*IOSR Journal of Mathematics (IOSR-JM)*
*e-ISSN: 2278-0661, p-ISSN: 2278-8727, Volume 20, Issue 6, Ser. 1 (Nov. – Dec. 2024), PP 37-41*
*www.iosrjournals.org*

# A New Separation Axioms In Intuitionistic Topological Spaces

F. Monisha[1], Dr. J. Arul Jesti[2]
*Research Scholar, Reg No:20112212092001, Department Of Mathematics, St. Mary's College (Autonomous), Thoothukudi. Affiliated To Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli, India.*
*Assistant Professor, Department Of Mathematics, St. Mary's College(Autonomous),Thoothukudi. Affiliated To Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli, India.*

***Abstract***
*The purpose of this paper is to introduce a new concept of $\mathfrak{I}\widehat{w}$- separation axioms in intuitionistic topological spaces. After giving some characterization of $\mathfrak{I}\widehat{w}\ T_0$, $\mathfrak{I}\widehat{w}\ T_1$, $\mathfrak{I}\widehat{w}\ T_2$- spaces separation axioms in intuitionistic topological spaces. We explore the fundamental properties of separation axioms and counter examples in intuitionistic topological spaces.*
***Keywords:*** *Intuitionistic sets, Intuitionistic topological spaces, $\mathfrak{I}\widehat{w}\ T_0$- space, $\mathfrak{I}\widehat{w}\ T_1$- space, $\mathfrak{I}\widehat{w}\ T_2$- space.*

---

Date of Submission: 15-11-2024 Date of Acceptance: 25-11-2024

---

## I. Introduction

The concept of intuitionistic sets in topological spaces was first introduced by Coker [3] in 1996. He also introduced the concept of intuitionistic points and investigated some fundamental properties of closed sets in intuitionistic topological spaces. Later he[5] defined $T_1$ and $T_2$ separation axioms and discussed some properties. In this paper, to define a new type of separation axiom based on $\mathfrak{I}\widehat{w}$- open sets in intuitionistic topological spaces. We introduce the concepts of $\mathfrak{I}\widehat{w}\ T_0$- space, $\mathfrak{I}\widehat{w}\ T_1$- space,$\mathfrak{I}\widehat{w}\ T_2$- spaces using $\mathfrak{I}\widehat{w}$- open sets and discuss the relationship between them.

## II. Preliminaries

**Definition 2.1 [3]:** Let $\mathcal{M}$ be a non-empty set. **An intuitionistic set** ( shortly $\mathfrak{I}\boldsymbol{S}$ ) $\mathcal{A}$ is an object having the form $\mathcal{A} = <\mathcal{M}, \mathcal{A}_1, \mathcal{A}_2>$ Where $\mathcal{A}_1, \mathcal{A}_2$ are subsets of $\mathcal{M}$ satisfying $\mathcal{A}_1 \cap \mathcal{A}_2 = \varphi$. The set $\mathcal{A}_1$ called the set of members of $\mathcal{A}$, while $\mathcal{A}_2$ is called set of nonmembers of $\mathcal{A}$.

**Definition 2.2 [4]:** Let $\mathcal{M}$ be a nonempty set and $p \in \mathcal{M}$ be a fixed element. Then the $\mathfrak{I}S\tilde{p}$ defined by $\tilde{p} = <\mathcal{M}, \{\mathrm{p}\}, \{p\}^c>$ is called an **intuitionistic point** ( shortly $\mathfrak{I}P$).

**Definition 2.3 [1] :** An intuitionistic topological space $(\mathcal{M}, \mathfrak{I}\tau_1)$ is said to be

i. $\mathfrak{I}\boldsymbol{T_1(i)}$**- space** if for all $\mathscr{b}, \mathscr{k} \in \mathcal{M}$ ( $\mathscr{b} \neq \mathscr{k}$ ) there exist an $\mathfrak{I}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\mathscr{b}} \in \mathcal{V}$ , $\tilde{\mathscr{k}} \notin \mathcal{V}$ and $\tilde{\mathscr{k}} \in \mathcal{W}$ , $\tilde{\mathscr{b}} \notin \mathcal{W}$ .
ii. $\mathfrak{I}\boldsymbol{T_1(ii)}$**- space** if for all $\mathscr{b}, \mathscr{k} \in \mathcal{M}$ ( $\mathscr{b} \neq \mathscr{k}$ ) there exist an $\mathfrak{I}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\tilde{\mathscr{b}}} \in \mathcal{V}$ , $\tilde{\tilde{\mathscr{k}}} \in \mathcal{V}$ and $\tilde{\tilde{\mathscr{k}}} \in \mathcal{W}$, $\tilde{\tilde{\mathscr{b}}} \notin \mathcal{W}$.
iii. $\mathfrak{I}\boldsymbol{T_1(iii)}$**- space** if for all $\mathscr{b}, \mathscr{k} \in \mathcal{M}$ ( $\mathscr{b} \neq \mathscr{k}$ ) there exist an $\mathfrak{I}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\mathscr{b}} \in \mathcal{V} \subseteq \bar{\bar{\tilde{\mathscr{k}}}}$ and $\tilde{\mathscr{k}} \in \mathcal{W} \subseteq \bar{\bar{\tilde{\mathscr{b}}}}$.
iv. $\mathfrak{I}\boldsymbol{T_1(iv)}$**- space** if for all $\mathscr{b}, \mathscr{k} \in \mathcal{M}$ ( $\mathscr{b} \neq \mathscr{k}$ ) there exist an $\mathfrak{I}$ - open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\tilde{\mathscr{b}}} \in \mathcal{V} \subseteq \bar{\bar{\tilde{\tilde{\mathscr{k}}}}}$, $\tilde{\tilde{\mathscr{k}}} \in \mathcal{W} \subseteq \bar{\bar{\tilde{\tilde{\mathscr{b}}}}}$.
v. $\mathfrak{I}\boldsymbol{T_1(v)}$**- space** if for all $\mathscr{b}, \mathscr{k} \in \mathcal{M}$ ( $\mathscr{b} \neq \mathscr{k}$ ) there exist an $\mathfrak{I}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\mathscr{k}} \notin \mathcal{V}$ and $\tilde{\mathscr{b}} \notin \mathcal{W}$.
vi. $\mathfrak{I}\boldsymbol{T_1(vi)}$**- space** if for all $\mathscr{b}, \mathscr{k} \in \mathcal{M}$ ( $\mathscr{b} \neq \mathscr{k}$ ) there exist an $\mathfrak{I}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\tilde{\mathscr{k}}} \notin \mathcal{V}$ and $\tilde{\tilde{\mathscr{b}}} \notin \mathcal{W}$.

**Definition 2.4 [1] :** An intuitionistic topological space $(\mathcal{M}, \mathfrak{I}\tau_1)$ is said to be

i. $\mathfrak{I}\ \boldsymbol{T_2(i)}$**- space** if for all $\mathscr{b}, \mathscr{k} \in \mathcal{M}$ ( $\mathscr{b} \neq \mathscr{k}$ ) there exist $\mathfrak{I}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\mathscr{b}} \in \mathcal{V}$ , $\tilde{\mathscr{k}} \in \mathcal{W}$ and $\mathcal{V} \cap \mathcal{W} = \tilde{\varphi}$ .

ii. $\mathfrak{T}\, T_2(ii)$**- space** if for all $\mathcal{b}, \mathcal{k} \in \mathcal{M}$ ( $\mathcal{b} \neq \mathcal{k}$ ) there exist $\mathfrak{T}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\tilde{\mathcal{b}}} \in \mathcal{V}$ , $\tilde{\tilde{\mathcal{k}}} \in \mathcal{W}$ and $\mathcal{V} \cap \mathcal{W} = \tilde{\varphi}$ .

iii. $\mathfrak{T}\, T_2(iii)$**- space** if for all $\mathcal{b}, \mathcal{k} \in \mathcal{M}$ ( $\mathcal{b} \neq \mathcal{k}$ ) there exist $\mathfrak{T}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\mathcal{b}} \in \mathcal{V}$ , $\tilde{\mathcal{k}} \in \mathcal{W}$ and $\mathcal{V} \subseteq \overline{\mathcal{W}}$.

iv. $\mathfrak{T}\, T_2(iv)$**- space** if for all $\mathcal{b}, \mathcal{k} \in \mathcal{M}$ ( $\mathcal{b} \neq \mathcal{k}$ ) there exist $\mathfrak{T}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\tilde{\mathcal{b}}} \in \mathcal{V}$ , $\tilde{\tilde{\mathcal{k}}} \in \mathcal{W}$ and $\mathcal{V} \subseteq \overline{\mathcal{W}}$.

v. $\mathfrak{T}\, T_2(v)$**- space** if for all $\mathcal{b}, \mathcal{k} \in \mathcal{M}$ ( $\mathcal{b} \neq \mathcal{k}$ ) there exist $\mathfrak{T}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\mathcal{b}} \in \mathcal{V} \subseteq \overline{\tilde{\mathcal{k}}}$, $\tilde{\mathcal{k}} \in \mathcal{W} \subseteq \overline{\tilde{\mathcal{b}}}$ and $\mathcal{V} \subseteq \overline{\mathcal{W}}$ .

vi. $\mathfrak{T}\, T_2(vi)$**- space** if for all $\mathcal{b}, \mathcal{k} \in \mathcal{M}$ ( $\mathcal{b} \neq \mathcal{k}$ ) there exist $\mathfrak{T}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\tilde{\mathcal{b}}} \in \mathcal{V} \subseteq \overline{\tilde{\tilde{\mathcal{k}}}}$, $\tilde{\tilde{\mathcal{k}}} \in \mathcal{W} \subseteq \overline{\tilde{\tilde{\mathcal{b}}}}$ and $\mathcal{V} \subseteq \overline{\mathcal{W}}$.

**Definition 2.5 [6]:** An intuitionistic topological space $(\mathcal{M}, \mathfrak{T}\tau_1)$ is called $\mathfrak{T}T_1$**- space** if for all $a, b \in \mathcal{M}$ $(a \neq b)$ there exist $\mathfrak{T}$- open set $\mathbb{U}$, $\mathbb{V}$ such that $a \in \mathbb{U}_1$, $b \notin \mathbb{U}_1$ and $b \in \mathbb{V}_1$ , $a \notin \mathbb{V}_1$.

**Definition 2.6 [6]:** An intuitionistic topological space $(\mathcal{M}, \mathfrak{T}\tau_1)$ is called $\mathfrak{T}T_2$**- space** if for all $\mathcal{b}, \mathcal{k} \in \mathcal{M}$ with ( $\mathcal{b} \neq \mathcal{k}$ ) there exist $\mathfrak{T}$- open set $\mathcal{V}, \mathcal{W}$ such that $\mathcal{b} \in \mathcal{V}_1$, $\mathcal{k} \notin \mathcal{V}_1$ and $\mathcal{k} \in \mathcal{W}_1$, $\mathcal{b} \notin \mathcal{W}_1$ and $\mathcal{V} \cap \mathcal{W} = \tilde{\varphi}$.

## III. $\mathfrak{T}\widehat{w}$- Separation Axioms

**Definition 3.1:** An intuitionistic topological space $(\mathcal{M}, \mathfrak{T}\tau_1)$ is said to be

a) $\mathfrak{T}\widehat{w}\, T_0(i)$**- space** if for all $a, b \in \mathcal{M}$ $(a \neq b)$ there exist $\mathfrak{T}\widehat{w}$- open set $U$ such that $\tilde{a} \in U$ and $\tilde{b} \notin U$ or $\tilde{b} \in U$ and $\tilde{a} \notin U$.

b) $\mathfrak{T}\widehat{w}\, T_0(ii)$**- space** if for all $a, b \in \mathcal{M}$ $(a \neq b)$ there exist $\mathfrak{T}\widehat{w}$- open set $U$ such that $\tilde{\tilde{a}} \in U$ and $\tilde{\tilde{b}} \notin U$ and $\tilde{\tilde{b}} \in U$ and $\tilde{\tilde{a}} \notin U$.

**Definition 3.2:** An intuitionistic topological space $(\mathcal{M}, \mathfrak{T}\tau_1)$ is called $\mathfrak{T}\widehat{w}\, T_0$**- space** if for all $a, b \in \mathcal{M}$ with $(a \neq b)$ there exist $\mathfrak{T}\widehat{w}$- open set $U$ such that $a \in U_1, b \in U_2$ or $b \in U_1, a \in U_2$.

**Theorem 3.3 :** Every intuitionistic $T_0$ space is $\mathfrak{T}\widehat{w}\, T_0$- space but not conversely.

**Proof:** Since every intuitionistic open is $\mathfrak{T}\widehat{w}$- open, the proof follows.

**Example 3.4 :** Let $\mathcal{M} = \{a, \mathcal{b}\}$ with the family $\mathfrak{T}\tau_1 = \{\tilde{\varphi}, \widetilde{\mathcal{M}}, <\mathcal{M}, \varphi, \varphi>, <\mathcal{M}, \{\mathcal{b}\}, \varphi>\}$. $\mathfrak{T}\widehat{w}$- $OS$ $(\mathcal{M}, \mathfrak{T}\tau_1) = \{\tilde{\varphi}, \widetilde{\mathcal{M}}, <\mathcal{M}, \varphi, \varphi>, <\mathcal{M}, \{a\}, \varphi>, <\mathcal{M}, \varphi, \{a\}>, <\mathcal{M}, \{\mathcal{b}\}, \varphi>, <\mathcal{M}, \{\mathcal{b}\}, \{a\}>\}$. Here $\mathcal{b} \in <\mathcal{M}, \{\mathcal{b}\}, \{a\}>$ and $a \notin <\mathcal{M}, \{\mathcal{b}\}, \varphi>$. Hence $(\mathcal{M}, \mathfrak{T}\tau_1)$ is $\mathfrak{T}\widehat{w}\, T_0$- space. But there exist no intuitionistic open set $U$ such that $a \in U$ and $\mathcal{b} \notin U$ or $\mathcal{b} \in U$ and $a \notin U$. Hence $(\mathcal{M}, \mathfrak{T}\tau_1)$ is not an intuitionistic $T_0$ space .

**Theorem 3.5:** Let $(\mathcal{M}, \mathfrak{T}\tau_1)$ and $(\mathbb{Y}, \mathfrak{T}\tau_2)$ be two intuitionistic topological spaces. Let $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ be a one-one, onto and $\mathfrak{T}\widehat{w}$- open map. If $(\mathcal{M}, \mathfrak{T}\tau_1)$ is a $\mathfrak{T}T_0$ space then $(\mathbb{Y}, \mathfrak{T}\tau_2)$ is $\mathfrak{T}\widehat{w}\, T_0$- space.

**Proof :** Suppose $a, b \in \mathbb{Y}$ with $a \neq b$. Since $\mathfrak{F}$ is onto, then there exist $\mathcal{p}, \mathcal{r} \in \mathcal{M}$ such that $\mathfrak{F}(\mathcal{p}) = a$ and $\mathfrak{F}(\mathcal{r}) = b$. Then $\mathfrak{F}(\mathcal{p}) \neq \mathfrak{F}(\mathcal{r}) \Rightarrow \mathcal{p} \neq \mathcal{r}$ as $\mathfrak{F}$ is one-one. Since $\mathcal{p}, \mathcal{r} \in \mathcal{M}, \mathcal{p} \neq \mathcal{r}$ and $(\mathcal{M}, \mathfrak{T}\tau_1)$ is $\mathfrak{T}\, T_0$ space, there exist $\mathfrak{T}$ -open set $U$ in $\mathcal{M}$ such that $\mathcal{p} \in U_1$, $\mathcal{r} \notin U_1$. As $\mathfrak{F}$ is $\mathfrak{T}\widehat{w}$- open, $F(U)$ is $\mathfrak{T}\widehat{w}$- open in $(\mathbb{Y}, \mathfrak{T}\tau_2)$. Since $\mathfrak{F}(U) = <\mathbb{Y}, \mathfrak{F}(U_1), \mathfrak{F}(U_2)>, a = \mathfrak{F}(\mathcal{p}) \in \mathfrak{F}(U_1)$ and $b = \mathfrak{F}(\mathcal{r}) \notin \mathfrak{F}(U_1)$. Finally, we get $a, b \in \mathbb{Y}$ with $a \neq b$ there exist $\mathfrak{T}\widehat{w}$- open set $\mathfrak{F}(U) \in (\mathbb{Y}, \mathfrak{T}\tau_2)$ such that $a = \mathfrak{F}(\mathcal{p}) \in \mathfrak{F}(U_1)$, $b = \mathfrak{F}(\mathcal{r}) \notin \mathfrak{F}(U_1)$. Hence $(\mathbb{Y}, \mathfrak{T}\tau_2)$ is $\mathfrak{T}\widehat{w}\, T_0$- space.

**Theorem 3.6:** Let $(\mathcal{M}, \mathfrak{T}\tau_1)$ and $(\mathbb{Y}, \mathfrak{T}\tau_2)$ be two intuitionistic topological spaces. Let $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ be a one-one, onto and $\mathfrak{T}\widehat{w}$- continuous map. If $(\mathbb{Y}, \mathfrak{T}\tau_2)$ is a $TT_0$ space then $(\mathcal{M}, \mathfrak{T}\tau_1)$ is $\mathfrak{T}\widehat{w}\, T_0$- space.

**Proof :** Let $\mathcal{X}, \mathcal{Y} \in \mathcal{M}$ with $\mathcal{X} \neq \mathcal{Y}$ implies $\mathfrak{F}(\mathcal{X}), \mathfrak{F}(\mathcal{Y}) \in \mathbb{Y}$ with $\mathfrak{F}(\mathcal{X}) \neq \mathfrak{F}(\mathcal{Y})$ as $\mathfrak{F}$ is one-one. Since $\mathfrak{F}(\mathcal{X}), \mathfrak{F}(\mathcal{Y}) \in \mathbb{Y}$ and $(\mathbb{Y}, \mathfrak{T}\tau_2)$ is $\mathfrak{T}T_0$ space, there exist a $T$- open set $U$ in $\mathbb{Y}$ such that $\mathfrak{F}(\mathcal{X}) \in U_1$, $\mathfrak{F}(\mathcal{Y}) \notin U_1$ or $\mathfrak{F}(\mathcal{Y}) \in U_1$, $\mathfrak{F}(\mathcal{X}) \notin U_1$. Since $\mathfrak{F}$ is $\mathfrak{T}\widehat{w}$- continuous map, $\mathfrak{F}^{-1}(U)$ is $\mathfrak{T}\widehat{w}$- open in $(\mathcal{M}, \mathfrak{T}\tau_1)$. Now, $\mathfrak{F}(\mathcal{X}) \in U_1$ implies $\mathfrak{F}^{-1}(\mathfrak{F}(\mathcal{X})) \in \mathfrak{F}^{-1}(U_1)$ which implies $\mathcal{X} \in \mathfrak{F}^{-1}(U_1)$ and $\mathfrak{F}(\mathcal{Y}) \in U_1$ implies $\mathfrak{F}^{-1}(\mathfrak{F}(\mathcal{Y})) \in \mathfrak{F}^{-1}(U_1)$ which implies $\mathcal{Y} \in \mathfrak{F}^{-1}(U_1)$. Similarly, $\mathcal{Y} \notin \mathfrak{F}^{-1}(U_1)$, $\mathcal{X} \notin \mathfrak{F}^{-1}(U_1)$. Thus if $\mathcal{X}, \mathcal{Y} \in \mathcal{M}$ with $\mathcal{X} \neq \mathcal{Y}$, there exist $\mathfrak{T}\widehat{w}$- open set $\mathfrak{F}^{-1}(U)$ such that $\mathcal{X} \in \mathfrak{F}^{-1}(U_1)$, $\mathcal{Y} \notin \mathfrak{F}^{-1}(U_1)$ or $\mathcal{Y} \in \mathfrak{F}^{-1}(U_1)$, $\mathcal{X} \notin \mathfrak{F}^{-1}(U_1)$. Hence $(\mathcal{M}, \mathfrak{T}\tau_1)$ is $\mathfrak{T}\widehat{w}$-$T_0$- space.

**Definition 3.7 :** An intuitionistic topological space $(\mathcal{M}, \mathfrak{T}\tau_1)$ is said to be

i. $\mathfrak{T}\widehat{w}\, T_1(i)$**- space** if for all $\mathcal{b}, \mathcal{k} \in \mathcal{M}$ ( $\mathcal{b} \neq \mathcal{k}$ ) there exist $\mathfrak{T}\widehat{w}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\mathcal{b}} \in \mathcal{V}$, $\tilde{\mathcal{k}} \notin \mathcal{V}$ and $\tilde{\mathcal{k}} \in \mathcal{W}$, $\tilde{\mathcal{b}} \notin \mathcal{W}$ .

ii. $\mathfrak{T}\widehat{w}\, T_1(ii)$**- space** if for all $\mathcal{b}, \mathcal{k} \in \mathcal{M}$ ( $\mathcal{b} \neq \mathcal{k}$ ) there exist $\mathfrak{T}\widehat{w}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\tilde{\mathcal{b}}} \in \mathcal{V}$, $\tilde{\tilde{\mathcal{k}}} \notin \mathcal{V}$ and $\tilde{\tilde{\mathcal{k}}} \in \mathcal{W}$, $\tilde{\tilde{\mathcal{b}}} \notin \mathcal{W}$.

iii. $\mathfrak{T}\widehat{w}\, T_1(iii)$**- space** if for all $\mathcal{b}, \mathcal{k} \in \mathcal{M}$ ( $\mathcal{b} \neq \mathcal{k}$ ) there exist $\mathfrak{T}\widehat{w}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\mathcal{b}} \in \mathcal{V} \subseteq \overline{\tilde{\mathcal{k}}}$ and $\tilde{\mathcal{k}} \in \mathcal{W} \subseteq \overline{\tilde{\mathcal{b}}}$.

iv. **$\mathfrak{I}\hat{w}\ T_1(iv)$- space** if for all $\mathscr{b}, \mathscr{k} \in \mathcal{M}$ ( $\mathscr{b} \neq \mathscr{k}$ ) there exist $\mathfrak{I}\hat{w}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\tilde{\tilde{\mathscr{b}}}} \in \mathcal{V} \subseteq \tilde{\tilde{\tilde{\mathscr{k}}}}, \tilde{\tilde{\mathscr{k}}} \in \mathcal{W} \subseteq \tilde{\tilde{\tilde{\mathscr{b}}}}$.

v. **$\mathfrak{I}\hat{w}\ T_1(v)$- space** if for all $\mathscr{b}, \mathscr{k} \in \mathcal{M}$ ( $\mathscr{b} \neq \mathscr{k}$ ) there exist $\mathfrak{I}\hat{w}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\mathscr{k}} \notin \mathcal{V}$ and $\tilde{\mathscr{b}} \notin \mathcal{W}$.

vi. **$\mathfrak{I}\hat{w}\ T_1(vi)$- space** if for all $\mathscr{b}, \mathscr{k} \in \mathcal{M}$ ( $\mathscr{b} \neq \mathscr{k}$ ) there exist $\mathfrak{I}\hat{w}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\tilde{\mathscr{k}}} \notin \mathcal{V}$ and $\tilde{\tilde{\mathscr{b}}} \notin \mathcal{W}$.

**Theorem 3.8 :** Let $(\mathcal{M}, \mathfrak{I}\tau_1)$ be intuitionistic topological spaces. Then the following implications are valid but not conversely.

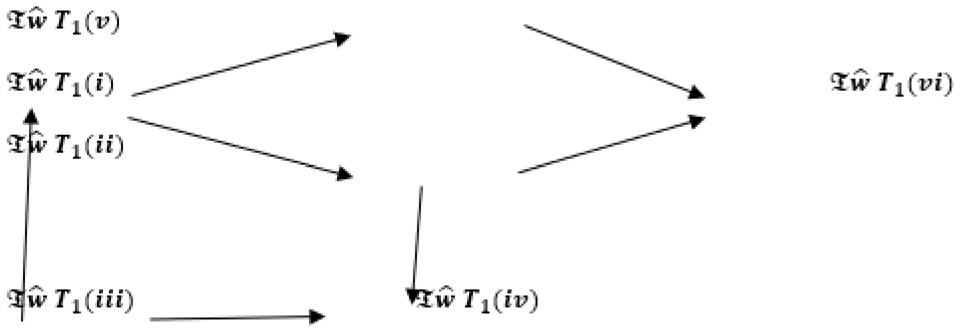


**Fig. 3.1**

**Proof :** Obvious.

**Example 3.9:** Let $\mathcal{M} = \{\mathscr{g}, \ell\}$ with the family $\mathfrak{I}\tau = \{\widetilde{\mathcal{M}}, \tilde{\varphi}, < \mathcal{M}, \{\ell\}, \varphi >, < \mathcal{M}, \varphi, \varphi >\}$. $\mathfrak{I}\hat{w}$ -$OS(\mathcal{M}, \mathfrak{I}\tau)$ $= \{\widetilde{\mathcal{M}}, \tilde{\varphi}, < \mathcal{M}, \varphi, \varphi >, < \mathcal{M}, \varphi, \{\mathscr{g}\} >, < \mathcal{M}, \{\mathscr{g}\}, \varphi >, < \mathcal{M}, \{\ell\}, \varphi >, < \mathcal{M}, \{\ell\}, \{\mathscr{g}\} >\}$.Here $(\mathcal{M}, \mathfrak{I}\tau)$ is $\mathfrak{I}\hat{w}\ T_1(i)$- space but not $\mathfrak{I}\hat{w}\ T_1(ii)$- space.

**Example 3.10:** Let $\mathcal{M} = \{\mathscr{g}, \ell\}$ with the family $\mathfrak{I}\tau = \{\widetilde{\mathcal{M}}, \tilde{\varphi}, < \mathcal{M}, \{\mathscr{g}\}, \{\ell\} >\}$. $\mathfrak{I}\hat{w}$ -$OS(\mathcal{M}, \mathfrak{I}\tau) = \{\widetilde{\mathcal{M}}, \tilde{\varphi}, < \mathcal{M}, \varphi, \varphi >, < \mathcal{M}, \varphi, \{\mathscr{g}\} >, < \mathcal{M}, \{\mathscr{g}\}, \varphi >, < \mathcal{M}, \varphi, \{\ell\} >, < \mathcal{M}, \{\mathscr{g}\}, \{\ell\} >\}$. Here $(\mathcal{M}, \mathfrak{I}\tau)$ is $\mathfrak{I}\hat{w}\ T_1(iv)$- space but not $\mathfrak{I}\hat{w}\ T_1(iii)$- space.

**Example 3.11 :** Let $\mathcal{M} = \{\mathscr{g}, \ell\}$ with the family $\mathfrak{I}\tau = \{\widetilde{\mathcal{M}}, \tilde{\varphi}, < \mathcal{M}, \{\mathscr{g}\}, \{\ell\} >, < \mathcal{M}, \{\mathscr{g}\}, \varphi >, < \mathcal{M}, \varphi, \varphi >, < \mathcal{M}, \varphi, \{\ell\} >\}$. $\mathfrak{I}\hat{w}$ - $OS$ $(\mathcal{M}, \mathfrak{I}\tau) = \mathfrak{I}\tau$. Here $(\mathcal{M}, \mathfrak{I}\tau)$ is $\mathfrak{I}\hat{w}\ T_1(v)$- space but not $\mathfrak{I}\hat{w}\ T_1(ii)$- space.

**Example 3.12 :** The above example, satisfied $\mathfrak{I}\hat{w}\ T_1(iv)$- space but not $\mathfrak{I}\hat{w}\ T_1(ii)$- space.

**Example 3.13 :** Let $\mathcal{M} = \{\mathscr{g}, \ell\}$ with the family $\mathfrak{I}\tau = \{\widetilde{\mathcal{M}}, \tilde{\varphi}, < \mathcal{M}, \{\mathscr{g}\}, \{l\} >, < \mathcal{M}, \{\mathscr{g}\}, \varphi >\}$. $\mathfrak{I}\hat{w}$ - $OS$ $(\mathcal{M}, \mathfrak{I}\tau) = \{\widetilde{\mathcal{M}}, \tilde{\varphi}, < \mathcal{M}, \varphi, \{\ell\} >, < \mathcal{M}, \{\mathscr{g}\}, \varphi >, \mathcal{M}, \{\mathscr{g}\}, \{\ell\} >\}$.Here$(\mathcal{M}, \mathfrak{I}\tau)$ is $\mathfrak{I}\hat{w}\ T_1(iv)$- space but not $\mathfrak{I}\hat{w}\ T_1(iii)$- space.

**Example 3.14 :** Let $\mathcal{M} = \{\mathscr{g}, \ell\}$ with the family $\mathfrak{I}\tau = \{\widetilde{\mathcal{M}}, \tilde{\varphi}, < \mathcal{M}, \{\mathscr{g}\}, \{\ell\} >, < \mathcal{M}, \{\mathscr{g}\}, \varphi >, < \mathcal{M}, \varphi, \varphi >, < \mathcal{M}, \varphi, \{\ell\} >\}$. $\mathfrak{I}\hat{w}$ - $OS$ $(\mathcal{M}, \mathfrak{I}\tau) = \mathfrak{I}\tau$. Here $(\mathcal{M}, \mathfrak{I}\tau)$ is $\mathfrak{I}\hat{w}\ T_1(v)$- space but not $\mathfrak{I}\hat{w}\ T_1(ii)$- space.

**Example 3.15 :** The above example satisfied $\mathfrak{I}\hat{w}\ T_1(iv)$- space but not $\mathfrak{I}\hat{w}\ T_1(ii)$- space.

**Example 3.16 :** Let $\mathcal{M} = \{\mathscr{g}, \ell\}$ with the family $\mathfrak{I}\tau = \{\widetilde{\mathcal{M}}, \tilde{\varphi}, < \mathcal{M}, \{\mathscr{g}\}, \{l\} >, < \mathcal{M}, \{\mathscr{g}\}, \varphi >\}$.$\mathfrak{I}\hat{w}$ - $OS(\mathcal{M}, \mathfrak{I}\tau) = \{\widetilde{\mathcal{M}}, \tilde{\varphi}, < \mathcal{M}, \varphi, \{\ell\} >, < \mathcal{M}, \{\mathscr{g}\}, \varphi >, \mathcal{M}, \{\mathscr{g}\}, \{\ell\} >\}$. Here $(\mathcal{M}, \mathfrak{I}\tau)$ is $\mathfrak{I}\hat{w}\ T_1(vi)$- space but not $\mathfrak{I}\hat{w}\ T_1(ii)$- space.

**Example 3.17:** In example 5.1.16, $\mathfrak{I}\hat{w}T_1(i)$- space is satisfied but not $\mathfrak{I}\hat{w}\ T_1(iii)$- space.

**Definition 3.18:** An intuitionistic topological space $(\mathcal{M}, \mathfrak{I}\tau_1)$ is called **$\mathfrak{I}\hat{w}T_1$- space** if for all $a, b \in \mathcal{M}$ $(a \neq b)$ there exist $\mathfrak{I}\hat{w}$- open set $\mathbb{U}, \mathbb{V}$ such that $a \in \mathbb{U}_1$ , $b \notin \mathbb{U}_1$ and $b \in \mathbb{V}_1$ , $a \notin \mathbb{V}_1$.

**Theorem3.19:** Every $\mathfrak{I}T_1$- space is $\mathfrak{I}\hat{w}\ T_1$- space but not conversely.

**Proof:** Since every intuitionistic open is $\mathfrak{I}\hat{w}$- open, the proof follows.

**Example 3.20:** Let $\mathcal{M} = \{d, \mathscr{f}\}$ with the family $\mathfrak{I}\tau = \{\widetilde{\mathcal{M}}, \tilde{\varphi}, < \mathcal{M}, \{\mathscr{f}\}, \varphi >\}$. Then $d \in < \mathcal{M}, \{d\}, \varphi >$, $\mathscr{f} \notin < \mathcal{M}, \{d\}, \varphi >$ and $\mathscr{f} \in < \mathcal{M}, \{\mathscr{f}\}, \{d\} >$, $d \notin < \mathcal{M}, \{\mathscr{f}\}, \{d\} >$. Here $(\mathcal{M}, \mathfrak{I}\tau_1)$ is $\mathfrak{I}\hat{w}\ T_1(i)$- space. But there exist no intuitionistic open set $\mathbb{U}$ and $\mathbb{V}$ such that $d \in \mathbb{U}$ and $\mathscr{f} \notin \mathbb{U}$ and $\mathscr{f} \in \mathbb{V}$ and $d \notin \mathbb{V}$. Hence $(\mathcal{M}, \mathfrak{I}\tau_1)$ is not $\mathfrak{I}T_1$- space .

**Theorem 3.21:** Let $(\mathcal{M}, \mathfrak{I}\tau_1)$ and $(\mathbb{Y}, \mathfrak{I}\tau_2)$ be two intuitionistic topological spaces. Let $\mathfrak{F} : (\mathcal{M}, \mathfrak{I}\tau_1) \to (\mathbb{Y}, \mathfrak{I}\tau_2)$ be a one- one, onto and $\mathfrak{I}\hat{w}$- open map. If $(\mathcal{M}, \mathfrak{I}\tau_1)$ is a $\mathfrak{I}T_1$ space then $(\mathcal{M}, \mathfrak{I}\tau_1)$ is $\mathfrak{I}\hat{w}\ T_1$- space.

**Proof :** Suppose $r, u \in \mathbb{Y}$ with $r \neq u$. Since $\mathfrak{F}$ is onto, then there exist $\mathscr{b}, \mathscr{f} \in \mathcal{M}$ such that $\mathfrak{F}(\mathscr{b}) = r$ and $\mathfrak{F}(\mathscr{f}) = u$. Then $\mathfrak{F}(\mathscr{b}) \neq \mathfrak{F}(\mathscr{f}) \Rightarrow \mathscr{b} \neq \mathscr{f}$, as $\mathfrak{F}$ is one - one. Since $\mathscr{b}, \mathscr{f} \in \mathcal{M}$, $\mathscr{b} \neq \mathscr{f}$ and $(\mathcal{M}, \mathfrak{I}\tau_1)$ is a $\mathfrak{I}T_1$ space, then there exist $T$- open set $\mathbb{U}$ and $\mathbb{V}$ in $\mathcal{M}$ such that $\mathscr{b} \in \mathbb{U}_1$, $\mathscr{f} \notin \mathbb{U}_1$ and $\mathscr{b} \in \mathbb{V}_1$, $\mathscr{f} \notin \mathbb{V}_1$. As $\mathfrak{F}$ is

$\mathfrak{T}\widehat{w}$- open, $\mathfrak{F}(\mathbb{U})$ is $\mathfrak{T}\widehat{w}$- open in $(\mathbb{Y},\mathfrak{T}\tau_2)$. Since $\mathfrak{F}(\mathbb{U}) = <\mathbb{Y},\mathfrak{F}(\mathbb{U}_1),\mathfrak{F}(\mathbb{U}_2)>, r = \mathfrak{F}(b) \in \mathfrak{F}(\mathbb{U}_1)$ and $u = \mathfrak{F}(f) \notin \mathfrak{F}(\mathbb{U}_1)$. Finally, we get $r, u \in \mathbb{Y}$ with $r \neq u$ there exist $\mathfrak{T}\widehat{w}$- open set $\mathfrak{F}(\mathbb{U}) \in (\mathbb{Y},\mathfrak{T}\tau_2)$ such that $r = \mathfrak{F}(b) \in \mathfrak{F}(\mathbb{U}_1)$, $u = \mathfrak{F}(f) \notin \mathfrak{F}(\mathbb{U}_1)$. Hence $(\mathbb{Y},\mathfrak{T}\tau_2)$ is $\mathfrak{T}\widehat{w}\ T_1$- space.

**Theorem 3.22 :** Let $(\mathcal{M},\mathfrak{T}\tau_1)$ and $(\mathbb{Y},\mathfrak{T}\tau_2)$ be two intuitionistic topological spaces. Let $\mathfrak{F}:(\mathcal{M},\mathfrak{T}\tau_1) \rightarrow (\mathbb{Y},\mathfrak{T}\tau_2)$ be a one- one, onto and $\mathfrak{T}\widehat{w}$- continuous map. If $(\mathbb{Y},\mathfrak{T}\tau_2)$ is a $\mathfrak{T}T_1$ space then $(\mathcal{M},\mathfrak{T}\tau_1)$ is $\mathfrak{T}\widehat{w}\ T_1$- space.

**Proof:** Let $d, h \in \mathcal{M}$ with $d \neq h$ implies $\mathfrak{F}(d), \mathfrak{F}(h) \in \mathbb{Y}$ with $\mathfrak{F}(d) \neq \mathfrak{F}(h)$ as $\mathfrak{F}$ is one- one. Since $\mathfrak{F}(d), \mathfrak{F}(h) \in \mathbb{Y}$ and $(\mathbb{Y},\mathfrak{T}\tau_2)$ is an intuitionistic $T_1$ space, there exist $\mathfrak{T}$ -open set $\mathbb{U}$ and $\mathbb{V}$ in $\mathbb{Y}$ such that $\mathfrak{F}(d) \in \mathbb{U}_1$, $\mathfrak{F}(h) \notin \mathbb{U}_1$ or $\mathfrak{F}(h) \in \mathbb{V}_1$, $\mathfrak{F}(d) \notin \mathbb{V}_1$. Now, $\mathfrak{F}(d) \in \mathbb{U}_1$ implies $\mathfrak{F}^{-1}(\mathfrak{F}(d)) \in \mathfrak{F}^{-1}(\mathbb{U}_1)$ which implies $d \in \mathfrak{F}^{-1}(\mathbb{U}_1)$. As $\mathfrak{F}(h) \in \mathbb{V}_1$, $\mathfrak{F}^{-1}(\mathfrak{F}(h)) \in \mathfrak{F}^{-1}(\mathbb{V}_1)$ which implies $h \in \mathfrak{F}^{-1}(\mathbb{V}_1)$. Similarly, $h \notin \mathfrak{F}^{-1}(\mathbb{U}_1)$, $d \notin \mathfrak{F}^{-1}(\mathbb{V}_1)$. Finally, we get $d, h \in \mathcal{M}$ with $d \neq h$, there exist $\mathfrak{T}\widehat{w}$- open set $\mathfrak{F}^{-1}(\mathbb{U})$ and $\mathfrak{F}^{-1}(\mathbb{V})$ such that $d \in \mathfrak{F}^{-1}(\mathbb{U}_1)$, $h \notin \mathfrak{F}^{-1}(\mathbb{U}_1)$ or $h \in \mathfrak{F}^{-1}(\mathbb{V}_1)$, $d \notin \mathfrak{F}^{-1}(\mathbb{V}_1)$. Hence $(\mathcal{M},\mathfrak{T}\tau_1)$ is $\mathfrak{T}\widehat{w}\ T_1$- space.

**Definition 3.23 :** An intuitionistic topological space $(\mathcal{M},\mathfrak{T}\tau_1)$ is said to be

vii. **$\mathfrak{T}\widehat{w}\ T_2(i)$- space** if for all $b, k \in \mathcal{M}$ ( $b \neq k$ ) there exist $\mathfrak{T}\widehat{w}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{b} \in \mathcal{V}, \tilde{k} \in \mathcal{W}$ and $\mathcal{V} \cap \mathcal{W} = \tilde{\varphi}$ .

viii. **$\mathfrak{T}\widehat{w}\ T_2(ii)$- space** if for all $b, k \in \mathcal{M}$ ( $b \neq k$ ) there exist $\mathfrak{T}\widehat{w}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\tilde{b}} \in \mathcal{V}$, $\tilde{\tilde{k}} \in \mathcal{W}$ and $\mathcal{V} \cap \mathcal{W} = \tilde{\varphi}$ .

ix. **$\mathfrak{T}\widehat{w}\ T_2(iii)$- space** if for all $b, k \in \mathcal{M}$ ( $b \neq k$ ) there exist $\mathfrak{T}\widehat{w}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{b} \in \mathcal{V}, \tilde{k} \in \mathcal{W}$ and $\mathcal{V} \subseteq \overline{\mathcal{W}}$.

x. **$\mathfrak{T}\widehat{w}\ T_2(iv)$- space** if for all $b, k \in \mathcal{M}$ ( $b \neq k$ ) there exist $\mathfrak{T}\widehat{w}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\tilde{b}} \in \mathcal{V}$, $\tilde{\tilde{k}} \in \mathcal{W}$ and $\mathcal{V} \subseteq \overline{\mathcal{W}}$.

xi. **$\mathfrak{T}\widehat{w}\ T_2(v)$- space** if for all $b, k \in \mathcal{M}$ ( $b \neq k$ ) there exist $\mathfrak{T}\widehat{w}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{b} \in \mathcal{V} \subseteq \bar{\tilde{k}}$, $\tilde{k} \in \mathcal{W} \subseteq \bar{\tilde{b}}$ and $\mathcal{V} \subseteq \overline{\mathcal{W}}$ .

xii. **$\mathfrak{T}\widehat{w}\ T_2(vi)$- space** if for all $b, k \in \mathcal{M}$ ( $b \neq k$ ) there exist $\mathfrak{T}\widehat{w}$- open set $\mathcal{V}, \mathcal{W}$ such that $\tilde{\tilde{b}} \in \mathcal{V} \subseteq \bar{\tilde{\tilde{k}}}$, $\tilde{\tilde{k}} \in \mathcal{W} \subseteq \bar{\tilde{\tilde{b}}}$ and $\mathcal{V} \subseteq \overline{\mathcal{W}}$.

**Definition 3.24:** An intuitionistic topological space $(\mathcal{M},\mathfrak{T}\tau_1)$ is called **$\mathfrak{T}\widehat{w}\ T_2$- space** if for all $b, k \in \mathcal{M}$ with ( $b \neq k$ ) there exist $\mathfrak{T}\widehat{w}$- open set $\mathcal{V}, \mathcal{W}$ such that $b \in \mathcal{V}_1, k \notin \mathcal{V}_1$ and $k \in \mathcal{W}_1$, $b \notin \mathcal{W}_1$ and $\mathcal{V} \cap \mathcal{W} = \tilde{\varphi}$

**Theorem 3.25 :** Every intuitionistic $T_2$- space is $\mathfrak{T}\widehat{w}\ T_2$- space but not conversely.

**Proof:** Since every intuitionistic open is $\mathfrak{T}\widehat{w}$- open, the proof follows.

**Example 3.26:** Let $\mathcal{M} = \{a, b\}$ with the family $\mathfrak{T}\tau = \{ \widetilde{\mathcal{M}}, \tilde{\varphi}, <\mathcal{M},\varphi,\varphi> \}$.Then $a \in \mathcal{V} = <\mathcal{M},\{a\},\{b\}>$, $b \notin \mathcal{V}$ and $b \in \mathcal{W} = <\mathcal{M},\{b\},\{a\}>, a \notin \mathcal{W}$. Also, $\mathcal{V} \cap \mathcal{W} = \tilde{\varphi}$. Here $(\mathcal{M},\mathfrak{T}\tau_1)$ is $\mathfrak{T}\widehat{w}\ T_2$- space. But there exist not an intuitionistic open set $\mathcal{V}, \mathcal{W}$ such that $b \in \mathcal{V}_1, k \notin \mathcal{V}_1$ and $k \in \mathcal{W}_1$, $b \notin \mathcal{W}_1$ and $\mathcal{V} \cap \mathcal{W} = \tilde{\varphi}$.

**Theorem 3.27:** Let $(\mathcal{M},\mathfrak{T}\tau_1)$ and $(\mathbb{Y},\mathfrak{T}\tau_2)$ be two intuitionistic topological spaces. Let $\mathfrak{F}:(\mathcal{M},\mathfrak{T}\tau_1) \rightarrow (\mathbb{Y},\mathfrak{T}\tau_2)$ be one- one, onto and $\mathfrak{T}\widehat{w}$- open map. If $(\mathcal{M},\mathfrak{T}\tau_1)$ is $\mathfrak{T}T_2$- space then $(\mathbb{Y},\mathfrak{T}\tau_2)$ is $\mathfrak{T}\widehat{w}\ T_2$- space.

**Proof:** Suppose $b, k \in \mathbb{Y}$ with ( $b \neq k$ ). Since $\mathfrak{F}$ is onto, then there exist $\mathfrak{g}, \mathfrak{m} \in \mathcal{M}$ such that $\mathfrak{F}(\mathfrak{g}) = b$ and $\mathfrak{F}(\mathfrak{m}) = k$. Then $\mathfrak{F}(\mathfrak{g}) \neq \mathfrak{F}(\mathfrak{m})$ which implies $\mathfrak{g} \neq \mathfrak{m}$, as $\mathfrak{F}$ is one-one. Since $\mathfrak{g}, \mathfrak{m} \in \mathcal{M}$, $\mathfrak{g} \neq \mathfrak{m}$ and $(\mathcal{M},\mathfrak{T}\tau_1)$ is $\mathfrak{T}T_2$- space, then there exist $\mathfrak{T}$- open set $\mathcal{V}$ in $\mathcal{M}$ such that $\mathfrak{g} \in \mathcal{V}_1$, $\mathfrak{m} \notin \mathcal{V}_1$ and $\mathfrak{m} \in \mathcal{W}_1$, $\mathfrak{g} \notin \mathcal{W}_1$ and $\mathcal{V} \cap \mathcal{W} = \tilde{\varphi}$. Since, $\mathfrak{F}$ is $\mathfrak{T}\widehat{w}$- open, $\mathcal{V}$ and $\mathcal{W} \in (\mathcal{M},\mathfrak{T}\tau_1)$ implies $\mathfrak{F}(\mathcal{V})$ and $\mathfrak{F}(\mathcal{W})$ is $\mathfrak{T}\widehat{w}$- open in $(\mathbb{Y},\mathfrak{T}\tau_2)$. As $\mathfrak{F}(\mathcal{V}) = <\mathbb{Y}, \mathfrak{F}(\mathcal{V}_1), \mathfrak{F}(\mathcal{V}_1)>$, $\mathfrak{F}(\mathcal{W}) = <\mathbb{Y},\mathfrak{F}(\mathcal{W}_1), \mathfrak{F}(\mathcal{W}_1)>, b = \mathfrak{F}(\mathfrak{g}) \in \mathfrak{F}(\mathcal{V}_1)$ and $k = \mathfrak{F}(\mathfrak{m}) \in \mathfrak{F}(\mathcal{W}_1)$. Also, $\mathfrak{m} \notin \mathcal{V}_1$ implies $k = \mathfrak{F}(\mathfrak{m})) \notin \mathfrak{F}(\mathcal{V}_1)$ and $\mathfrak{g} \notin \mathcal{W}_1$ implies $b = \mathfrak{F}(p) \notin \mathfrak{F}(\mathcal{W}_1)$ . Consider $\mathfrak{F}(\mathcal{V}) \cap \mathfrak{F}(\mathcal{W}) \neq \tilde{\varphi}$ which implies $\mathfrak{F}(\mathcal{V}_1) \cap \mathfrak{F}(\mathcal{W}_1) \neq \varphi$ then there exists at least one $c \in \mathbb{Y}$ for which $c \in \mathfrak{F}(\mathcal{V}_1) \cap \mathfrak{F}(\mathcal{W}_1)$ which implies $c \in \mathfrak{F}(\mathcal{V}_1)$ and $c \in \mathfrak{F}(\mathcal{W}_1)$. Then there exists $u \in \mathcal{V}_1$ and $v \in \mathcal{W}_1$ such that $\mathfrak{F}(u) = \mathfrak{F}(v) = c \Rightarrow u = v$ as $\mathfrak{F}$ is one-one $\Rightarrow u = v \in \mathcal{V}_1 \cap \mathcal{W}_1$ which is a contradiction to the fact that $\mathcal{V} \cap \mathcal{W} = \tilde{\varphi}$. Therefore, we get $\mathfrak{F}(\mathcal{V}) \cap \mathfrak{F}(\mathcal{W}) = \tilde{\varphi}$. Finally, we get $b, k \in \mathbb{Y}$ with ( $b \neq k$ ) there exist $\mathfrak{T}\widehat{w}$- open set $\mathfrak{F}(\mathcal{V})$, $\mathfrak{F}(\mathcal{W}) \in (\mathbb{Y},\mathfrak{T}\tau_2)$ such that $b = \mathfrak{F}(\mathfrak{g}) \in \mathfrak{F}(\mathcal{V}_1)$, $k = \mathfrak{F}(\mathfrak{m}) \notin \mathfrak{F}(\mathcal{V}_1)$ and $k = \mathfrak{F}(\mathfrak{m}) \in \mathfrak{F}(\mathcal{W}_1)$, $b = \mathfrak{F}(\mathfrak{g}) \notin \mathfrak{F}(\mathcal{W}_1)$ and $\mathfrak{F}(\mathcal{V}) \cap \mathfrak{F}(\mathcal{W}) = \tilde{\varphi}$. Hence $(\mathbb{Y},\mathfrak{T}\tau_2)$ is $\mathfrak{T}\widehat{w}\ T_2$- space.

**Theorem 3.28:** Let $(\mathcal{M},\mathfrak{T}\tau_1)$ and $(\mathbb{Y},\mathfrak{T}\tau_2)$ be two intuitionistic topological spaces. Let $\mathfrak{F}:(\mathcal{M},\mathfrak{T}\tau_1) \rightarrow (\mathbb{Y},\mathfrak{T}\tau_2)$ be a one- one, onto and $\mathfrak{T}\widehat{w}$- continuous map. If $(\mathbb{Y},\mathfrak{T}\tau_2)$ is $\mathfrak{T}T_2$- space then $(\mathcal{M},\mathfrak{T}\tau_1)$ is $\mathfrak{T}\widehat{w}\ T_2$- space.

**Proof:** Let $d, h \in \mathcal{M}$ with $d \neq h$ implies $\mathfrak{F}(d), \mathfrak{F}(h) \in \mathbb{Y}$ with $\mathfrak{F}(d) \neq \mathfrak{F}(h)$ as $\mathfrak{F}$ is one- one. Since $\mathfrak{F}(d), \mathfrak{F}(h) \in \mathbb{Y}$ and $(\mathbb{Y},\mathfrak{T}\tau_2)$ is $\mathfrak{T}T_2$- space, then there exist an intuitionistic open set $\mathcal{V}$ and $\mathcal{W}$ in $\mathbb{Y}$ such that $\mathfrak{F}(d) \in \mathcal{V}_1$, $\mathfrak{F}(h) \notin \mathcal{V}_1$ or $\mathfrak{F}(h) \in \mathcal{W}_1$, $\mathfrak{F}(d) \notin \mathcal{W}_1$ and $\mathcal{V} \cap \mathcal{W} = \tilde{\varphi}$ which implies $\mathcal{V}_1 \cap \mathcal{W}_1 = \tilde{\varphi}$. Now, $\mathfrak{F}(d) \in \mathcal{V}_1$ implies $\mathfrak{F}^{-1}(\mathfrak{F}(d)) \in \mathfrak{F}^{-1}(\mathcal{V}_1)$ which implies $d \in \mathfrak{F}^{-1}(\mathcal{V}_1)$. And $\mathfrak{F}(h) \in \mathcal{V}_1$ implies $\mathfrak{F}^{-1}(\mathfrak{F}(h)) \in \mathfrak{F}^{-1}(\mathcal{V}_1)$ which implies $h \in \mathfrak{F}^{-1}(\mathcal{V}_1)$. As $\mathfrak{F}(h) \in \mathcal{W}_1$, $\mathfrak{F}^{-1}(\mathfrak{F}(h)) \in \mathfrak{F}^{-1}(\mathcal{W}_1)$ which implies $h \in \mathfrak{F}^{-1}(\mathcal{W}_1)$. Similarly, $h \notin \mathfrak{F}^{-1}(\mathcal{V}_1)$, $d \notin \mathfrak{F}^{-1}(\mathcal{W}_1)$. Suppose $\mathfrak{F}^{-1}(\mathcal{V}) \cap \mathfrak{F}^{-1}(\mathcal{W}) \neq \tilde{\varphi}$ which implies $\mathfrak{F}^{-1}(\mathcal{V}_1) \cap \mathfrak{F}^{-1}(\mathcal{W}_1) \neq \varphi$ which implies $\mathfrak{F}(\mathfrak{F}^{-1}(G_1)) \cap \mathfrak{F}(\mathfrak{F}^{-1}(H_1)) \neq \varphi$ which implies $\mathcal{V}_1 \cap \mathcal{W}_1 \neq \varphi$ which is a contradiction. Therefore $\mathfrak{F}^{-1}(\mathcal{V}) \cap \mathfrak{F}^{-1}(\mathcal{W}) = \tilde{\varphi}$. Finally, we get $d, h \in \mathcal{M}$ with $d \neq h$ there exist $\mathfrak{T}\widehat{w}$- open set $\mathfrak{F}^{-1}(\mathcal{V})$ such that $d \in \mathfrak{F}^{-1}(\mathcal{V}_1)$, $h \notin \mathfrak{F}^{-1}(\mathcal{V}_1)$ or $h \in \mathfrak{F}^{-1}(\mathcal{W}_1)$, $d \notin \mathfrak{F}^{-1}(\mathcal{W}_1)$ and $\mathfrak{F}^{-1}(\mathcal{V}) \cap \mathfrak{F}^{-1}(\mathcal{W}) = \tilde{\varphi}$. Hence $(\mathcal{M}, \mathfrak{T}\tau_1)$ is $\mathfrak{T}\widehat{w}\, T_2$- space.

**Theorem 3.29 :** Let $(\mathcal{M}, \mathfrak{T}\tau_1)$ be intuitionistic topological spaces. Then the following implications are valid but not conversely.

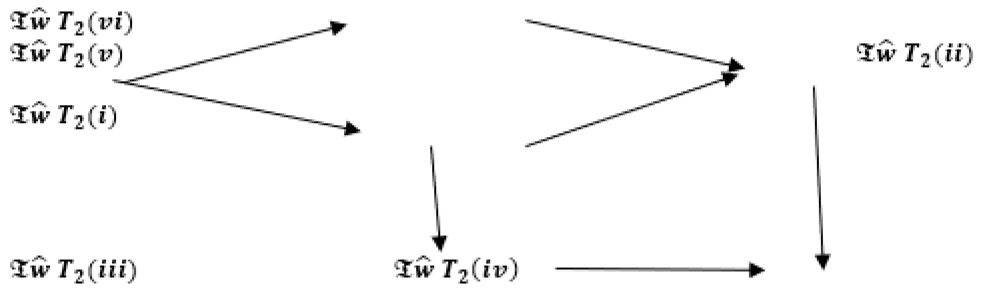


**Fig. 3.2**

**Proof:** Obvious.

**Example 3.30:** Let $\mathcal{M} = \{ g, \ell\}$ with the family $\mathfrak{T}\tau = \{ \widetilde{\mathcal{M}}, \tilde{\varphi}, < \mathcal{M}, \{g\}, \{ l\} >\}$. $\mathfrak{T}\widehat{w}$ $-OS(\mathcal{M}, \mathfrak{T}\tau) = \{\widetilde{\mathcal{M}}, \tilde{\varphi}, < \mathcal{M}, \varphi, \varphi >, < \mathcal{M}, \varphi, \{g\} >, < \mathcal{M}, \{g\}, \varphi >, < \mathcal{M}, \varphi, \{\ell\} >, < \mathcal{M}, \{g\}, \{\ell\} >\}$. Here $(\mathcal{M}, \mathfrak{T}\tau)$ is $\mathfrak{T}\widehat{w}$ $T_2(ii)$- space but not $\mathfrak{T}\widehat{w}\, T_2(i)$- space.

**Example 3.31 :** In example 3.30, $\mathfrak{T}\widehat{w}T_2(ii)$- space is satisfied but not $\mathfrak{T}\widehat{w}$ $T_1(v)$- space.

**Example 3.32:** Let $\mathcal{M} = \{ g, \ell\}$ with the family $\mathfrak{T}\tau = \{ \widetilde{\mathcal{M}}, \tilde{\varphi}, < \mathcal{M}, \{g\}, \{ l\} >\}$. $\mathfrak{T}\widehat{w}$ - $OS$ $(\mathcal{M}, \mathfrak{T}\tau) = \{\widetilde{\mathcal{M}}, \tilde{\varphi}, < \mathcal{M}, \varphi, \varphi >, < \mathcal{M}, \{g\}, \varphi >, < \mathcal{M}, \varphi, \{\ell\} >$, M, $\{g\}, \{\ell\} >\}$. $\mathfrak{T}\widehat{w}$ - $OS$ $(\mathcal{M}, \mathfrak{T}\tau) = \mathfrak{T}\tau$. Here $(\mathcal{M}, \mathfrak{T}\tau)$ is $\mathfrak{T}\widehat{w}\, T_2(iv)$- space but not $\mathfrak{T}\widehat{w}\, T_2(i)$- space.

**Example 3.33:** The above example satisfies $\mathfrak{T}\widehat{w}\, T_2(iv)$- space but not $\mathfrak{T}\widehat{w}\, T_2(ii)$- space and $\mathfrak{T}\widehat{w}\, T_2(iii)$- space.

**Example 3.34 :** Let $\mathcal{M} = \{ g, \ell\}$ with the family $\mathfrak{T}\tau = \{ \widetilde{\mathcal{M}}, \tilde{\varphi}, < \mathcal{M}, \{g\}, \{ l\} >\}$. $\mathfrak{T}\widehat{w}$ $-OS(\mathcal{M}, \mathfrak{T}\tau) = \{\widetilde{\mathcal{M}}, \tilde{\varphi}, < \mathcal{M}, \varphi, \varphi >, < \mathcal{M}, \varphi, \{g\} >, < \mathcal{M}, \{g\}, \varphi >, < \mathcal{M}, \varphi, \{\ell\} >, \mathcal{M}, \{g\}, \{\ell\} >\}$. Here $(\mathcal{M}, \mathfrak{T}\tau)$ is $\mathfrak{T}\widehat{w}\, T_2(vi)$- space but not $\mathfrak{T}\widehat{w}\, T_2(v)$- space.

## References:

[1] Bayhan, S. And Coker, D. 2001, On Separation Axioms In Intuitionistic Topological Spaces, Int. J. Math. Science, Vol. 27, No.10, Pp. 621-630.

[2] Bayhan, S. And Coker, D. 2005, Pairwise Separation Axioms In Intuitionistic Topological Spaces, Hacettepe Journal Of Mathematics And Statistics, Vol. 34, Pp. 101-114.

[3] Coker. D, An Introduction To Intuitionistic Topological Spaces. Busefal. 2000; 81: 51.

[4] Coker, D. 1995, An Introduction To Intuitionistic Topological Spaces, Akdeniz University, Mathematics Department, 07058-Antalya/Turkey.

[5] Coker, D. 2005, Pairwise Separation Axioms In Intuitionistic Topological Spaces, Journal Of Mathematics And Statistics, Vol.34S, Pp. 101-114.

[6] Tamanna Tasnim Prova And Md. Sahadat Hossain. 2022, Separation Axioms In Intuitionistic Topological Spaces, Italian Journal Of Pure And Applied Mathematics, No. 48, Pp 986-995.