+\frac{1}{3^2.10^2}+\frac{1}{3^3.10^3}+\cdots+\frac{1}{3^{n-1}.10^{n-1}}+\cdots\right]$$
$$=\frac{1}{3.10}+\frac{1}{3^2.10^2}+\frac{1}{3^3.10^3}+\cdots+\frac{1}{3^n.10^n}+\cdots$$

Let us consider the number of terms and their sum till we get the desired result of the fraction in one complete block as follows in table-8:

Table-8

n^{th} term	Value of nth term	Sum of n terms	Value Correct to Decimal Places
1	0.0333333333333333333	0.0333333333333333333	2
2	0.0011111111111111111	0.0344444444444444444	4
3	0.0000370370370370370	0.0344814814814814751 4	5
4	$1.23456790123457\times10^{-6}$	0.03448271604938271605	7
5	$4.11522633744856\times10^{-8}$	0.03448275720164609053	8
6	$1.37174211248285\times10^{-9}$	0.03448275857338820302	9
7	$4.57247370827618\times10^{-11}$	0.03448275861911294010	10
8	$1.52415790275873\times10^{-12}$	0.03448275862063709800	12

In table-8 we see that as we increase the number of terms in the expansion, we reach accordingly to the nearest decimal value of the fraction. It justifies that the sutra and its two methods have been derived from the Binomial series expansion technique for negative integral index '-1'.

Similarly we can find the decimal value of the fraction in which the denominator is not ending with 9 using the series expansion procedures as follows:

$$\frac{1}{42}=\frac{1}{[(4+1)10-8]}=\frac{1}{(4+1)10}\left[1-\frac{8}{(4+1)10}\right]^{-1}$$
$$=\frac{1}{(4+1)10}+\frac{8}{(4+1)^2 10^2}+\frac{8^2}{(4+1)^3 10^3}+\frac{8^3}{(4+1)^4 10^4}+\cdots$$
$$=0.02+0.0032+0.000512+0.00008192+\cdots=0.02379392\ldots$$

Obviously it is not the correct value but is correct to three decimal places and if it is round-off it will be correct to four decimal places as 0.0238, whereas its real value is 0.0238095 and 238095 makes one block in non-terminating decimal representation. Therefore Binomial theorem for negative integral index '-1' is very useful in finding the decimal representation of both unit and non-unit fractions. Thus we can conclude that the two procedures multiplication and division method of Vedic Ekadhikena Sutra has been derived from the Binomial theorem for negative integral index '-1'.

At last one more point is to be noted that in finding the decimal representation of fractions like $\frac{1}{13},\frac{1}{19},\frac{1}{29},\frac{1}{39},\frac{1}{49}$, we need 6, 18, 28, 6, 42 digits respectively for one block of recurring decimal value. Therefore we cannot propound an exact formula to determine exactly the number of digits required to write an equivalent decimal form of any given fraction.

IV. Conclusion

The multiplication and division methods based on Vedic Mathematics Ekadhikena Purvena Sutra has been derived from the Binomial theorem for negative integral index '-1' and is not applicable for all fractions. The series expansion based on Binomial theorem for negative integral index is helpful in finding the equivalent decimal representation of all types of fractions. Both methods would be useful to apply if the denominator of the fraction has two to three digits as the concept of Vedic sutras are known for short-cut and fast method. These methods can also be applied to non-unit fractions satisfying the condition that the last digit of the denominator should be 1, 3, 7, or 9. These methods have also been applied to unit and non-unit fractions whose denominator ends with the digits 0, 2, 4, 5, 6, 8 and it has been found that they are not giving the correct result. The short-cut method for both the multiplication and division methods varies with examples. So it cannot be generalized, yet a generalized working rule has been propounded in the paper with four special cases of the denominators of the fractions ending with 1, 3, 7 or 9.

V. Limitations

The Vedic Mathematics Ekadhikena Purvena Sutra to find the equivalent decimal representation of a fraction, whose denominator ends with 1, 3, 7 or 9 and standard numerator 1, using two methods: multiplication and division, is applicable to some particular cases only and they didn't give correct result for another fractions. The short-cut method of the two techniques is not applicable for all types of fraction, even for those having denominator ending with the digit 9. But the biggest demerit of this rule is that it is applied to only those fractions, which has 1, 3, 7, or 9 as the last digit in its denominator.

VI. Future Scope Of Research

A short-cut method has not been found for general fractions with a general rule. So there is a possibility to work in this direction. Using numerical technique some work can be done on both sutras to find out the number of terms required in the Binomial expansion to express the fraction up to a certain degree of accuracy.

References

[1]. Bose, S. (2021). Vedic Mathematics, V & S Publishers, New Delhi, Pp. 25-28.
[2]. Ganesh, R.S., Hemamalini, K., Indhu, V. & Parbha, S. K. (2018). Review Of Vedic Sutras, International J. Of Creative Research Thoughts (Ijcrt), 6(1), Pp. 820-830.
[3]. Grewal, B. S. (2014). Numerical Methods In Engineering And Science, Khanna Publishers, New Delhi, Pp.9.
[4]. Gupta, V. K. (2015) Vedic Mathematics And The Mathematics Of Vedic Period – An Analysis And Application, 26 July – Dec, Pp- 01-12.
[5]. Gupta, A. (2018). The Power Of Vedic Maths, Jaico Publishing House, Mumbai, Pp. 79-83.
[6]. Halai, C. (2018). Vedic Mathematics Inside Out, The Write Place, Pune, Pp. 111-113, 117-119.
[7]. James & James (2001). Mathematics Dictionary, 4th Edition, Cbs Publishers & Distributors, New Delhi, Pp. 98-99, 156.
[8]. Joshi, D. R. (2017). Vedic Mathematics In Modern Era, International Journal Of Research In All Subjects In Multi Language (Ijrsml),5(6), Pp-1.
[9]. Khare, H. C. (2006). Vedic Mathematics, Motilal Banarsidass Publishers Pvt. Ltd. Delhi, Pp. 41-42.
[10]. Khatru, A. (2022). Simplified Vedic Mathematics, Brahmee Publications, Odisha, Pp. 74.
[11]. Knnojiya, D. S. & Yadav, D. K. (2008). Algorithm Of Aanuruppen Binomial Method, International J. Of Math. Sci. & Engg. Appls. (Ijmsea), 2(4), Pp.101-112.
[12]. Lal, R. R. & Yadav, D. K. (2023). Relevance Of Vedic Mathematics Ekanyunena Purvena And Ekadhikena Purvena Sutras In Calculus, Scope, 13(3), Pp.950-957.
[13]. Maharaja, J.S.S.B.K.T. (2015). Vedic Mathematics, Motilal Banarsidass Publishers Private Limited, New Delhi, Pp. 01-10, 27-29, 191-201.
[14]. Murthy, T. S. B. (2009). A Modern Introduction To Ancient India Mathematics, New Age International Publishers, Delhi, Pp. 67-78.
[15]. Parajuli, K. K. (2020). Basic Operations On Vedic Mathematics: A Study On Special Parts, Nepal Journal Of Mathematical Science (Njms), 1, Pp.71-76.
[16]. Parajuli, K. K. (2021). Three Separate Methods For Squaring: A Connective Prospective On Lilavati And Trachtenberg, International Journal Of Statistics And Applied Mathematics, 6(2), Pp. 43-47.
[17]. Pethkar, P. A. (2020). Vedic Maths – Ekadhikena Purvena, International Journal Of Engineering, Science And Mathematics, 9(1), Pp. 176-180.
[18]. Priya, Goel, P., Kumer, A. (2021). A Brief Into Vedic Mathematics-Its Origin, Features And Sutras, Journal Of Mathematics Problems, Equations And Statistics, 2(1), Pp-36-39.
[19]. Selvaraj, P., Ashwin, Sgs (2021), Vedic Mathematics, Notionpress.Com, Chennai, Tamil Nadu, Pp. 6.
[20]. Sharma, T., Khubnani, R. & Subramanyam C. (2022). Study Of Mathematics Through Indian Veda's: A Review, Futuristic Research In Modeling Of Dynamics System (Frmds 2022), Pp. 1-8.
[21]. Shashtri, P. R. (2011) Vedic Mathematics Made Easy, Arihant Publication (I) Pvt. Ltd. Meerut (Up) , Pp. 56-57.
[22]. Shembalker, S., Dhole, S., Yadav, T. & Thakre, P. (2017). Vedic Mathematics Sutras – A Review. International Conference On Recent Trends In Engineering Science & Technology (Icrtest 2017), 5(1), Pp.148-155.
[23]. Tekriwal, G. (2017). Maths Sutras: From Around The World, Penguin Random House India Pvt. Ltd, Haryana, Pp. 131-133.
[24]. Thakur, R. (2013). The Essentials Of Vedic Mathematics, Rupa Publication India Pvt. Ltd, New Delhi, Pp. 129-32.
[25]. Tripathi, M. (2022). Vedic Mathematics For Students, Parbhat Paperbacks, New Delhi, Pp. 37-40.
[26]. Srivastava, C. M. (2011). Vedic Ganit Paddati, Manoj Publication, Delhi, Pp.139-144.
[27]. Yadav, D. K. (2007). Aanuruppen Binomial Method To Find The Nth Power Of Integers And Rational Numbers In Terminating Decimal Forms, Acta Ciencia Indica, 33 (2), Pp.647-655.
[28]. Yadav, D. K. & Lal, R. R. (2023). Analysis Of Vedic Mathematics Ekadhikena Purvena Sutra In Squaring And Multiplication, Proceedings Of The 8th International Conference On Communication And Electronics System (Icces 2023), Dvd Part Number: Cfp23awo-Dvd; Isbn: 979-8-3503-9662-1, Pp.12-19.