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An Inventory Model For Weibull Distributed Deteriorating Items With Stochastic Demand And Time-Varying Holding Cost.

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Abstract:
In the present paper, we deal with an inventory model for deteriorating items with stochastic demand in which shortages are allowed with fully backlogged. The holding cost follows the linear trended in time. Here rate of deterioration follows a two-parameter Weibull (Swedish engineer Wallodi Weibull) distribution. The demand pattern is assumed to be linearly dependent on to time with a stochastic error. The model is maximized to the total average profit by finding optimal values. The developed model is illustrated by a numerical example and finally the sensitivity analysis for the optimal solutions towards the changes in the values of key parameters has been presented.

Keywords: *Inventory, Weibull deteriorating items, stochastic demand, holding cost and shortages.*

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I. Introduction:

Almost all items deteriorate with time. It is a natural phenomenon in many inventory systems. The rate of deterioration of such items like steel, hardware, glassware etc is very small that there is hardly any need to assume the effect of deterioration. But many items like fruits, vegetables, milk, fish, fashion goods, medicine, electronic goods, alcohol, food grains etc having deterioration considerably high and therefore this realistic factor should be considered in inventory modelling. It is noticed that

- Deterioration is defined as falling from a higher to a lower level in quality, it also simply implies a change, decay, obsolescence, collapse, spoilage, loss of utility or loss of marginal value of goods that results in a decrease of the usefulness of the original item
- Deterioration is always considered over time
- The rate of deterioration is nearly negligible for commodities like hardware, glassware, toys and steel, but it is very much effective for products such as fruits, vegetables, medicines etc
- The rate of consumption are very large for highly deteriorated items like gasoline, alcohol, turpentine etc, and they deteriorate continuously through the process of mortification.
- The deteriorated items like radioactive substances, electronic goods, grain, photographic film etc., deplete over time through a process of evaporation and also they deteriorate through a gradual loss of potential or utility from one time to another.

Therefore deterioration of items plays a vital role in the determination of an inventory model and has to be taken into account. These assumptions were followed by many researchers like Ghare and Schrader [1], Covert & Philip [2] , A. K.Jalan et al [3], Mandal B[4], Dr. Biswaranjan Mandal [5], P. R. Tadikamalla [6], R.KavithaPriya1 et al [7], A. Hatibaruah et al [8] and many more.

The assumption of constant demand rate may not be always appropriate for many inventory theories. For examples, milk items, vegetables, fruits, cosmetics etc have a negative impact on demand due to loss of confidence of consumers on the quality of such products for their age of inventory. Also we noticed that the demand of seasonal foods and garments is highly dependent on time. So it can be concluded that demand for items varies with respect to time. However, in order to match with real-life criteria, many authors have developed new types of inventory models with a variable demand rate. Also the acceptance of some constant demand rate is not reasonable for many inventory items such as electronic goods, fashionable garments, tasty foods, volatile liquids etc , as they fluctuate in the demand rate. As a result, linear trended dependent rate on to time with a stochastic error has a prominent role in inventory control system. Researchers like U Dave et al [9], T K Datta et al [10], J T Teng et al [11], Biswaranjan[12], M Mallick et al [13] A Kundu [14] are mentioned a few.

In a classical inventory management system, it is assumed that holding cost is fixed as constant. But in reality, inventory is stored up to meet the demand of the customers or for future usage. So variable holding cost is attracting most of the related researchers 'attention because maintaining inventory is very much crucial.

Holding cost is also high and dependent on time for many deteriorating items like fruits, vegetables, medicine etc. Therefore variable holding cost is utmost important in inventory management system. In this matter, few researchers like M Goh [15], A Roy [16], V K Mishra [17], Tripathi et al [18], M Sharma [19] are noteworthy.

The word shortage means a state or situation in which the needed items cannot be obtained in sufficient amounts or totally absent. It has a great importance for many models, especially when delay in payment is considered. When a shortage occurs but the company offers delay in payment, it can gain more orders from the customers. So shortages have an important role on optimization in inventory theory.

In view of the above sort of situations and facts, the present paper deals with an inventory model for Weibull deteriorating items having time dependent demand rate with a stochastic error in which shortages are allowed and fully backlogged. The holding cost follows the linear trended in time. The model is maximized to the total average profit by finding optimal values and finally it is illustrated by a numerical example along with the sensitivity analysis for the optimal solutions towards the changes in the values of system parameters.

II. Assumptions And Notations:

Assumptions:

The present inventory model is developed on the basis of the following assumptions

i. Lead time is zero.
ii. Replenishment rate is infinite but size is finite.
iii. The time horizon is finite.
iv. There is no repair of deteriorated items occurring during the cycle.
v. Rate of deterioration follows a two parameter Weibull distribution. .
vi. The demand rate is a linear function of time with stochastic error.
vii. Holding cost follows linear trended in time..
viii. Shortages are allowed and completely backlogged.

Notations:

The following notations are used in the proposed model:

i. Q : On hand inventory at time t.
ii. $\theta(t)=\alpha\beta t^{\beta-1}, t\geq 0$ is the two parameter Weibull distribution deterioration rate function where $\alpha(0<\alpha<<1)$ is a scale parameter and $\beta(>0)$ is a shape parameter.
iii. t_1 : The time length in which the stock is completely diminished.
iv. T : The fixed length of each production cycle.
v. A_0 : The ordering cost per order during the cycle period.
vi. p_c : The purchasing cost per unit item.
vii. d_c : The deterioration cost per unit item.
viii. c_s : The shortage cost per unit item.
ix. r_c : Sales revenue cost per unit time.
x. h(t) : Holding cost $h(t)=h+\delta t$, $h,\delta>0$.
xi. D(t) : Demand rate D(t) = $a+bt+\varepsilon$, where a, b>0 so that the demand is positive throughout the demand, ε (stochastic error). Here the shape of the demand curve is deterministic while the scaling parameter representing the size of the market is random. From practical stand point if "a" is large relative to the variance of ε, unbounded probability distribution such as the normal distribution provides adequate approximation. We assume that F(.) and f(.) represent the cumulative distribution and probability density function of ε, respectively having mean μ and standard deviation δ.
xii. q(t) : The level of inventory $q(t)=\left|\begin{matrix} q_1(t), 0\leq t\leq t_1 \\ q_2(t), t_1\leq t\leq T\end{matrix}\right\rangle$
xiii. ATP : Average total profit per unit time.
xiv. <ATP> : Expected average total profit per unit time.

III. Mathematical Formulation And Solution:

In this model, we consider the variation of the inventory level during the period $[0,T]$. The inventory level is depleted only due to demand and deterioration and ultimately falls to zero at t $=t_1$. The shortages occur during time period [t_1, T] which are completely backlogged. The differential equations pertaining to the above situations are given by

$$\frac{dq_1(t)}{dt}+\alpha\beta t^{\beta-1}q_1(t)=-[a+bt+\varepsilon],0\le t\le t_1 \tag{3.1}$$

And $$\frac{dq_2(t)}{dt}=-(a+bt+\varepsilon),t_1\le t\le T \tag{3.2}$$

The initial conditions are $q_1(0)=Q_1$, $q_1(t_1)=0$ and $q_2(t_1)=0$ (3.3)

Since $\alpha(0<\alpha<<1)$, we ignore the terms $O(\alpha^2)$, then the solutions of the equations (3.1) and (3.2) using (3.3) are given by the following

$$q_1(t)=(a+\varepsilon)[(t_1-t)(1-\alpha t^{\beta})+\frac{\alpha}{\beta+1}(t_1^{\beta+1}-t^{\beta+1})]$$

$$+\quad b[\frac{1}{2}(t_1^2-t^2)(1-\alpha t^{\beta})+\frac{\alpha}{\beta+2}(t_1^{\beta+2}-t^{\beta+2})],\ 0\le t\le t_1 \tag{3.4}$$

And $$q_2(t)=(a+\varepsilon)(t_1-t)+\frac{b}{2}(t_1^2-t^2),\quad t_1\le t\le T \tag{3.5}$$

Since $q_1(0)=Q$, we get the following expression of on-hand inventory from the equations (3.4), neglecting second and higher order powers of α ,

$$Q=(a+\varepsilon)[t_1+\frac{\alpha}{\beta+1}t_1^{\beta+1}]+b[\frac{1}{2}t_1^2+\frac{\alpha}{\beta+2}t_1^{\beta+2}] \tag{3.6}$$

IV. Cost Components:

The total profit over the period [0, T] consists of the following cost components:

(1). Ordering cost **(OC)** over the period [0,T] = A_0 (fixed) (4.1)

(2). Purchasing cost **(PC)** over the period [0,T] = p_c Q

Or, $$PC=p_c[(a+\varepsilon)\{t_1+\frac{\alpha}{\beta+1}t_1^{\beta+1}\}+b\{\frac{1}{2}t_1^2+\frac{\alpha}{\beta+2}t_1^{\beta+2}\}] \tag{4.2}$$

(3). Holding cost for carrying inventory **(HC)** over the period [0,T]

HC = $[\int_0^{t_1}(h+\delta t)q_1(t)dt]$

Putting the values of $q_1(t)$ from (3.4), and integrating and then substituting the value of Q from (3.6), we get the following expression after neglecting second and higher order powers of α

HC = $$h(a+\varepsilon)[\frac{1}{2}t_1^{\,2}+\frac{\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2}]+hb[\frac{1}{3}t_1^3+\frac{\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3}]$$

$$+\ \delta(a+\varepsilon)[\frac{1}{6}t_1^{\ 3}+\frac{\alpha\beta}{2(\beta+2)(\beta+3)}t_1^{\beta+3}]+\delta b[\frac{1}{8}t_1^4+\frac{\alpha\beta}{2(\beta+2)(\beta+4)}t_1^{\beta+4}] \quad (4.3)$$

(4). Cost due to deterioration **(CD)** over the period [0,T]

CD = $d_c\int_0^{t_1}\alpha\beta t^{\beta-1}q_1(t)dt$

Putting the value $q_1(t)$ from (3.4), and then integrating and neglecting second and higher order powers of α , we get the following

$$\text{CD } = d_c\alpha[\frac{a+\varepsilon}{\beta+1}t_1^{\beta+1}+\frac{b}{\beta+2}t_1^{\beta+2}] \quad (4.4)$$

(5). Cost due to shortage **(CS)** over the period [0,T]

CS = $-c_s\int_{t_1}^{T}q_2(t)dt$

Putting the value $q_2(t)$ from (3.5), and then integrating we get the following

$$\text{CS} = c_s[\frac{a+\varepsilon}{2}(T^2-2Tt_1+t_1^2)+\frac{b}{6}(T^3-3Tt_1^{\ 2}+2t_1^3)] \quad (4.5)$$

(6). Sales Revenue Cost **(SRC)** over the period [0,T]

SRC = $r_c\int_0^{t_1}(a+bt+\varepsilon)dt$

$$= \quad r_c(at_1+\frac{b}{2}t_1^2+\varepsilon t_1) \quad (4.6)$$

Thus the average total profit per unit time of the system during the cycle [0,T] will be

ATP(t_1) = $\frac{1}{T}$ **[SRC-OC –PC-HC-CD-CS]**

$$=\frac{1}{T}\,[\,r_c(at_1+\frac{b}{2}t_1^2+\varepsilon t_1)\ -\ A_0\ -\ -p_c[(a+\varepsilon)\{t_1+\frac{\alpha}{\beta+1}t_1^{\beta+1}\}+b\{\frac{1}{2}t_1^2+\frac{\alpha}{\beta+2}t_1^{\beta+2}\}]$$

$$-\ h(a+\varepsilon)[\frac{1}{2}t_1^{\ 2}+\frac{\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2}]-hb[\frac{1}{3}t_1^3+\frac{\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3}]$$

$$-\delta(a+\varepsilon)[\frac{1}{6}t_1^{\ 3}+\frac{\alpha\beta}{2(\beta+2)(\beta+3)}t_1^{\beta+3}]-\delta b[\frac{1}{8}t_1^4+\frac{\alpha\beta}{2(\beta+2)(\beta+4)}t_1^{\beta+4}]$$

$$-\ d_c\alpha[\frac{a+\varepsilon}{\beta+1}t_1^{\beta+1}+\frac{b}{\beta+2}t_1^{\beta+2}]\ -\ c_s[\frac{a+\varepsilon}{2}(T^2-2Tt_1+t_1^2)+\frac{b}{6}(T^3-3Tt_1^{\ 2}+2t_1^3)]\ \ ] \quad (4.7)$$

Let us suppose that

$f(\varepsilon)=\frac{1}{k_1-k_0}$, $k_1\le\varepsilon\le k_0$

= 0, elsewhere

Where (μ,σ) are mean and standard deviation.

Therefore the expected average total profit (EATP) is given by

EATP = $< ATP(t_1) > = \frac{1}{T}$ [$r_c(at_1 + \frac{b}{2}t_1^2 + \mu t_1)$ - A_0 $-p_c[(a+\mu)\{t_1 + \frac{\alpha}{\beta+1}t_1^{\beta+1}\} + b\{\frac{1}{2}t_1^2 + \frac{\alpha}{\beta+2}t_1^{\beta+2}\}]$

- $h(a+\mu)[\frac{1}{2}t_1^{\ 2} + \frac{\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2}] - hb[\frac{1}{3}t_1^3 + \frac{\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3}]$

$-\delta(a+\mu)[\frac{1}{6}t_1^{\ 3} + \frac{\alpha\beta}{2(\beta+2)(\beta+3)}t_1^{\beta+3}] - \delta b[\frac{1}{8}t_1^4 + \frac{\alpha\beta}{2(\beta+2)(\beta+4)}t_1^{\beta+4}]$

- $d_c\alpha[\frac{a+\mu}{\beta+1}t_1^{\beta+1} + \frac{b}{\beta+2}t_1^{\beta+2}]$ - $c_s[\frac{a+\mu}{2}(T^2 - 2Tt_1 + t_1^2) + \frac{b}{6}(T^3 - 3Tt_1^{\ 2} + 2t_1^3)]$] (4.8)

To maximize the profit, the necessary condition is $\frac{d < ATP(t_1) >}{dt_1} = 0$

This gives

$r_c(a + bt_1 + \mu)$ $-p_c[(a+\mu)\{1+\alpha t_1^{\beta}\} + b\{t_1 + \alpha t_1^{\beta+1}\}]$

- $h(a+\mu)[t_1 + \frac{\alpha\beta}{(\beta+1)}t_1^{\beta+1}] - hb[t_1^2 + \frac{\alpha\beta}{(\beta+1)}t_1^{\beta+2}]$

$-\delta(a+\mu)[\frac{1}{2}t_1^{\ 2} + \frac{\alpha\beta}{2(\beta+2)}t_1^{\beta+2}] - \delta b[\frac{1}{2}t_1^3 + \frac{\alpha\beta}{2(\beta+2)}t_1^{\beta+3}]$

- $d_c\alpha[(a+\mu)t_1^{\beta} + bt_1^{\beta+1}]$ - $c_s[(a+\mu)(t_1 - T) + b(t_1^{\ 2} - Tt_1)]$ = 0 (4.9)

Or, r_c $-p_c(1+\alpha t_1^{\beta})$ - $h(t_1 + \frac{\alpha\beta}{\beta+1}t_1^{\beta+1})$ $-\frac{\delta}{2}(t_1^{\ 2} + \frac{\alpha\beta}{\beta+2}t_1^{\beta+2})$ - $d_c\alpha t_1^{\beta}$ - $c_s(t_1 - T)$ = 0

(4.10)

For maximum, the sufficient condition $\frac{\mathrm{d}^2 \mathrm{<ATP(t_1)>}}{\mathrm{dt_1}^2}$ <0 would be satisfied.

Let $t_1 = t_1^{\ *}$ be the optimum value of t_1.

The optimal values Q^* of Q and $< ATP^* >$ of <ATP> are obtained from the expressions (3.6) and (4.8) by putting the value $t_1 = t_1^{\ *}$.

V. Some Special Cases:

(a). **Absence of deterioration:**

If the deterioration of items is switched off i.e. $\alpha = 0$, then the expressions (3.6) and (4.8) of on-hand inventory(Q) and expected average total profit per unit time (<ATP(t_1)>) during the period [0,T] become

$$Q = (a+\varepsilon)t_1 + \frac{b}{2}t_1^2 \tag{5.1}$$

And $< ATP(t_1) > = \frac{1}{T}$ [$r_c(at_1 + \frac{b}{2}t_1^2 + \mu t_1)$ - A_0 - $-p_c[(a+\mu)t_1 + \frac{b}{2}t_1^2]$ - $\frac{h(a+\mu)}{2}t_1^{\ 2} - \frac{hb}{3}t_1^3$

$-\frac{\delta(a+\mu)}{6}t_1^{\ 3} - \frac{\delta b}{8}t_1^4$ - $c_s[\frac{a+\mu}{2}(T^2 - 2Tt_1 + t_1^2) + \frac{b}{6}(T^3 - 3Tt_1^{\ 2} + 2t_1^3)]$] (5.2)

The equation (4.10) becomes

$$r_c \; - p_c \text{ - } \; ht_1 - \frac{\delta}{2} t_1^{\,2} \text{ - } c_s(t_1 - T) = 0 \qquad (5.3)$$

This gives the optimum value of t_1.

(b). **Constant deterioration rate:**

If the demand rate is constant in nature i.e. β =1, then the expressions (3.6) and (4.8) of on-hand inventory (Q) and expected average total profit per unit time (<ATP(t_1)>) during the period [0,T] become

$$Q = (a+\varepsilon)[t_1 + \frac{\alpha}{2}t_1^2] + b[\frac{1}{2}t_1^2 + \frac{\alpha}{3}t_1^3] \qquad (5.4)$$

And $$< ATP(t_1) > = \frac{1}{T}\,[\, r_c(at_1 + \frac{b}{2}t_1^2 + \mu t_1) \text{ - } A_0 \text{ - } - p_c[(a+\mu)\{t_1 + \frac{\alpha}{2}t_1^2\} + b\{\frac{1}{2}t_1^2 + \frac{\alpha}{3}t_1^3\}]$$

$$\text{- } h(a+\mu)[\frac{1}{2}t_1^{\,2} + \frac{\alpha}{6}t_1^3] - hb[\frac{1}{3}t_1^3 + \frac{\alpha}{8}t_1^4] - \delta(a+\mu)[\frac{1}{6}t_1^{\,3} + \frac{\alpha}{24}t_1^4] - \delta b[\frac{1}{8}t_1^4 + \frac{\alpha}{30}t_1^5]$$

$$\text{- } d_c\alpha[\frac{a+\mu}{2}t_1^2 + \frac{b}{3}t_1^3] \text{ - } c_s[\frac{a+\mu}{2}(T^2 - 2Tt_1 + t_1^2) + \frac{b}{6}(T^3 - 3Tt_1^{\,2} + 2t_1^3)] \;\;] \qquad (5.5)$$

The equation (4.10) becomes

$$r_c \; - p_c(1+\alpha t_1^{\beta}) \text{ - } \; h(t_1 + \frac{\alpha}{2}t_1^2) - \frac{\delta}{2}(t_1^{\,2} + \frac{\alpha}{3}t_1^3) \text{ - } d_c\alpha t_1 \text{ - } c_s(t_1 - T) = 0 \qquad (5.6)$$

This gives the optimum value of t_1.

VI. Numerical Analysis:

To exemplify the above model numerically, let the values of parameters be as follows:

a = 100; b = 0.5; α = 0.01; β = 2; A_0 = \$ 300 per order; p_c = \$ 5 per unit, h = \$ 15 per unit; δ = 0.02; d_c = \$9 per unit; c_s = \$10 per unit; r_c = \$ 12 per unit and T = 1 year

Also we assume that a uniformly-distributed random demand component exhibited an error span of u= 20 with $[k_0, k_1]$=[10,20], and a mean μ =20.

Solving the equation (4.10) with the help of computer using the above values of parameters, we find the following optimum outputs

t_1^* = 0.68 year; Q^* = 81.41 units and $< ATP^* >$ = Rs. 532.84

It is checked that this solution satisfies the sufficient condition for optimality.

For Special Cases:

Nature of deterioration	t_1^*	Q^*	$< ATP^* >$
Absence of deterioration	0.68	81.69	533.02
Constant deterioration	0.67	81.36	533.34

VII. Sensitivity Analysis And Discussion:

We now study the effects of changes in the system α = 0.01, β = 2, a = 100, b = 0.5, r_c =12, p_c = 5, d_c = 9, h = 15, δ = 0.02, c_s = 10 and μ =20 on the optimal ordering quantity (Q^*) and expected average total profit per unit time (<ATP(t_1)>) in the present inventory model. The sensitivity analysis is performed by changing each of the parameters by – 50%, – 20%, +20% and +50%, taking one parameter at a time and keeping remaining parameters unchanged. The results are furnished in table A.

Table A: Effect of changes in the parameters on the model

Changing parameter	% change in the system parameter	% change in Q^*	% change in $< ATP^* >$
α	-50	0.18	0.017
	-20	0.07	0.007
	+20	- 0.07	- 0.007
	+50	- 0.17	- 0.016
β	-50	- 0.058	0.094
	-20	- 0.041	0.019
	+20	0.047	- 0.007
	+50	0.116	- 0.006
a	-50	-41.61	7.21
	-20	-16.64	2.88
	+20	16.64	-2.88
	+50	41.61	-7.21
b	-50	- 0.07	-0.017
	-20	- 0.02	-0.006
	+20	0.03	0.006
	+50	0.07	0.017
r_c	-50	-35.25	34.35
	-20	-14.12	15.04
	+20	14.14	- 15.76
	+50	35.40	- 35.13
p_c	-50	14.81	- 32.09
	-20	5.92	- 12.16
	+20	- 5.91	11.26
	+50	- 14.75	26.47
, d_c	-50	0.11	0.05
	-20	0.05	0.02
	+20	- 0.05	-0.02
	+50	- 0.11	-0.05
h	-50	42.82	- 29.54
	-20	13.61	- 9.41
	+20	- 10.69	7.40
	+50	- 33.02	25.95
δ	-50	0.012	-0.004
	-20	0.005	-0.001
	+20	- 0.005	0.001
	+50	- 0.012	0.004
c_s	-50	- 11.87	-14.54
	-20	- 4.14	- 5.06
	+20	3.53	4.32
	+50	7.95	9.73
μ	-50	-8.32	- 29.04
	-20	-3.33	- 11.61
	+20	3.32	11.61
	+50	8.32	29.04

Analyzing the results of table A, the following observations may be made:

(i) The optimum ordering quantity Q^* increase or decrease with the increase or decrease in the values of the system parameters β , a, b, r_c , c_s and μ . On the other hand Q^* increase or decrease with the decrease or increase in the values of the system parameters α , p_c , d_c , h , and δ . The results obtained show that Q^* is very highly sensitive to changes in the value of parameters a, r_c , p_c , h, c_s and μ ; and less sensitive to the changes of parameters α , β , b, d_c and δ .

(ii) The optimum expected average total profit $< ATP^* >$ increase or decrease with the increase or decrease in the values of the system parameters, b, p_c , h , δ , c_s and μ . On the other hand $< ATP^* >$ increase or decrease with the decrease or increase in the values of the system parameters α , β , a, r_c and d_c . The results obtained show that $< ATP^* >$ is very highly sensitive to changes in the value of parameters a, r_c , p_c , h, c_s and μ ; and less sensitive to the changes of parameters α , β , b, d_c and δ .

From the above analysis, it is seen that a, r_c, p_c, h, c_s and μ are very sensitive parameters in the sense that any error in the estimation of these parameters result in significant errors in the optimal solution. Hence estimation of these parameters needs adequate attention.

Scope of future work:

In the present paper, we have developed an inventory model for two-parameter Weibull distribution deterioration with stochastic demand with fully backlogged shortages. The holding cost follows the linear trended in time. The demand pattern is assumed to be linearly dependent on to time with a stochastic error. The model is maximized to the total average profit by finding optimal values. Eventually, a researcher can extend this model considering trade credits, inflation with time discounting under shortages which are partially backlogged.

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