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Contraction And C(α)- Suboperator Classes

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***Abstract**:*
In this paper we characterize the generalisation of contraction in Hilbert space and $C(\alpha)$- Suboperator class and establish some new result in the class of $C(\alpha)$- Suboperator [1] and its contractive extensions of a $C(\alpha)$- Suboperator class in a complex Hilbert space. The characterizations include a quadratic form inequality. The bounded linear operator T in complex Hilbert space which satisfy $T = \beta I + (1-\beta)U$ where $\beta \in (0,1)$ and U is a contraction ($\|U\| \leq 1$).
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I. Introduction:

Let H be a complex Hilbert space with the inner product $(.,.)$ and the norm $\|.\|$. Here an operator in H means a linear map $T: D(T) \subset H \to H$ whose domain $D(T)$ is a linear subspace of H. Also $\sigma(T)$ is the spectrum of T and $R(T)$ is the range, moreover $L(H)$ denotes the space of all bounded linear operator on H . P_{H_0} denote the orthogonal projection in a Hilbert space H on to its subspace H_0.

Definition 1.1: Let $\alpha \in \left(0, \frac{\pi}{2}\right)$ and let a linear operator T in H be defined on the subspace H_0 with the condition

$$\|T sin\alpha \pm i cos\alpha I\| \leq 1 \qquad (1)$$

If in the case $H_0 = H$, we say that T belongs to the class $C(\alpha)$ and if in the case $H_0 \neq H$, the operator T is called a $C(\alpha)$- Suboperator. It is clear that if $T \in C(\alpha)$ iff $T^* \in C(\alpha)$. It is easy to see that the condition (1) is equivalent to

$$sin\alpha(\|h\|^2 - \|Th\|^2) \geq 2cos\alpha|Im(Th,h)| \quad \text{for all } h \in H_0 \qquad (2)$$

It is clear that the operators from the class $C(\alpha)$ and $C(\alpha)$- Suboperators are contractions and this type of operator define the class $C(0)$ as the set of all self-adjoint contractions in H and a nondensely defined Hermitian contraction we will call a $C(0)$- Suboperator.

Let $0 < t < \pi$, we define open sector as $S_t = \left\{z \in \mathbb{C}/_{(0)} : |argz| < t\right\}$ and its closure $\bar{S}_t = \left\{z \in \mathbb{C}/_{(0)} : |argz| \leq t\right\}$ we consider the following arg with value in $[-\pi, \pi]$.

Definition 1.2: Let $-1 < u < 0$ and $0 \leq v < \pi$, we denoted by$\theta_v^u(H)$,for the set of all closed linear operator $T: D(T) \subset H \to H$ which satisfy (a) $\sigma(T) \subset S_v$ (b) For any $v < t < \pi$, there exit a positive constant c_t such that $\|(z-T)^{-1}\| \leq c_t|z|^u$, for any $z \notin S_t$ then this linear operator T is called sectorial operator in H if $T \in \theta_v^u(H)$ .It is clearly it having non-densely domain and range.

$C(\alpha)$- Suboperators or operator of the class $C(\alpha)$ naturally arise in the Fractional linear transformation of the form $(I-S)(I+S)^{-1}$ of sectorial linear relation S with vertex at the origin and the semi angle α [2].

Let T be a non-densely defined contraction in the complex Hilbert space H with $D(T) = H_0$. By M.G. Crandall [3], gave a parametric form of all contractive extension on H of the operator T in the operator form as

$$\bar{T}_M = TP_{H_0} + (I - TT^*)^{1/2} MP_R \qquad (3)$$

Where $T^*: H \to H_0$ is adjoint of T and $R = H \ominus H_0$, and P_{H_0}, P_R are orthogonal projection in H on to H_0 and R respectively and $M: R \to \overline{ran}(I - TT^*)^{1/2}$ is a contractive parameter. This description of all such extension of a non-densely defined contraction T is in the form of block operator matrices. We will often use the following well known result of R.G. Dauglus [4].

Theorem 1.3: [4] For every $S, T \in L(H)$ the following statement are equivalent:
(a) $ran\ S \subset ran\ T$
(b) $S = TU$ for some $U \in L(H)$
(c) $SS^* \leq \lambda TT^*$ for some $\lambda \geq 0$.
In this case there is a unique U satisfying $\|U\| = [inf\{\lambda: SS^* \leq TT^*\}]^{1/2}$ and $ran\ U \subset \overline{ran}\, T^*$.

Let T be contraction operator which satisfy the inequality $\|h\|^2 - \|Th\|^2 \geq \alpha Re\big((I-T)h,h\big)$ (4)
For some $\alpha > 0$ and all $h \in H$, then T has a form
$T = \mu I + (1-\mu)U$ (5)

Where U is a contraction ($\|U\| \leq 1$) and $\mu \in (0,1)$ this means that the spectrum of T is contained in a disk $(z \in C: |z-\mu| \leq 1-\mu)$ and this type of inequality is called quadratic form inequality , and one has $\|T^n h\| \leq \|e^{-\epsilon n(I-T)}h\|$ for some $\epsilon \in (0,1)$ and $h \in H$, $n \in N$. This is a type of domination of the discrete semigroup $(T^n)_{n\in N}$ by the continuous time semigroup $\left(e^{-t(I-T)}\right)_{t\geq 0}$. A generalization of (5) had formulated by Nevanlinna [5] who obtained the following results.

Theorem 1.4 : Let H be complex Hilbert space and $T \in L(H)$,the following two condition are equivalent.

(a) there exist $\mu \in (0,1)$, $U \in L(H)$ such that $sup_{n\in N}\|U^n\| < \infty$ and $T = \mu I + (1-\mu)U$

(b) there exist constant $a, b > 0$ such that $\|e^{zT}\| \leq ae^{|z|(1-b\theta^2)}$ for all $z \in C$ with $z = |z|e^{i\theta}$, $\theta \in [-\pi,\pi]$, Moreover, if these condition hold, then $sup_{n\in N}\|T^n\| < \infty$, $sup_{n\in N} n^{1/2}\|T^n - T^{n+1}\| < \infty$.

II. Contractions And Their Contractive Extensions

Let H and H' be two Hilbert space and let suppose H_0 is a subspace of H and $T: H_0 \to H'$ is a non-densely contraction. The operator $\tilde{T}$ defined on H is called a contractive extension of T if $\tilde{T} \supset T$ and $\|\tilde{T}\| \leq 1$. let $T \in L(H_0, H')$ and $T^* \in L(H', H_0)$ be its adjoint operator and $R = H \ominus H_0$. the following theorem considered by M.G Crandall [6].

Theorem 2.1: The result $\tilde{T} = TP_{H_0} + (I - T^*T)^{1/2}KP_R$ establishes 1-1 correspondense between all contractive extension of T and all contraction K from R to closure of the range $(I - T^*T)^{1/2}$.
Proof: Let the operator $\tilde{T}$ be given in this result, where K is a contraction, then

$$\widetilde{T^*} = T^* + K^*(I - T^*T)^{1/2}$$

Let for all $u \in H'$ and $(I - T^*T)^{1/2} = \beta(say)$,then

$$\|\widetilde{T^*}u\|^2 = \|T^*u\|^2 + \|K^*\beta u\|^2$$
$$\leq \|T^*u\|^2 + \|\beta u\|^2 = \|u\|^2$$

Thus $\widetilde{T^*}$ is a contraction. Hence the operator $\tilde{T}$ is contraction and $\tilde{T} \restriction H_0 = T$.

Conversely, if $\tilde{T}$ is a contractive extension of T then its adjoint $\tilde{T}^*: H \to H$ is also a contraction. Because $\tilde{T} \supset T$, we get $P_{H_0}\tilde{T}^* = T^*$. hence the operator $\tilde{T}^*$ is in the form $\tilde{T}^* = T^* + S$, where the range of the operator S is in the contained $R = H \ominus H_0$, it follow that $\|\tilde{T}^*u\|^2 = \|T^*u\|^2 + \|Su\|^2$, for all $u \in H'$. Since $\tilde{T}^*$ is a contraction ,we obtain

$\|Su\|^2 \leq \|u\|^2 - \|T^*u\|^2$, $u \in H'$,by theorem 1.3 , we have $S^* = (I - T^*T)^{1/2} K$, where K is a contraction from R to C_{T^*} , where C_{T^*} is the closure of the range space of $(I - T^*T)^{1/2}$.

We extend this result as a consequence for $\tilde{T} = TP_{H_0} + (I - T^*T)^{1/2} KP_R$ with non-densely contraction $K \in L(R, C_{T^*})$ has the following relations

$$\left\|\left(I - \tilde{T}^*\tilde{T}\right)^{1/2}v\right\|^2 = \left\|\left[(I - T^*T)^{1/2} P_{H_0} - T^*KP_R\right]v\right\|^2$$
$$+ \left\|(I - K^*K)^{1/2}P_R v\right\|^2 , v \in H \quad (6)$$

and $\left\|\left[I - \left(\tilde{T}^*\right)^*\tilde{T}^*\right]u\right\|^2 = \|[I - (K^*)^*K^*][I - (T^*)^*T^*]u\|^2$, $u \in H'$ (7)
since $T^*C_{T^*} \subset C_T$, from relation (5) gives,

$$inf\left\{\left\|\left(I - \tilde{T}^*\tilde{T}\right)v - \left(I - \tilde{T}^*\tilde{T}\right)w\right\|^2 : w \in H_0\right\} = \left\|(I - K^*K)^{1/2}P_R v\right\|^2, \text{ for all } v \in H$$

This implies that, $Range\left(I - \tilde{T}^*\tilde{T}\right)^{1/2} \cap R = range(I - K^*K)^{1/2}$ and by (7) implies that

$$Range\left[I - \left(\tilde{T}^*\right)^*\tilde{T}^*\right]^{1/2} = [I - (T^*)^*T^*]^{1/2}\, range[I - (K^*)^*K^*]^{1/2} .$$

III. Matrix Representation Of Contractive Extensions

If possible, the Hilbert space H' is decomposed in the way as $H' = H_0' \oplus M$, then we write $T = C^* + T_0$, where $T_0 = P_{H_0'}T \in L\left(H_0, H_0'\right)$ and $C^* = P_M T \in L(H_0, M)$.

Then, we transform T in matrix form $=\begin{bmatrix} C^* \\ T_0 \end{bmatrix}$, but T is a contraction, we have $\|T_0h_0\|^2 + \|C^*h_0\|^2 \le \|h_0\|^2$ for all $h_0 \in H_0$, then

$$C^* = K_0(I - T_0^*T_0)^{1/2} \qquad (8)$$

Where K_0 is operator from closure of the range $(I - T_0^*T_0)^{1/2}$ to M, which is also contraction. A bounded extension $\tilde{T}$ of T also has the matrix representation as

$$\tilde{T} = \begin{bmatrix} C^* & B \\ T_0 & D \end{bmatrix} : \begin{bmatrix} H_0 \\ R \end{bmatrix} \to \begin{bmatrix} H_0' \\ M \end{bmatrix}$$

In this representation, the block matrix description of B and D of all contractive extension $\tilde{T}$ was obtained in [7] . here we propose another approach based on the Crandall,s form [6]

Theorem 3.1: The consequence $\tilde{T} = \begin{bmatrix} T_0 & d_{T_0^*}S \\ K_0 d_{T_0} & -K_0T_0^*S + d_{K_0^*}Ud_S \end{bmatrix} : \begin{bmatrix} H_0 \\ R \end{bmatrix} \to \begin{bmatrix} H_0' \\ M \end{bmatrix}$ (9) establishes 1-1 correspondence between all contractive extension $\tilde{T}$ of the contraction $T = T_0 + K_0 d_{T_0}$ and all pairs (S, U) of contractive operator where $S \in L(R, C_{T_0^*})$ and $U \in L(C_S, C_{K_0^*})$ and non-negative square root $d_T = (I - T^*T)^{1/2}$ which is also called defect operator of T.

Proof: since from theorem 2.1, we have that $T^* = T_0^*P_{H_0'} + d_{T_0}K_0^*P_M$,then for all $u \in H'$, we have , $\|u\|^2 - \|T^*u\|^2 = \|P_{H_0'}u\|^2 + \|P_M u\|^2 - \|(T_0^*P_{H_0'} + d_{T_0}K_0^*P_M)u\|^2$

$$= \|P_M u\|^2 + \|P_{H_0'}u\|^2 - \|T_0^*P_{H_0'}u\|^2 - \|d_{T_0}K_0^*P_M u\|^2 - 2Re(T_0^*P_{H_0'}u, d_{T_0}K_0^*P_M u)$$

$$= \|d_{T_0^*}P_{H_0'}u\|^2 - \|K_0^*P_M u\|^2 + \|T_0K_0^*P_M u\|^2 - 2Re(d_{T_0^*}P_{H_0'}u, T_0K_0^*P_M u) + \|P_M u\|^2$$

$$= \|d_{T_0^*}P_{H_0'}u - T_0K_0^*P_M u\|^2 + \|d_{K_0^*}P_M u\|^2$$

And hence, $\|d_{T^*}u\|^2 = \|d_{T_0^*}P_{H_0'}u - T_0K_0^*P_M u\|^2 + \|d_{K_0^*}P_M u\|^2$ (10)

Since the equality $T_0 d_{T_0} = d_{T_0^*}T_0$, so we get $T_0C_{T_0} \subset C_{T_0^*}$ and since $range(K_0^*) \subset C_{T_0}$ thus from (10)

$\|d_{K_0^*}P_M u\|^2 = inf\{\|d_{T^*}(u - v)\|^2 : v \in H_0\}$ (11)

Let $\varkappa_0' = \overline{d_{T^*}H_0'}$ and $M_0' = C_{T^*} \ominus \varkappa_0'$, so $M_0' = (m \in C_{T^*} : d_{T^*}m \in M)$, and from (10) and (11) we get , $\|P_{\varkappa_0'}d_{T^*}u\|^2 = \|d_{T_0^*}P_{H_0'}u - T_0K_0^*P_M u\|^2$, and

$\|P_{M_0'}d_{T^*}u\|^2 = \|d_{K_0^*}P_M u\|^2 \qquad u \in H'$ (12)

Thus, $\|P_{\varkappa_0'}d_{T^*}w\|^2 = \|d_{T_0^*}w\|^2, \ w \in H_0'$

From (12), we define a unitary operator $U_0 \in L(\varkappa_0', C_{T_0^*})$ and $Z_0 \in L(M_0', C_{K_0^*})$ such a way that

$U_0P_{\varkappa_0'}d_{T^*}u = d_{T_0^*}P_{H_0'}u - T_0K_0^*P_M u$

$Z_0P_{M_0'}d_{T^*}u = d_{K_0^*}P_M u , \qquad u \in H'$ (13)

Since adjoint of unitary operator is equal to its inverse that means $U_0^* = U_0^{-1}$ and $Z_0^* = Z_0^{-1}$, then from (13) we have

$$d_{T^*} = U_0^{-1}(d_{T_0^*}P_{H_0'} - T_0K_0^*P_M) + Z_0^{-1}d_{K_0^*}P_M$$

$= (d_{T_0^*} - K_0T_0^*)U_0P_{\varkappa_0'} + d_{K_0^*}Z_0P_{M_0'}$ (14)

Since $K \in L(R, C_{T^*})$ is contraction, then $K = P_{\varkappa_0'}K + P_{M_0'}K$, put $S = U_0P_{\varkappa_0'}K, Y = Z_0P_{M_0'}K$

$\bar{K} = S + Y$, it follows that $K = U_0^{-1}S + Z_0^{-1}Y$ and $\|Kr\|^2 = \|\bar{K}r\|^2 = \|Sr\|^2 + \|Yr\|^2$ for all $r \in R$. Since K is contraction then $\bar{K} \in L(M, C_{T_0^*}\oplus C_{K_0^*})$ is also contraction iff $Y = Ud_S$ where $U \in L(C_S, C_{K_0^*})$ is also contraction.

Again since $K \in L(R, C_{T^*})$ is contraction, then by (14) and all $r \in R$ we get

$d_{T^*}Kr = (d_{T_0^*} - K_0T_0^*)Sr + d_{K_0^*}Ud_S r$ (15)

Let $\bar{T} = TP_{H_0} + d_{T^*}KP_R$, then by (5) and (15) gives (9), if the $\bar{T}$ is given by (6) with contraction $S \in L(R, d_{T_0^*})$ and $U \in L(C_S, C_{K_0^*})$ then the operators $\bar{K} = S + Ud_S$ and $K = U_0^{-1}S + Z_0^*Ud_S$ are contractions. Since $K \in L(R, C_{T^*})$ and hence we obtain

$$\bar{T} = (T_0 + K_0d_{T_0})P_{H_0} + [(d_{T_0^*} - K_0T_0^*)S + d_{K_0^*}Ud_N]P_R$$

$$= TP_{H_0} + [(d_{T_0^*} - K_0T_0^*)U_0P_{\varkappa_0'} + d_{K_0^*}Z_0P_{R_0'}]KP_R$$

$$= TP_{H_0} + d_{T^*}KP_R$$

And hence $\bar{T}$ is a contractive extension of T.

Theorem 3.2: Let T be a $C(\alpha)$-suboperator in H with $\mathrm{dom}(T) = H_0 \subset H$ and $S = (I - T^*)(I + T)$ be sectorial operator with the vertex at the origin and the semi angle θ. Define the contractive extension $\bar{T}_0 = TP_{H_0}$ and let

$\tilde{\varphi}(z) = \left[-\bar{T}_0 + z\bar{T}_0^*(I - z\bar{T}_0^*)^{-1}d_{\bar{T}_0}\right] \upharpoonright C_{\bar{T}_0}$ be the characteristic function [8] of $\bar{T}_0$. Then there exist strong unitary limits $\tilde{\varphi}(\pm 1) = S - \lim_{z\to\pm 1} \bar{\varphi}(z)$, $\tilde{\varphi}^* = S - \lim_{z\to\pm} \tilde{\varphi}^*(z)$, (which is nontangential to the imazinary axis) and moreover, the operator $\tilde{\varphi}(\pm 1) \upharpoonright R$ are linear isometries.

Proof: since $\bar{T}_0^* = T^*$, then $d_{T^*} = d_{\bar{T}_0^*}$, $d_{\bar{T}_0} = d_T P_{H_0} + P_R$ and hence $C_{\bar{T}_0^*} = C_{T^*}$, $C_{\bar{T}_0} = C_T \oplus R$, then we have,

$$d_{T^*}(I - zT_0^*)^{-1}d_{Tu} = U_0^{-1}d_{T_0^*}(I - zT_0^*)^{-1}d_{T_0}d_{K_0}V_0u, \quad u \in C_T \qquad (16)$$

Where V_0 is an isometry from C_T on to C_{K_0}. Consequently,

$$\tilde{\varphi}(z)u = -Tu + U_0^{-1}[\varphi_0(z) + T_0]d_{K_0}V_0u, \qquad u \in C_T \qquad (17)$$

Let $r \in R$, then,

$$d_{T^*}(I - zT^*)^{-1}r = d_{T^*}(I - zT^*)^{-1}(r - zT^*r + zT^*r)$$
$$= d_{T^*}r + zd_{T^*}(I - zT^*)^{-1}T^*r = d_{T^*}r + zd_{T^*}(I - zT^*)^{-1}d_{T_0}K_0^*r$$
$$= d_{T^*}r + U_0^{-1}[\varphi(z) + T_0]K_0^*r$$

Therefore, $\tilde{\varphi}(z)r = zd_{T^*}r + zU_0^{-1}[\varphi(z) + T_0]K_0^*r, \qquad r \in R \qquad (18)$

Since T_0^* and T_0 being the class $C(\alpha)$ in the subspace $H_0 \subset H$,there exist unitary strong limiting values $\varphi(\pm 1)$ and $\varphi^*(\pm 1)$ of $\varphi(z)$ and $\varphi^*(z)$ respectively, this implies that there exist unitary nontangential strong limiting value $\tilde{\varphi}(\pm 1)$, $\tilde{\varphi}^*(\pm 1)$ and

$$\tilde{\varphi}(\pm 1) = \left[-T + U_0^{-1}[\varphi(\pm 1) + T_0]d_{K_0}V_0\right]P_{H_0} \pm \left[d_{T^*} + U_0^{-1}[\varphi(\pm 1) + T_0]K_0^*\right]P_R$$

Next ,we prove that $\tilde{\varphi}(\pm 1) \upharpoonright R$ are linear isometry. It is easy to say that

$$\|r\|^2 - \|\tilde{\varphi}(z)r\|^2 = (1 - |z|^2)\|(I - zT^*)^{-1}r\|^2, \qquad r \in R$$

For $r \in R$ from the equality $T^*P_R = d_{T_0}K_0^*P_R$, we have,

$$(1 - |z|^2)^{1/2}(I - zT^*)^{-1}r = (1 - |z|^2)^{1/2}\, r + z(1 - |z|^2)^{1/2}(I - zT^*)^{-1}T^*r$$
$$= (1 - |z|^2)^{1/2}\, r + z(1 - |z|^2)^{1/2}(I - zT^*)^{-1}d_{T_0}K_0^*r$$

Since, $\|v\|^2 - \|\varphi(z)v\|^2 = (1 - |z|^2)\left\|(I - zT^*)^{-1}d_{T_0}v\right\|^2$, $\quad v \in C_{T_0}$ and the operator $\varphi(\pm 1)$ are unitary in C_{T_0}. Consequently we have,

$$S - \lim_{z\to\pm 1}(1 - |z|^2)^{1/2}(I - zT^*)^{-1}d_{T_0} = 0$$

Therefore, for all $r \in R$, we have ,

$$\lim_{z\to\pm 1}(\|r\|^2 - \|\tilde{\varphi}(z)r\|^2) = \lim_{z\to\pm 1}\left\|(1 - |z|^2)^{1/2}\left(r + z(I - zT^*)^{-1}d_{T_0}K_0^*r\right)\right\|^2$$

And hence operator $\tilde{\varphi}(\pm 1) \upharpoonright R$ are linear isometry.

Theorem 3.3: Let $T \in C(\alpha)$ - sub operator class and $\tilde{T}$ be its contractive extension such that there exist $\lambda > 0$ with

$$\|h\|^2 - \left\|\tilde{T}h\right\|^2 \geq \lambda\left|\left[(I - \tilde{T})h, h\right]\right| \qquad (19)$$

For all $h \in H$, then , $Sup_{n\in N}\, n\left|\tilde{T}^n - \tilde{T}^{n+1}\right| < \infty$.

Proof: This inequality implies that $\|h\|^2 - \left\|\tilde{T}h\right\|^2 \geq 0$, so that $\tilde{T}$ is contraction. Thus this implies $\left\|\tilde{T}\right\| \leq 1$ and there exist $\lambda > 0$ such that $\|h\|^2 - \left\|\tilde{T}h\right\|^2 \geq \lambda\, Re\left[(I - \tilde{T})h, h\right]$ for all $h \in H$, Consequently ,

$$\|h\|^2 - \left\|\tilde{T}h\right\|^2 + \left\|(I - \tilde{T})h\right\|^2 = 2\, Re\left[(I - \tilde{T})h, h\right] \qquad (20)$$

Hence, from (19) and (20) we get, $\left|\left[(I - \tilde{T})h, h\right]\right| \leq 2\lambda^{-1}\, Re\left[(I - \tilde{T})h, h\right]$

This inequality is a sectorial estimate with quadratic form which implies that the semi group $\left(e^{-t(I-\tilde{T})}\right)_{t\geq 0}$ is bounded holomorphic and hence one has a result

$\left\|(I - \tilde{T})e^{-t(I-\tilde{T})}\right\| \leq \lambda t^{-1}$ for all $t > 0$

(But we know a result for all contraction T, $\|T^n h\| \leq \left\|e^{-\varepsilon n(I-T)}h\right\|$, $h \in domT$)

Using this result, we obtain a bound,

$$\left\|(I - \tilde{T})\tilde{T}^n\right\| \leq \left\|(I - \tilde{T})e^{-\varepsilon n(I-\tilde{T})}\right\| \leq \lambda n^{-1}, \quad \text{for all } n \in N$$

And hence we get , $Sup_{n\in N}\, n\left|\tilde{T}^n - \tilde{T}^{n+1}\right| < \infty$.

IV. Conclusions:

In this article we make use of the results from previously the known results and establish some new results on extension of $C(\alpha)$-sub operator classes and also extend some results and application.

References:

[1] Yu. M. Arlinskii, A class of contractions in a Hilbert space, Translated from Ukrainski Matematicheskii zhurnal, vol.39, No.6, (1987), 691-696.
[2] M.M. Malamud, On some classes of extensions of sectorial operators and dual pair of contractions, Operator theory: Advances and appl. 124 (2001), 401-448.
[3] M.G. Crandall, Norm preserving extensions of linear transformation in Hilbert space, Proc. Amer. Math. Soc. 21 (1969), 335-340.
[4] R.G. Douglus, On majorization, factorization and range inclusion of operators in Hilbert space, Proc. Amer. Math. Soc. 17 (1966), 413-416.
[5] O. Nevanlinna, Resolvent conditions and powers of operators, Studia math, 145 (2001) 113-14.
[6] Gr. Arsene and A. Gheondea, Completing matrix contraction, J. Operator theory, 7 (1982) 179-198.
[7] T. Kato, Perturbation theory of linear operator, 2nd ed, springer, 1980.
[8] W. Rudin, Functional analysis, 2nd ed, McGraw-Hill Inc, New york, 1991.
[9] N. Dungey, Time regularity for random walks on locally compact group, Probab. Theory Related fields, 137 (2007), 404-410.
[10] S. Sarkar, Pure contractive multipliers of some reproducing kernel Hilbert spaces and applications, arXiv:2112. 08332v1[Math.FA], 2021.
[11] J. W. He, Y. Zhou. , Non-autonomous fractional Cauchy problems with almost sectorial operators, B. sci. Math, 191 (2024).
[12] T. Ozawa, J. E. Restrepo, D. Suragan, Inverse abstract Cauchy problems, Appl. Anal. 101(14) (2022) 4965-4969.
[13] A. B. Yadav, Characterization of S_ξ –super strictly singular operator, IOSR. J. of Mathematics, Vol-19, (2023) 30-33.