IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-0661, p-ISSN: 2278-8727, Volume 21, Issue 2, Ser. 2 (Mar. – Apr. 2025), PP 47-58
www.iosrjournals.org

Th Artin,s cokernel of The Group $Q_{2l}\times C_7$ Where $l\neq 2$and l is prime number

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Abstract:

In this research, we found the circular partition of the fractional Abelian group $AC(Q_{2l}\times C_7)$ after knowing $Ar(Q_{2l}\times C_7)$, and the result was as follows:

$$AC(Q_{2l}\times C_7) = \bigoplus_{i=1}^{4} C_2 .$$

Introduction:

Representation theory is a branch of mathematics that studies abstract algebraic structures by representing their elements as linear transformations of vector spaces, and studying modularity over these abstract algebraic structures. In essence, the representation makes an abstract algebraic object more concrete by describing its elements by matrices and algebraic operations (e.g., matrix addition, matrix multiplication). In 1995, N. S. Jassim [12] studied the factor group $cf(G,Z)$ / $\overline{R}(G)$ for the special linear group SL(2, p). In 1995, N. R. Mahmood [10] studied the factor group $cf(Q_{2m},Z)/\overline{R}(Q_{2m})$ and he found the rational valued characters table of the Quaternion group Q_{2m} . In 1996, K. Knwabues [12]gave some definitions of Artin's Exponent of finite group . In 2001, A. M. Ibraheem [13] studied a nother definition of Artin Exponent $A(G)$. In 2002, K. Sekiguchi [12] studied the irreducible Artin characters of p-group. In 2006, A.S. Abed [2] found the Artin characters table of dihedral group D_n when n is an odd number. In 2007, A. H. Mohammed [14] found the Artin cokernel of the dihedral group D_n when n is an even number. In 2013, N. A. Rahi [15]. studied The cyclic decomposition of the factor group $cf(Q_{2m}\times C_3, Z)/\overline{R}(Q_{2m}\times C_3)$when m is an odd number. In this research, we will find the cyclic decomposition of the factor group $AC(Q_{2l}\times C_7)$ when m is an odd number, such that the group$(Q_{2l}\times C_7)$ is the direct product group of the quaternion group Q_2l of order 4l and the cyclic group C_7 of order 7, then the order of The group$(Q_2l\times C_7)$ is 28l .

1-Preliminars :

In this section, we will review a set of important theories that will help us obtain the results of our research. I will symbolize in this research, prime number with the symbol (pr) , principal character with the symbol (pc) , positive integer number with the symbol (pin) , Artin characters with the symbol (Arc) and Γ-classes with the symbol (Γ-c). The Artin characters table of G denoted by $Ar(G)$it is a table in which the third row is the size of the centralized $|C_G(CL_\alpha)|$; The second row is the number of elements in each conjugate class ,The first row is Γ-conjugate classes and other rows contains the values of (Arc) .

Theorem:(1.1): [2]

$Ar(Cl^s)$ of the group Cl^s when l is a (pr) and s is (pin) will be like this :-

$Ar(Cl^s)$=

Γ - C	[1]	$[a^{l^{s-1}}]$	$[a^{l^{s-2}}]$	$[a^{l^{s-3}}]$	$\cdots$	[a]
$\lvert CL_\alpha \rvert$	1	1	1	1	$\cdots$	1
$\lvert C_{l^s}(CL_\alpha)\rvert$	l^s	l^s	l^s	l^s	$\cdots$	l^s
φ'_1	l^s	0	0	0	$\cdots$	0
φ'_2	l^{s-1}	l^{s-1}	0	0	$\cdots$	0
φ'_3	l^{s-2}	l^{s-2}	l^{s-2}	0	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
φ'_s	l	l	l	l	$\cdots$	0
φ'_{s+1}	1	1	1	1	$\cdots$	1

Table(1,1)

Theorem: (1.2) :[2]

If it was $m = l_1^{\alpha_1} . l_2^{\alpha_2} .... l_n^{\alpha_n}$ So that it has the greatest common divisor $(l_i, l_j) = 1$, if $i \neq j$, l_i's are (pr) and α_i any (pin) for all $1 \leq i \leq n$, then :

$$Ar(C_m) = Ar(C_{l_1^{\alpha_1}}) \otimes Ar(C_{l_2^{\alpha_2}}) \otimes \ldots \otimes Ar(C_{l_n^{\alpha_n}}).$$

Example :(1.3)

$Ar(C_{2l})$ for the cyclic group C_{2l} with the help of the theorem(1.1), $Ar(C_2)$ and $Ar(C_l)$ it will be:

$Ar(C_2)$=

Γ - C	[1]	[a]
$\lvert CL_\alpha \rvert$	1	1
$\lvert C_{C_2}(CL\alpha)\rvert$	2	2
φ'_1	2	0
φ'_2	1	1

Table(1,2)

$Ar(C_l)$=

Γ - C	[1]	[a]
$\lvert CL_\alpha \rvert$	1	1
$\lvert C_{C_l}(CL\alpha)\rvert$	l	l
φ'_1	l	0
φ'_2	1	1

Table(1,3)

and by theorem (1.2) it will be

$Ar(C_{2l})$=$Ar(C_2)$ $\otimes$ $Ar(C_l)$=

Γ - C	[1]	$[a^l]$	$[a^2]$	[a]
$\lvert CL_\alpha \rvert$	1	1	1	1
$\lvert C_{C_{2l}}(CL\alpha)\rvert$	$2l$	$2l$	$2l$	$2l$
φ'_1	$2l$	0	0	0
φ'_2	l	l	0	0
φ'_3	2	0	2	0
φ'_4	1	1	1	1

Table(1,4)

Theorem(1.4):[3]

$Ar(Q_{2l})$ when l is an (pr) will be like this:-

$Ar(Q_{2l})$ =

	Γ- C of C_{2l}								
Γ- C	a^{2r}				a^{2r+1}				[b]
$\lvert CL_a \rvert$	1	2	…	2	1	2	…	2	2l
$\lvert C_{Q_{2l}}(CL_a) \rvert$	4l	2l	…	2l	4l	2l	…	2l	2
$\emptyset_1$	2.Ar(C_{2l})								0
$\emptyset_2$									0
⋮									⋮
$\emptyset_n$									0
$\emptyset_{n+1}$	1	0	…	0	1	0	…	0	1

Table (1.5)

So that n is the number of Γ-C of C_{2l} , $0 \leq r \leq l-1$ and ϕ_τ are (Arc) of the quaternion group Q_{2l}, for all. $1 \leq \tau \leq n+1$

Example (1.5):

$Ar(Q_{14}) \equiv Ar(Q_{2.7})$ we note that l=7. using theorem(1.5) it will be:

$Ar(Q_{14}) \equiv$

	Γ-C of C_{14}				
Γ- C	[1]	[a^2]	[a^7]	[a]	[b]
$\lvert CL_a \rvert$	1	2	1	2	10
$\lvert C_{Q14}(CL\alpha) \rvert$	28	14	28	14	2
Φ_1	28	0	0	0	0
Φ_2	4	4	0	0	0
Φ_3	14	0	14	0	0
Φ_4	2	2	2	2	0
Φ_5	7	0	7	0	1

Table(1,6)

Theorem (1.6): [1]

$Ar(Q_{2l}\times C_7)$ where $l\neq 2$ and l is (pr) ; is given as follows:

$Ar(Q_{2l}\times C_7)$ =

	Γ-C of $Q_{2l}\times\{I\}$					Γ-C of $Q_{2l}\times\{z\}$				
Γ-C	[1,I]	[a^2,I]	[a^l,I]	[a,I]	[b,I]	[1,z]	[a^2,z]	[a^l,I]	[a,z]	[b,z]
$\lvert CL_\alpha\rvert$	1	2	1	2	$2l$	1	2	1	2	$2l$
$\lvert C_{(Q_{2l}\times C_7)}(CL\alpha)\rvert$	$28l$	$14l$	$28l$	$14l$	14	$20l$	$10l$	$20l$	$10l$	14
$\theta_{(1,1)}$	7 Ar(Q_{2l})					0				
$\theta_{(2,1)}$										
$\theta_{(3,1)}$										
$\theta_{(4,1)}$										
$\theta_{(5,1)}$										
$\theta_{(1,2)}$	Ar(Q_{2l})					Ar(Q_{2l})				
$\theta_{(2,2)}$										
$\theta_{(3,2)}$										
$\theta_{(4,2)}$										
$\theta_{(5,2)}$										

Table(1,7)

square matrix of capacity 10×10

Example (1.7):

$Ar(Q_{14}\times C_7)$= $Ar(Q_{2l}\times C_7)$, l=7 , Using theorem:(1.4) it will be :-

$Ar(Q_{14})$=

Γ-C	[1]	[a^2]	[a^l]	[a]	[b]
$\lvert CL_\alpha\rvert$	1	2	1	2	10
$\lvert C_{Q_{14}\times C_7}(CL_\alpha)\rvert$	28	14	28	14	2
θ_1	28	0	0	0	0
θ_2	4	4	0	0	0
θ_3	14	0	14	0	0
θ_4	2	2	2	2	0
θ_5	7	0	7	0	1

Table(1,8)

Now with the help theorem (1.6) $Ar(Q_{14}\times C_7)$ is:-
$Ar(Q_{14}\times C_7)$=

Γ- C	[1,I]	[a²,I]	[a⁷,I]	[a,I]	[b,I]	[1,z]	[a²,z]	[a⁷,z]	[a,z]	[b,z]
$\|CL_\alpha\|$	1	2	1	2	14	1	2	1	2	14
$\|C_{Q_{14}\times C_7}(CL_\alpha)\|$	196	98	196	98	14	140	70	140	70	10
$\theta_{(1,1)}$	196	0	0	0	0	0	0	0	0	0
$\theta_{(2,1)}$	28	28	0	0	0	0	0	0	0	0
$\theta_{(3,1)}$	98	0	98	0	0	0	0	0	0	0
$\theta_{(4,1)}$	14	14	14	14	0	0	0	0	0	0
$\theta_{(5,1)}$	49	0	49	0	7	0	0	0	0	0
$\theta_{(1,2)}$	28	0	0	0	0	28	0	0	0	0
$\theta_{(2,2)}$	4	4	0	0	0	4	4	0	0	0
$\theta_{(3,2)}$	14	0	14	0	0	14	0	14	0	0
$\theta_{(4,2)}$	2	2	2	2	0	2	2	2	2	0
$\theta_{(5,2)}$	7	0	7	0	1	7	0	7	0	1

Table(1,9)

2. Factor Group AC(G):

In this part, we aim to explain the basic definitions and important theories of the group AC(G).

Definition (2.1):[6]

*The finite factor abelian group $AC(G)=\overline{R}(G)/T(G)$ is **called Artin cokernel of G**, in which $T(G)$ is the subgroup of $\overline{R}(G)$ generated by (Atc) and $T(G)$ is a normal subgroup of $\overline{R}(G)$.*

Definition (2.2): [7]

*Let A be a matrix with entries in a principal ideal domain R. A **k – minor of A** is the determinate of k×k sub-matrix preserving row and column order.*

Definition (2.3):[7]

*A **k-th determinant divisor of A** is the greatest common divisor of all the k – minor, this is denoted by $D_K(A)$.*

Theorem (2.4):[7]

Let A be an n×n matrix with entries in a principal ideal domain R, then there exist matrices B and C such that:

1 - B and C are invertible.
2 - B A C = D.
3 - D is a diagonal matrix.
4 - If we denote D_{jj} by d_j then there exists a natural number m ; $0 \le m \le n$ such that $j > m$ implies $d_j = 0$ and $j \le m$ implies $d_j \neq 0$ and $1 \le j \le m$ implies $d_j \mid d_{j+1}$.

Definition (2.5): [7]

*Let A be matrix with entries in a principal ideal domain R such that A is equivalent to matrix $D = diag\{d_1, d_2, \ldots, d_m, 0, 0, \ldots, 0\}$ where $d_j \mid d_{j+1}$ for $1 \le j < m$. We call **D the invariant factor matrix of A** and $d_1, d_2, \ldots, d_m$ the invariant factors of A.*

Remark (2.6): [6]

Let l be the number of all distinct Γ- C of G then Ar(G) and $\equiv^{*}(G)$ are of rank l. According to the Artin's theorem there exists an invertible matrix $A^{-1}(G)$ with entries in Q such that :

$$\equiv^{*}(G) = A^{-1}(G) \cdot Ar(G) \quad \text{and this implies} \quad A(G) = Ar(G) \cdot (\equiv^{*}(G))^{-1}$$

A (G) is the matrix expressing the T(G) basis in terms of the $\bar{R}(G)$ basis. By Theorem (3. 4), there exist two matrices B(G) and C(G) with determinant ± 1 such that:

$B(G).A(G).C(G) = diag\{d_1, d_2, \ldots, d_l\} = D(G)$ where $d_i = \pm D_i(G)/D_{i-1}(G)$ This process yields a new basis for T(G) and $\bar{R}(G)$, $\{v_1, v_2, \ldots, v_l\}$ and $\{u_1, u_2, \ldots, u_l\}$ respectively, with the property $v_i = d_i u_i$.

Theorem (2.7):[6]

$AC(G) = \bigoplus_{i=1}^{l} C_{d_i}$ in which $d_i = \pm D_i(G)/D_{i-1}(G)$ and l is the number of all distinc Γ- c of G.

Corollary (2.8):[6]

$|AC(G)| = |\det(A(G))|$.

Lemma (2.9):[6]

Let S of rank ℓ and Y of rank ℓ be two invertible matrices , over a principal ideal domain R and let: $B_1 S C_1 = D(S) = diag\{d_1(S), d_2(S), \ldots, d_l(S)\}$ and

$B_2 Y C_2 = D(Y) = diag\{d_1(Y), d_2(Y), \ldots, d_m(Y)\}$ the invariant factor matrices of S and Y then:

$(B_1 \otimes B_2)(S \otimes Y)(C_1 \otimes C_2) = D(S) \otimes D(Y)$ and from this the invariant factor matrices of $S \otimes Y$ can be written $dig(S \otimes Y) = d_1(S).d_1(Y), d_2(S).d_2(Y), \ldots\ldots\ldots, d_l(S).d_m(Y)$

Lemma (2.10):[6]

If H_1 and H_2 are two matrices of degree m and t respectively, then:

$\det(H_1 \otimes H_2) = (\det(H_1))^t . (\det(H_2))^m$.

Proposition(2.11):[7]

Let H_1 and H_2 be two p-groups then the matrix which expresses the $T(H_1\times H_2)$ basis of $\bar{R}(H_1\times H_2)$ basis is $A_1 \otimes A_2$

3. The Main Results

In this section we will study of the $AC(Q_{2l})$ in which l is an odd number and the cyclic decomposition of the group $(Q_{2l}\times C_7)$ where ,$l\neq 2$ and l is (pr).

Proposition (3. 1): [9]

If l is a (pr) and s is a (pin), then

$$M(C_{l^s}) = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & \cdots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

Capacity square matrix of $(s+1)\times(s+1)$.

Proposition(3.2):[10]

In general, the general formula for the matrices $B(C_{l^s})$ and $C(C_{l^s})$ are :

$$B(C_{l^s}) = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ & & & & & \ddots & & \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix}$$

Capacity square matrix of $(s+1)\times(s+1)$.

$D(C_{l^s}) = diag\{\underbrace{1,1,\dots\dots,1}_{s+1}\}$ and $C(C_{l^s}) = I_{s+1}$, I_{s+1} is an identity matrix

Remark (3.3): [10]

In general if so $m = l_1^{\alpha 1} \cdot l_2^{\alpha 2} \cdots\cdots l_n^{\alpha n}$ where the biggest common denominator $(l_i, l_j)=1$, if $i \neq j$ and for all $i, 1 \leq i \leq n$, $l_i \neq 2$ are (pr) and α_i any (pin) is all $i = 1, 2, \dots, n$;then :

$C_m = C_{l_1^{\alpha_1}} \times C_{l_2^{\alpha_2}} \times \dots \times C_{l_n^{\alpha_n}}$,

1- By Lemma (2.9) we get:

$A(C_m) = A(C_{l_1^{\alpha_1}}) \otimes A(C_{l_2^{\alpha_2}}) \otimes \dots \otimes A(C_{l_n^{\alpha_n}})$.

Now you can write $A(C_m)$ as:

$$A(C_m) = \begin{bmatrix} & & & & & 1 \\ & & R(C_m) & & & 1 \\ & & & & & \vdots \\ & & & & & 1 \\ 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix}$$

In which $R(C_m)$ is a matrix obtained by deleting the last column {1,1,...,1} and last row {0, 0,...,0, 1} from the tensor product, $A(C_{l_1^{\alpha_1}}) \otimes A(C_{l_2^{\alpha_2}}) \otimes \dots \otimes A(C_{l_n^{\alpha_n}})$, Where $A(C_m)$ is of order, $[(\alpha_1+1)(\alpha_2+1)\dots(\alpha_n+1) \times (\alpha_1+1)(\alpha_2+1)\dots(\alpha_n+1)]$ square matrix.

2- Using Lemma (2.10) we get,

a- $B(C_m) = B(C_{l_1^{\alpha_1}}) \otimes B(C_{l_2^{\alpha_2}}) \otimes \dots \otimes B(C_{l_n^{\alpha_n}})$.

b- $C(C_m) = C(C_{l_1^{\alpha_1}}) \otimes C(C_{l_2^{\alpha_2}}) \otimes \dots \otimes C(C_{l_n^{\alpha_n}})$.

Proposition (3.4):[3]

If $m = l_1^{\alpha 1}\cdot l_2^{\alpha 2}\cdots\cdots l_n^{\alpha n}$ *where the biggest common denominator* $(l_i, l_j)=1$, *if* $i\neq j$ *and* l_i *'s are (pr), and* α_i *any (pin), then the matrix* $A(Q_{2m})$ *of the quaternion group* Q_{2m} *is :*

$$A(Q_{2m}) = \left[\begin{array}{ccccc|ccccc|c}
 & 2R(C_m) & & & 1 & & 2R(C_m) & & & 1 & 1 \\
 & & & & 1 & & & & & 1 & 1 \\
 & & & & \vdots & & & & & \vdots & \vdots \\
 & & & & 1 & & & & & 1 & 1 \\
0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 1 & 1 \\ \hline
 & & & & 1 & 0 & 0 & \cdots & 0 & 1 & 0 \\
 & 2R(C_m) & & & 1 & 0 & 0 & \cdots & 0 & 1 & 0 \\
 & & & & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
 & & & & 1 & 0 & 0 & \cdots & 0 & 1 & 0 \\
0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 1 & 0 \\ \hline
1 & 1 & \cdots & 1 & 1 & 0 & 0 & \cdots & 0 & 0 & 0
\end{array}\right]$$

Which is $[2(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_n+1)+1]\times[2(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_n+1)+1]$ *square matrix,*
$R(C_m)$ is similar to the matrix in observations (3.3).

Proposition (3.5):[3]

If $m = l_1^{\alpha 1}\cdot l_2^{\alpha 2}\cdots\cdots l_n^{\alpha n}$ *where the biggest common denominator* $(l_i, l_j)=1$, *if* $i\neq j$ *and* l_i *'s are (pr), and* α_i *any (pin), then:*

1- The matrix $B(Q_{2m})$ *taking the form*

$$B(Q_{2m}) = \left[\begin{array}{ccccc|ccccc|c}
 & B(C_m) & & & & & -B(C_m) & & & & 0 \\
 & & & & & & & & & & 0 \\
 & & & & & & & & & & \vdots \\
 & & & & & & & & & & 0 \\
 & & & & & & & & & & 0 \\ \hline
0 & 0 & \cdots & 0 & 0 & & & & & & 0 \\
0 & 0 & \cdots & 0 & 0 & & B(C_m) & & & & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & & & & & & \vdots \\
0 & 0 & \cdots & 0 & 0 & & & & & & 0 \\
0 & 0 & \cdots & 0 & 0 & & & & & & 0 \\ \hline
0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1
\end{array}\right]$$

2- The matrix $C(Q_{2m})$ *taking the form*

$$C(Q_{2m}) = \left[\begin{array}{ccccc|cccccc|c}
0 & 0 & \cdots & 0 & 0 & & & & & & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & & & I_k & & & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & & & & & & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & & & & & & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & -1 & -1 & -1 & -1 & & 0 & 1 \\ \hline
 & & & & 0 & 0 & 0 & \cdots & 0 & & 0 & 0 \\
 & I_k & & & 0 & 0 & 0 & \cdots & 0 & & 0 & 0 \\
 & & & & \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots \\
 & & & & 0 & 0 & 0 & \cdots & 0 & & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 1 & 1 & \cdots & 1 & & 1 & -1 \\ \hline
0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & & 0 & 0
\end{array}\right]$$

Where $k=(\alpha_1+1)\cdot(\alpha_2+1)\cdot(\alpha_3+1)\cdots(\alpha_n+1)-1$. *They are* $[2(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_n+1)+1]\times[2(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_n+1)+1]$ *square matrix.*

Theorem (3.6):[3]

If $m = l_1^{\alpha_1} \cdot l_2^{\alpha_2} \cdots\cdots l_n^{\alpha_n}$ *where the biggest common denominator* $(l_i, l_j)=1$, *if* $i \neq j$ *and* l_i *'s are (pr) and* α_i *any (pin), then the cyclic decomposition of the factor group* $AC(Q_{2m})$ *is :*

$$AC(Q_{2m}) = \bigoplus_{i=1}^{2(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_n+1)-2} C_2$$

Example (3.7):

Consider the groups Q_{54} , then : $AC(Q_{54}) = AC(Q_{2\cdot 3^3}) = \bigoplus_{i=1}^{2(3+1)-2} C_2 = \bigoplus_{i=1}^{6} C_2$

Theorem (3.8):

If $m = l$: *l is prime number, the matrix* $A(Q_{2l} \times C_7)$ *of the group* $Q_{2l} \times C_7$ *is:*

$$A(Q_{2l} \times C_7) = \left[\begin{array}{c|c} A(Q_{2l}) & A(Q_{2l}) \\ \hline 0 & A(Q_{2l}) \end{array}\right]$$

Which is 10×10 *square matrix* , $A(Q_{2l})$ *It is the same matrix in the Proposition (3.4)*

proof :

Using matrix definition $A(G)$ *we find the matrix* $A(Q_{2l} \times C_7)$:

$A(Q_{2l} \times C_7) = Ar(Q_{2l} \times C_7) \cdot (\equiv^*(Q_{2l} \times C_7))^{-1} =$

$$\left[\begin{array}{ccccc|ccccc}
2 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\
2 & 1 & 0 & 1 & 0 & 2 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 2 & 1 & 2 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0
\end{array}\right] = \left[\begin{array}{c|c} A(Q_{2l}) & A(Q_{2l}) \\ \hline 0 & A(Q_{2l}) \end{array}\right]$$

Capacity square matrix of (10×10)

Example(3.9):

We find the matrix $A(Q_{10} \times C_7)$ *in two ways:*

First: *Using the definition of* $A(G)$

$A(Q_{10} \times C_7) = A(Q_{2\cdot 5} \times C_7) = Ar(Q_{2\cdot 5} \times C_7) \cdot (\equiv^*(Q_{2\cdot 5} \times C_7))^{-1}$

$$Ar(Q_{2\cdot 5} \times C_7) = \begin{bmatrix}
140 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
28 & 28 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
70 & 0 & 70 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
14 & 14 & 14 & 14 & 0 & 0 & 0 & 0 & 0 & 0 \\
35 & 0 & 35 & 0 & 7 & 0 & 0 & 0 & 0 & 0 \\
20 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 \\
4 & 4 & 0 & 0 & 0 & 4 & 4 & 0 & 0 & 0 \\
10 & 0 & 10 & 0 & 0 & 10 & 0 & 10 & 0 & 0 \\
2 & 2 & 2 & 2 & 0 & 2 & 2 & 2 & 2 & 0 \\
5 & 0 & 5 & 0 & 1 & 5 & 0 & 5 & 0 & 1
\end{bmatrix}$$

And $(\equiv^*(Q_{2\cdot 5}\times C_7))^{-1}=$

$$\begin{bmatrix}
1/70 & 1/140 & 1/70 & 1/140 & 1/140 & 1/70 & 1/140 & 1/70 & 1/140 & 1/140 \\
-1/70 & 1/35 & -1/70 & 1/35 & 1/35 & -1/70 & 1/35 & -1/70 & 1/35 & 1/35 \\
1/70 & 1/140 & -1/70 & 1/140 & -1/140 & 1/70 & 1/140 & 1/70 & 1/140 & -1/140 \\
-1/70 & 1/35 & 1/70 & 1/35 & -1/35 & -1/70 & 1/35 & -1/70 & 1/35 & -1/35 \\
0 & 1/14 & 0 & -1/14 & 0 & 0 & 1/14 & 0 & -1/14 & 0 \\
-1/70 & -1/140 & -1/70 & -1/140 & -1/140 & 2/35 & 1/35 & 2/35 & 1/35 & 1/35 \\
1/70 & -1/35 & 1/70 & -1/35 & -1/35 & -2/35 & 4/35 & -2/35 & 4/35 & 4/35 \\
-1/70 & -1/140 & 1/70 & -1/140 & 1/140 & 2/35 & 1/35 & -2/35 & 1/35 & -1/35 \\
1/70 & -1/35 & -1/70 & -1/35 & 1/35 & -2/35 & 4/35 & 2/35 & 4/35 & -4/35 \\
0 & -1/14 & 0 & 1/14 & 0 & 0 & 2/35 & 0 & -2/35 & 0
\end{bmatrix}$$

Then $Ar(Q_{2\cdot 5}\times C_7)\cdot(\equiv^*(Q_{2\cdot 5}\times C_7))^{-1}=\left[\begin{array}{ccccc|ccccc}
2 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\
2 & 1 & 0 & 1 & 0 & 2 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ \hline
0 & 0 & 0 & 0 & 0 & 2 & 1 & 2 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0
\end{array}\right]=A(Q_{10}\times C_7)$

Capacity square matrix of (10× 10)

Second: *By Proposition(4.3) , then* $A(Q_{10})=\begin{bmatrix} 2 & 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$. *and by Theorem (4.7):*

$$A(Q_{10}\times C_7)=\left[\begin{array}{c|c} A(Q_{10}) & A(Q_{10}) \\ \hline 0 & A(Q_{10}) \end{array}\right]=\left[\begin{array}{ccccc|ccccc}
2 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\
2 & 1 & 0 & 1 & 0 & 2 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ \hline
0 & 0 & 0 & 0 & 0 & 2 & 1 & 2 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0
\end{array}\right]$$

Capacity square matrix of (10× 10)

Proposition (3.10)

If $m = l$ *; and* l *is prime number , then:*

1- The matrix $B(Q_{2l}\times C_7)$ *of the group* $Q_{2l}\times C_7$ *taking the form*

$$B(Q_{2l}\times C_7)= F\cdot\left[\begin{array}{c|c} 0 & B(Q_{2l}) \\ \hline B(Q_{2l}) & -B(Q_{2l}) \end{array}\right]$$

Capacity square matrix of (10× 10)

2-The matrix $C(Q_{2l}\times C_7)$ *of the group* $Q_{2l}\times C_7$ *taking the form*

$$C(Q_{2l}\times C_7)=\left[\begin{array}{c|c} 0 & C(Q_{2l}) \\ \hline C(Q_{2l}) & 0 \end{array}\right]\cdot F$$

. *Capacity square matrix of (10× 10)*

Where $$F=\begin{bmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

Capacity square matrix of (10× 10)

Proof:

By Theorem (3.8) we get in the form of $A\,(Q_{2l}\times C_7)$. and above form $B(Q_{2l}\times C_7)$ and $C(Q_{2l}\times C_7)$ then:

$B(Q_{2l}\times C_7)\cdot A(Q_{2l}\times C_7)\cdot C(Q_{2l}\times C_7)$ =diag{2, 2, 2 ,2, 1, 1, 1, 1, 1, 1}=$D(Q_{2l}\times C_7)$

Capacity square matrix of (10× 10)

Example(3.11)

We find the matrices $B(Q_{10}\times C_7)$ and $C(Q_{10}\times C_7)$ by Proposition (3.4) to find $B(Q_{10})$ and $C(Q_{10})$:

$$B(Q_{10}) = \begin{bmatrix} 1&-1&-1&1&0\\ 0&1&0&-1&0\\ 0&0&1&-1&0\\ 0&0&0&1&0\\ 0&0&0&0&1 \end{bmatrix},\ C(Q_{10}) = \begin{bmatrix} 0&0&1&0&0\\ 0&0&-1&0&1\\ 1&0&0&0&0\\ 0&0&1&1&-1\\ 0&1&0&0&0 \end{bmatrix}$$ *Then by Proposition (4.10):*

$$B(Q_{10}\times C_7)=F.\left[\begin{array}{c|c} 0 & B(Q_{10})\\ \hline B(Q_{10}) & -B(Q_{10}) \end{array}\right] = \begin{bmatrix} 0&0&0&0&0&1&-1&-1&1&0\\ 1&-1&-1&1&0&-1&1&1&-1&0\\ 0&0&0&0&0&0&0&1&-1&0\\ 0&0&1&-1&0&0&0&-1&1&0\\ 0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0&-1&0\\ 0&1&0&-1&0&0&-1&0&1&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&1&0&0&0&0&-1&0\\ 0&0&0&0&1&0&0&0&0&-1 \end{bmatrix}$$

$$C(Q_{10}\times C_7)=\left[\begin{array}{c|c} \mathbf{0} & C(Q_{10})\\ \hline C(Q_{10}) & \mathbf{0} \end{array}\right].F = \begin{bmatrix} 0&0&0&0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&-1&0&1\\ 0&0&-1&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&1&-1\\ 0&0&1&1&-1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0 \end{bmatrix}$$

Example(3.12)

We find $D(Q_{10}\times C_7)$ of the group $Q_{10}\times C_7$. from Example (3.11) we get matrices $B(Q_{10}\times C_7)$ and $C(Q_{10}\times C_7)$ and from Example (3.9) we get matrix $A(Q_{10}\times C_7)$, then:

$B(Q_{10}\times C_7)\cdot A(Q_{10}\times C_7)\cdot C(Q_{10}\times C_7)$=diag{2, 2, 2, 2, 1, 1, 1, 1, 1, 1}=$D(Q_{10}\times C_7)$

Theorem(3.13):

If $m = l$; l is a (pr), the cyclic decomposition of the $AC(Q_{2l}\times C_7)$ is:

$$AC\,(Q_{2l}\times C_7) = \bigoplus_{i=1}^{4} C_2$$

Proof:

By Theorem (3.8), we will find the matrix $A(Q_{2l}\times C_7)$ and by Proposition (3.10) we will find matrixs $B(Q_{2l}\times C_7)$ and $C(Q_{2l}\times C_7)$:

$B(Q_{2l}\times C_7)\cdot A(Q_{2l}\times C_7)\cdot C(Q_{2l}\times C_7)=diag\{2,2,2,2,1,1,1,1,1,1\}$ Then by Theorem (2.8) we have:

$$AC(Q_{2l}\times C_7) = \bigoplus_{i=1}^{4} C_2$$

Example (3.14):

Let's take the groups $Q_{222}\times C_7$ and $Q_{226}\times C_7$ then :

$$1 - AC(Q_{222}\times C_7) = AC(Q_{2.111}\times C_7) = \bigoplus_{i=1}^{4} C_2$$

$$2 - AC(Q_{226}\times C_7) = AC(Q_{2.113}\times C_7) = \bigoplus_{i=1}^{4} C_2$$

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