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Around Artin's cokernel of The Group($Q_{2m}\times C_7$) Where $m=r_1.r_2$, $r_1, r_2 \neq 2$ Where greatest common denominator $(r_1, r_2) = 1$ and r_1, r_2 are primes numbers

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1.Abstract:

Determining the cyclic decomposition of the abelian factor group AC(G) = (G)/T(G), where $G = Q_{2m}\times C_7$, $m=r_1.r_2$, so that $r_1, r_2\neq 2$, r_1, r_2 are primes numbers and the greatest common denominator$(r_1, r_2)=1$ is the primary objective of the study. the group of all Z-valued characters of G over all cyclic subgroups of the group of induced unit characteristics of G

We find that $AC(Q_{2m}\times C_7) = \bigoplus_{i=1}^{12} C_2$.

Key words: Cyclic group, Quaternion group, prime number and odd number

2.Introduction:

Representation theory delves into the study of abstract algebraic structures by portraying their elements as transformations of vector spaces, often using matrices to elucidate algebraic operations. In the realm of finite groups, members are typically depicted via invertible matrices, thereby aligning group operations with matrix multiplication. This branch boasts applications across various disciplines, including physics, chemistry, and other realms of mathematics. The Artin cokernel, denoted AC(G), is pertinent to finite groups and delineates the factor group (G) /T(G), where T(G) denotes the center of group G. AC(G) manifests as an abelian group, generated by the Z-valued characters of G

3-Preliminars (3,1) :[2]

Defined for a positive integer $m \geq 2$, the Q_{2m} Generalized Quaternion Group is shaped by generators x and y, expressed as $Q_{2m} =\{x^h y^k, 0\leq h\leq 2m-1, k=0,1\}$. This group, having an order of 4m, adheres to $x^{2m}=y^4=I$ and $yx^m y^{-1}=x^{-m}$. Within finite group G, Artin characters, induced by a principal character in a cyclic subgroup, are structured in the Artin characters table Ar(G). This table unfolds with rows detailing Γ-conjugate classes, elements in each class, centralizer size |CG(CL_a)|, and successive rows presenting Artin character values.

Definition (3.2)

The group $Q_{2m}\times C_7$ is the direct product group of the quaternion group Q_{2m} of order 4m and the cyclic group C_7 of order 7, then the order of The Group $Q_{2m}\times C_7$ is 28m

Theorem(3,3): [3]

When s is a positive integer number and r is a prime number, the general form of the Artin characters table of C_{r^s} is given by :-

$Ar(C_{r^s})=$

Γ-classes	[1]	$[x^{r^{s-1}}]$	$[x^{r^{s-2}}]$	$[x^{r^{s-3}}]$	…	[x]
$\lvert CL_\alpha \rvert$	1	1	1	1	…	1
$\lvert C_{r^s}(CL_\alpha) \rvert$	r^s	r^s	r^s	r^s	…	r^s
φ'_1	r^s	0	0	0	…	0
φ'_2	r^{s-1}	r^{s-1}	0	0	…	0
φ'_3	r^{s-2}	r^{s-2}	r^{s-2}	0	…	0
⋮	⋮	⋮	⋮	⋮	⋮	
φ'_s	r^1	r^1	r^1	r^1	…	0
φ'_{s+1}	1	1	1	1	…	1

Table(3,1)

Corollary (3,4) : [3]

Let $m = r_1^{\alpha_1} \cdot r_2^{\alpha_2} \cdots \cdots r_n^{\alpha_n}$ where greatest common denominator $(r_i, r_j)=1$, if $i \neq j$ and r_i's are primes numbers, and α_i any positive integers, then; $Ar(C_m) = Ar(C_{r_1^{\alpha_1}}) \otimes Ar(C_{r_2^{\alpha_2}}) \otimes \ldots \otimes Ar(C_{r_n^{\alpha_n}})$.

Example (3.5):

Artin characters table of the cyclic group $(C_{2 \cdot r_1 r_2})$, $r_1, r_2 \neq 2$, r_1, r_2 are primes numbers and greatest common denominator$(r_1, r_2) = 1$. using corollary (3,3) it is as following $Ar(C_{2 \cdot r_1 r_2}) = Ar(C_2) \otimes Ar(C_{r_1}) \otimes Ar(C_{r_2})$:

$Ar(C_{2 \cdot r_1 r_2})$=

Γ- classes	[1]	$[x^2]$	$[x^{2r_1}]$	$[x^{2r_2}]$	$[x^{r_1 r_2}]$	$[x^{r_1}]$	$[x^{r_2}]$	[x]
$\lvert CL_\alpha \rvert$	1	1	1	1	1	1	1	1
$\lvert C_{C_{2 \cdot r_1 r_2}}(CL_\alpha) \rvert$	$2r_1r_2$	$2r_1r_2$	$2r_1r_2$	$2r_1r_2$	$2r_1r_2$	$2r_1r_2$	$2r_1r_2$	$2r_1r_2$
φ'_1	$2r_1r_2$	0	0	0	0	0	0	0
φ'_2	2	2	2	2	0	0	0	0
φ'_3	$2r_1$	0	$2r_1$	0	0	0	0	0
φ'_4	$2r_2$	0	0	$2r_2$	0	0	0	0
φ'_5	r_1r_2	0	0	0	r_1r_2	0	0	0
φ'_6	r_1	0	r_1	0	0	P_1	0	0
φ'_7	r_2	0	0	r_2	0	0	r_2	0
φ'_8	1	1	1	1	1	1	1	1

Table(3,2)

Theorem(3.6):[2]

When m is an odd number, the Quaternion group Q_{2m} 's Artin characters table is provided as follows:

$Ar(Q_{2m})$ =

	Γ-Classes of C_{2m}	

Γ-Classes	x^{2r}				x^{2r+1}				[y]
$\|CL_\alpha\|$	1	2	…	2	1	2	…	2	m
$\|C_{Q_{2m}}(CL_\alpha)\|$	4m	2m	…	2m	4m	2m	…	2m	4
Φ_1	2.Ar(C_{2m})								0
Φ_2									0
⋮									⋮
Φ_ℓ									0
$\Phi_{\ell+1}$	m	0	…	0	m	0	…	0	1

Table (3.3)

So that $0 \le r \le m-1$, ℓ it is a number of Γ-classes of C_{2m} and Φ_τ Artin characters of the Q_{2m} , for every $1 \le \tau \le \ell+1$.

Example (3.7):

$Ar(Q_{2.r_1.r_2})$,r_1, r_2 are primes numbers , greatest common denominator$(r_1, r_2) = 1$, and $r_1, r_2 \neq 2$ by using theorem(3.5)

$Ar(Q_{2.r_1.r_2}) =$

Γ-Classes	[1]	$[x^2]$	$[x^{2r_1}]$	$[x^{2r_2}]$	$[x^{r_1r_2}]$	$[x^{r_1}]$	$[x^{r_2}]$	[x]	[y]
$\|CL_\alpha\|$	1	2	2	2	1	2	2	2	r_1r_2
$\|CQ_{2r_1r_2}(CL_\alpha)\|$	$4r_1r_2$	$2r_1r_2$	$2r_1r_2$	$2r_1r_2$	$4r_1r_2$	$2r_1r_2$	$2r_1r_2$	$2r_1r_2$	4
Φ_1	$4r_1r_2$	0	0	0	0	0	0	0	0
Φ_2	4	4	4	4	0	0	0	0	0
Φ_3	$4r_1$	0	$4r_1$	0	0	0	0	0	0
Φ_4	$4r_2$	0	0	$4r_2$	0	0	0	0	0
Φ_5	$2r_1r_2$	0	0	0	$2r_1r_2$	0	0	0	0
Φ_6	$2r_1$	0	2r1	0	0	$2r_1$	0	0	0
Φ_7	$2r_2$	0	0	$2r_2$	0	0	$2r_1$	0	0
Φ_8	2	2	2	2	2	2	2	2	0
Φ_9	r_1r_2	0	0	0	r_1r_2	0	0	0	1

Table (3.4)

Theorem (3,8) [1]

$Ar(Q_{2m}\times C_7)$ so that $m=r_1.r_2$, r_1, r_2 are primes numbers , greatest common denominator$(r_1, r_2) = 1$ and $r_1, r_2 \neq 2$ be as follows:

$Ar(Q_{2m}\times C_7)$=

	Γ-Classes of $Q_{2m}\times$<I>									Γ-Classes of $Q_{2m}\times$<z>								
Γ-Classes	[1,I]	$[x^2,I]$	$[x^{2r_1},I]$	$[x^{2r_2},I]$	$[x^{r_1r_2},I]$	$[x^{r_1},I]$	$[x^{r_2},I]$	[x,I]	[y,I]	[1,z]	$[x^2,z]$	$[x^{2r_1},z]$	$[x^{2r_2},z]$	$[x^{r_1r_2},z]$	$[x^{r_1},z]$	$[x^{r_2},z]$	[x,z]	[y,z]
$\|CL_\alpha\|$	1	2	2	2	1	2	2	2	m	1	2	2	2	1	2	2	2	m
$\|C_{Q_{2m}\times C_7}(CL_\alpha)\|$	28m	14m	14m	14m	28m	14m	14m	14m	28	28m	14m	14m	14m	28m	14m	14m	14m	28
$\Phi_{(1,1)}$	7Ar(Q_{2m})									0								
$\Phi_{(2,1)}$																		
$\Phi_{(3,1)}$																		
$\Phi_{(4,1)}$																		
$\Phi_{(5,1)}$																		
$\Phi_{(6,1)}$																		
$\Phi_{(7,1)}$																		
$\Phi_{(8,1)}$																		
$\Phi_{(9,1)}$																		
$\Phi_{(1,2)}$	Ar(Q_{2m})									Ar(Q_{2m})								
$\Phi_{(2,2)}$																		
$\Phi_{(3,2)}$																		
$\Phi_{(4,2)}$																		
$\Phi_{(5,2)}$																		
$\Phi_{(6,2)}$																		
$\Phi_{(7,2)}$																		
$\Phi_{(8,2)}$																		
$\Phi_{(9,2)}$																		

Table (4.1)
18×18 square matrix.

Example (3.9):

To find $Ar(Q_{30}\times C_7)=Ar(Q_{2.3.5}\times C_7)$, $r_1=3$, $r_2=5$, using the theorm(3,6) it is :-
$Ar(Q_{30})$=

Γ-Classes	[1]	$[x^2]$	$[x^6]$	$[x^{22}]$	$[x^{33}]$	$[x^3]$	$[x^{11}]$	[x]	[y]
$\lvert CL_\alpha \rvert$	1	2	2	1	1	2	2	2	15
$\lvert C_{Q_{30}}(CL_\alpha)\rvert$	60	30	30	30	60	30	30	30	4
Φ_1	60	0	0	0	0	0	0	0	0
Φ_2	4	4	4	4	0	0	0	0	0
Φ_3	12	0	12	0	0	0	0	0	0
Φ_4	20	0	0	44	0	0	0	0	0
Φ_5	30	0	0	0	66	0	0	0	0
Φ_6	6	0	6	0	6	6	0	0	0
Φ_7	10	0	0	22	22	0	22	0	0
Φ_8	2	2	2	2	2	2	2	2	0
Φ_9	15	0	0	0	15	0	0	0	1

Table(4,2)

With the help of theorem (3,8)it is :-

$Ar(Q_{30}\times C_7)$

Γ-Classes	[1,I]	$[x^2,I]$	$[x^6,I]$	$[x^{10},I]$	$[x^{15},I]$	$[x^3,I]$	$[x^5,I]$	[x,I]	[y,I]	[1,z]	$[x^2,z]$	$[x^6,z]$	$[x^{10},z]$	$[x^{15},z]$	$[x^3,z]$	$[x^5,z]$	[x,z]	[y,z]
$\lvert CL_\alpha\rvert$	1	2	2	2	1	2	2	2	15	1	2	2	2	1	2	2	2	15
$\lvert C_{Q_{30}\times C_7}(CL_\alpha)\rvert$	420	210	210	210	420	210	210	210	28	420	210	210	210	420	210	210	210	28
$\Phi_{(1,1)}$	420	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	28	28	28	28	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	84	0	84	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	140	0	0	140	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	210	0	0	0	210	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(6,1)}$	42	0	30	0	42	42	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(7,1)}$	70	0	0	70	70	0	70	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(8,1)}$	14	14	14	14	14	14	14	14	0	0	0	0	0	0	0	0	0	0
$\Phi_{(9,1)}$	105	0	0	0	105	0	0	0	7	0	0	0	0	0	0	0	0	0
$\Phi_{(1,2)}$	60	0	0	0	0	0	0	0	0	60	0	0	0	0	0	0	0	0
$\Phi_{(2,2)}$	4	4	4	4	0	0	0	0	0	4	4	4	4	0	0	0	0	0
$\Phi_{(3,2)}$	12	0	12	0	0	0	0	0	0	12	0	12	0	0	0	0	0	0
$\Phi_{(4,2)}$	20	0	0	20	0	0	0	0	0	20	0	0	20	0	0	0	0	0
$\Phi_{(5,2)}$	30	0	0	0	30	0	0	0	0	30	0	0	0	30	0	0	0	0
$\Phi_{(6,2)}$	6	0	6	0	6	6	0	0	0	6	0	6	0	6	6	0	0	0
$\Phi_{(7,2)}$	10	0	0	10	10	0	10	0	0	10	0	0	10	10	0	10	0	0
$\Phi_{(8,2)}$	2	2	2	2	2	2	2	2	0	2	2	2	2	2	2	2	2	0
$\Phi_{(9,2)}$	15	0	0	0	15	0	0	0	1	15	0	0	0	15	0	0	0	1

Table(4,3)

4. The Factor Group AC(G):

In this section, we aim to explain the basic definitions and important theorems of the group AC(G).

Definition (4.1):[6]

Let T(G) be the subgroup of $\bar{R}(G)$ generated by Artin characters .

T(G) is a normal subgroup of $\bar{R}(G)$,then the finite factor abelian group $\bar{R}(G)/T(G)$ is called **Artin cokernel of G**, denoted by AC(G).

Definition (4.2.): [7]

Let U be a matrix with entries in a principal ideal domain R . A **k – minor of U** is the determinate of k×k sub-matrix preserving row and column order .

Definition (4.3):[7]

A **k-th determinant divisor of U** is the greatest common divisor (g.c.d) of all the k – minor, this is denoted by $D_k(U)$.

Lemma (4.4):[7]

Let U , B and E be matrices with entries in the principal ideal domain R . Let B and E be invertible matrices, then :

$D_k(B.U.E) \equiv D_k(U)$ modulo the group of units of R .

Theorem (4.5):[7]

Let U be an n×n matrix with entries in a principal ideal domain R, then there exist matrices B and E such that :

1 - B and E are invertible.

2 - B U E = D.

3 - D is a diagonal matrix.

4 -If we denote D_{ii} by d_i then there exists a natural number m ; $0 \leq m \leq n$ such that j > m implies $d_j=0$ and $j \leq m$ implies $d_j \neq 0$ and $1 \leq j \leq m$ implies $d_j \mid d_{j+1}$.

Definition (4.6): [7]

Let U be matrix with entries in a principal ideal domain R such that U is equivalent to matrix $D = diag\{d_1, d_2, \ldots, d_m, 0, 0, \ldots, 0\}$ where $d_j \mid d_{j+1}$ for $1 \leq j < m$.We call D **the invariant factor matrix of** U and $d_1, d_2, \ldots, d_m$ the invariant factors of U.

Theorem (4.7):[7]

If U is a matrix with entries in a principal ideal domain R, then the invariant factors are unique .

Theorem (4.8): [7]

Let K be a finitely generated module over the principal domain R, then K is the direct sum of cyclic submodules with annihilating ideal

$<d_1>, <d_2>, \dots, <d_m>, d_j \mid d_{j+1}$ *for* $j = 1, 2, \dots, m-1$.

Proposition (4.9):[7]

AC(G) is a finitely generated Z-module.

Theorem (Artin's) (4.10):[8]

Every rational valued character of G can be written as a linear combination of Artin characters with coefficient rational numbers.

Remark (4.11): [6]

Let l be the number of all distinct Γ- classes of G then Ar(G) and

$\equiv^*(G)$ *are of rank l. According to the Artin's theorem there exists an invertible matrix* $U^{-1}(G)$ *with entries in Q such that :*

$\equiv^*(G) = U^{-1}(G).Ar(G)$ *and this implies,* $U(G) = Ar(G).(\equiv^*(G))^{-1}$

U (G) is the matrix expressing the T(G) basis in terms of the $\bar{R}(G)$ *basis. By Theorem (4. 5), there exist two matrices B(G) and E(G) with determinant ± 1 such that:*

$B(G).U(G).E(G) = diag\{d_1, d_2, \dots, d_l\} = D(G)$ *where* $d_i = \pm D_i(G)/D_{i-1}(G)$ *This process yields a new basis for T(G) and* $\bar{R}(G)$ *,*$\{v_1, v_2, \dots, v_l\}$ *and* $\{u_1, u_2, \dots, u_l\}$ *respectively, with the property* $v_j = d_j u_j$. *Hence, by Theorem (4.7) and Proposition (4.8) the Z-module AC (G) is the direct sum of cyclic sub modules with annihilating ideals* $<d_1>, <d_2>, \dots, <d_l>$.

Theorem (4.12):[6]

$AC(G) = \bigoplus_{i=1}^{l} C_{d_i}$ *where* $d_i = \pm D_i(G)/D_{i-1}(G)$ *and l is the number of all distinc Γ-classes of G.*

Corollary (4.13):[6]

$|AC(G)| = |\det(U(G))|$.

Lemma (4.14):[6]

If A and B are two matrices of degree m and t respectively, then:

$\det(X \otimes Y) = (\det(X))^t . (\det(Y))^m$.

Lemma (4.15):[6]

Let X and Y be two invertible matrices of rank l and m respectively, over a principal ideal domain R and let: $B_1 X E_1 = D(X) = diag\{d_1(X), d_2(X), \ldots, d_l(X)\}$ And $B_2 Y E_2 = D(Y) = diag\{d_1(Y), d_2(Y), \ldots, d_m(Y)\}$ the invariant factor matrices of X and Y then: $(B_1 \otimes B_2)(X \otimes Y)(E_1 \otimes E_2) = D(X) \otimes D(Y)$ and from this the invariant factor matrices of

$X \otimes Y$ can be written dig $(X \otimes Y) = d_1(X).d_1(Y), d_2(X).d_2(Y), \ldots\ldots, d_l(X).d_m(Y)$

Proposition(4.16):[6]

Let H_1 and H_2 be two p-groups then the matrix which expresses the $T(H_1\times H_2)$ basis of $\bar{R}(H_1\times H_2)$ basis is $U_1 \otimes U_2$

Proposition (4. 17): [9]

If r is a prime number and s is a positive integer, then

$$U(C_{r^s}) = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & 1 & 1 & \cdots & 1 \\ 0 & 0 & 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Which is order $(s+1)\times(s+1)$ square matrix .

Proposition(4.18):[10]

The general form of matrices $B(C_{r^s})$ and $E(C_{r^s})$ are :

$$B(C_{r^s}) = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ & & & & & \ddots & & \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

which is $(s+1)\times(s+1)$ square matrix .

$E(C_{r^s}) = I_{s+1}$, I_{s+1} is an identity matrix and $D(C_{r^s}) = diag\{\underbrace{1,1,\ldots\ldots,1}_{s+1}\}$.

Remark (4.19): [10]

In general if $m=r_1^{\alpha_1}.r_2^{\alpha_2}\cdots\cdots r_n^{\alpha_n}$ where greatest common denominator $(r_i, r_j)=1$, if $i \neq j$ and for all $i, 1\leq i \leq n$, $r_i \neq 2$ are prime numbers and α_i any positive integers is all $i = 1, 2, \ldots, n$;then :

$C_m = C_{r_1^{\alpha_1}} \times C_{r_2^{\alpha_2}} \times \ldots \times C_{r_n^{\alpha_n}}$, 1- By Proposition (4.17) we get: $r_1 \neq 2$

$U(C_m) = U(C_{r_1^{\alpha_1}}) \otimes U(C_{r_2^{\alpha_2}}) \otimes \ldots \otimes U(C_{r_n^{\alpha_n}})$.

Thus, we can write $U(C_m)$ as:

$$U(C_m) = \begin{bmatrix} & & & & & 1 \\ & & R(C_m) & & & 1 \\ & & & & & \vdots \\ & & & & & 1 \\ 0 & 0 & 0 & \ldots & 0 & 1 \end{bmatrix}$$

Where $R(C_m)$ is the matrix which is obtained by omitting the last row {0, 0,…,0, 1} and the last column {1,1,…,1} from the Tensor product, $U(C_{r_1^{\alpha_1}}) \otimes U(C_{r_2^{\alpha_2}}) \otimes \ldots \otimes U(C_{r_n^{\alpha_n}})$, Where $U(C_m)$ is of order, $[(\alpha_1+1)(\alpha_2+1)\ldots(\alpha_n+1) \times (\alpha_1+1)(\alpha_2+1)\ldots(\alpha_n+1)]$ square matrix.

2- By Lemma (4.15) we have

a- $B(C_m) = B(C_{r_1^{\alpha_1}}) \otimes B(C_{r_2^{\alpha_2}}) \otimes \ldots \otimes B(C_{r_n^{\alpha_n}})$.

b- $E(C_m) = E(C_{r_1^{\alpha_1}}) \otimes E(C_{r_2^{\alpha_2}}) \otimes \ldots \otimes E(C_{r_n^{\alpha_n}})$.

Proposition (4.20):[2]

If $m=r_1^{\alpha_1} . r_2^{\alpha_2} \cdots \cdots r_n^{\alpha_n}$ where α_i any positive integers if $i \neq j$, r_i's are primes numbers and greatest common denominator $(r_i\ r_j)=1$ then the matrix $U(Q_{2m})$ of the quaternion group Q_{2m} is :

$$U(Q_{2m}) = \left[\begin{array}{ccccc|ccccc|c} & 2R(C_m) & & & 1 & & 2R(C_m) & & & 1 & 1 \\ & & & & 1 & & & & & 1 & 1 \\ & & & & \vdots & & & & & \vdots & \vdots \\ & & & & 1 & & & & & 1 & 1 \\ 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 1 & 1 \\ \hline & & & & 1 & 0 & 0 & \cdots & 0 & 1 & 0 \\ & 2R(C_m) & & & 1 & 0 & 0 & \cdots & 0 & 1 & 0 \\ & & & & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ & & & & 1 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 1 & 0 \\ \hline 1 & 1 & \cdots & 1 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \end{array}\right]$$

Which is $[2(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_n+1)+1] \times [2(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_n+1)+1]$ square matrix , $R(C_m)$ is similar to the matrix in Remarks (4.19).

Proposition (4.21):[2]

If $m=r_1^{\alpha_1} . r_2^{\alpha_2} \cdots \cdots r_n^{\alpha_n}$ where greatest common denominator $(r_i\ r_j)=1$, if $i \neq j$ and r_i 's are prime numbers and α_i any positive integers, then the matrices $B(Q_{2m})$ and $E(Q_{2m})$ are taking the forms :

$$B(Q_{2m}) = \left[\begin{array}{ccccc|ccccc|c} & & B(C_m) & & & & & -B(C_m) & & & 0 \\ & & & & & & & & & & 0 \\ & & & & & & & & & & \vdots \\ & & & & & & & & & & 0 \\ & & & & & & & & & & 0 \\ \hline 0 & 0 & \cdots & 0 & 0 & & & & & & 0 \\ 0 & 0 & \cdots & 0 & 0 & & & B(C_m) & & & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & & & & & & \vdots \\ 0 & 0 & \cdots & 0 & 0 & & & & & & 0 \\ 0 & 0 & \cdots & 0 & 0 & & & & & & 0 \\ \hline 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{array}\right]$$

$$E(Q_{2m}) = \left[\begin{array}{ccccc|cccccc|c} 0 & 0 & \cdots & 0 & 0 & & & & & & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & & & I_k & & & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & & & & & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & & & & & & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & -1 & -1 & -1 & -1 & & 0 & 1 \\ \hline & & & & 0 & 0 & 0 & \cdots & 0 & & 0 & 0 \\ & & I_k & & 0 & 0 & 0 & \cdots & 0 & & 0 & 0 \\ & & & & \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots \\ & & & & 0 & 0 & 0 & \cdots & 0 & & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 & 1 & \cdots & 1 & & 1 & -1 \\ \hline 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & & 0 & 0 \end{array}\right]$$

Where $k = (\alpha_1+1)\cdot(\alpha_2+1)\cdot(\alpha_3+1)\cdots(\alpha_n+1)-1$. *They are* $[2(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_n+1)+1] \times$ $[2(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_n+1)+1]$ *square matrix .*

5.The Main Results

In this section, we give the cyclic decomposition of the factor group $AC(Q_{2m}\times C_7)$ when $m=r_1.r_2$, r_1, r_2 are primes numbers , greatest common denominator$(r_1, r_2) = 1$ and $r_1, r_2 \neq 2$

Theorem (5.1):

If $m=r_1.r_2$, $r_1, r_2 \neq 2$ Where greatest common denominator $(r_1,r_2) = 1$ and r_1, r_2 are primes numbers, then the matrix $U(Q_{2m}\times C_7)$ of the group $Q_{2m}\times C_7$ is :

$$U(Q_{2m}\times C_7) = \left[\begin{array}{c|c} U(Q_{2m}) & U(Q_{2m}) \\ \hline 0 & U(Q_{2m}) \end{array}\right]$$

where 4(1+1).(1+1)+2=18 Which is 18× 18 square matrix.

(Q_{2m}) is similarity the matrix of Proposition (4.20)

Proof :

By definition of U(G) we find the matrix $U(Q_{2m}\times C_7)$:

$U(Q_{2m}\times C_7) = Ar(Q_{2m}\times C_7) \cdot (\equiv^* (Q_{2m}\times C_7))^{-1} =$

$$\left[\begin{array}{ccccccc|ccccccc}
2R(C_m) & & 1 & 2R(C_m) & & 1 & 1 & 2R(C_m) & & 1 & 2R(C_m) & & 1 & 1 \\
 & & \vdots & & & \vdots & \vdots & & & \vdots & & & \vdots & \vdots \\
0\cdots & 0 & 1 & 0\cdots & 0 & 1 & 1 & 0\cdots & 0 & 1 & 0\cdots & 0 & 1 & 1 \\
2R(C_m) & & 1 & 0\cdots & 0 & 1 & 0 & 2R(C_m) & & 1 & 0\cdots & 0 & 1 & 0 \\
 & & \vdots & \ddots & \vdots & \vdots & \vdots & & & \vdots & \ddots & \vdots & \vdots & \vdots \\
0\cdots & 0 & 1 & 0\cdots & 0 & 1 & 0 & 0\cdots & 0 & 1 & 0\cdots & 0 & 1 & 0 \\
1\cdots & & 1 & 0\cdots & & 0 & 0 & 1\cdots & & 1 & 0\cdots & & 0 & 0 \\
\hline
0\cdots & \cdots & 0 & \cdots\cdots & \cdots & 0 & 0 & 2R(C_m) & & 1 & 2R(C_m) & & 1 & 1 \\
0\cdots & \cdots & 0 & \cdots\cdots & \cdots & 0 & 0 & & & \vdots & & & \vdots & \vdots \\
\vdots & & \vdots & \ddots & & \vdots & \vdots & 0\cdots & 0 & 1 & 0\cdots & 0 & 1 & 1 \\
\vdots & \ddots & \vdots & & & \vdots & \vdots & 2R(C_m) & & 1 & 0\cdots & 0 & 1 & 0 \\
\vdots & & \vdots & & \ddots & \vdots & \vdots & & & \vdots & \ddots & \vdots & \vdots & \vdots \\
0\cdots & 0 & 0 & \cdots\cdots & \cdots & 0 & 0 & 0\cdots & 0 & 1 & 0\cdots & 0 & 1 & 0 \\
0\cdots & 0 & 0 & \cdots\cdots & \cdots & 0 & 0 & 1\cdots & & 1 & 0\cdots & & 0 & 0
\end{array}\right]$$

where 4(1+1).(1+1)+2=18 Which is 18× 18 square matrix.

$$= \left[\begin{array}{c|c} U(Q_{2m}) & U(Q_{2m}) \\ \hline 0 & U(Q_{2m}) \end{array}\right] = U(Q_{2m}\times C_7)$$

Example(5.2):

When we take the matrix $U(Q_{30}\times C_7)$ using two methods:

First:

By definition of U(G)

$U(Q_{30}\times C_7) \equiv U(Q_{2\cdot3\cdot5}\times C_7) = Ar(Q_{2\cdot3\cdot5}\times C_7)\cdot(\equiv * (Q_{2\cdot3\cdot5}\times C_7))^{-1}$

$Ar(Q_{2\cdot3\cdot5}\times C_7) =$

$$\begin{bmatrix}
420 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
28 & 28 & 28 & 28 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
84 & 0 & 84 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
140 & 0 & 0 & 140 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
210 & 0 & 0 & 0 & 210 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
42 & 0 & 42 & 0 & 42 & 42 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
70 & 0 & 0 & 70 & 70 & 0 & 70 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
14 & 14 & 14 & 14 & 14 & 14 & 14 & 14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
105 & 0 & 0 & 0 & 105 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
60 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 60 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 4 & 4 & 4 & 0 & 0 & 0 & 0 & 0 & 4 & 4 & 4 & 4 & 0 & 0 & 0 & 0 & 0 \\
12 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 \\
20 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 \\
30 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 \\
6 & 0 & 6 & 0 & 6 & 6 & 0 & 0 & 0 & 6 & 0 & 6 & 0 & 6 & 6 & 0 & 0 & 0 \\
10 & 0 & 0 & 10 & 10 & 0 & 10 & 0 & 0 & 10 & 0 & 0 & 10 & 10 & 0 & 10 & 0 & 0 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 0 \\
15 & 0 & 0 & 0 & 15 & 0 & 0 & 0 & 1 & 15 & 0 & 0 & 0 & 15 & 0 & 0 & 0 & 1
\end{bmatrix}$$

And $(\equiv * (Q_{2\cdot3\cdot5}\times C_7))^{-1} =$

$$\begin{bmatrix}
1/210 & 1/210 & 1/210 & 1/420 & 1/210 & 1/210 & 1/210 & 1/420 & 1/420 & 1/210 & 1/210 & 1/210 & 1/420 & 1/210 & 1/210 & 1/210 & 1/420 & 1/420 \\
1/210 & 8/105 & -2/105 & 2/105 & 1/210 & 8/105 & -2/105 & 2/105 & 2/105 & 1/210 & 8/105 & -2/105 & 2/105 & 1/210 & 8/105 & -2/105 & 2/105 & 2/105 \\
-1/210 & -1/210 & 2/105 & 1/105 & -1/210 & -1/210 & 2/105 & 1/105 & 1/105 & -1/210 & -1/210 & 2/105 & 1/105 & -1/210 & -1/210 & 2/105 & 1/105 & 1/105 \\
-1/210 & -1/210 & -1/210 & 1/210 & -1/210 & -1/210 & -1/210 & 1/210 & 1/210 & -1/210 & -1/210 & -1/210 & 1/210 & -1/210 & -1/210 & -1/210 & 1/210 & 1/210 \\
1/210 & 1/210 & 1/210 & 1/420 & -1/210 & -1/210 & -1/210 & 1/420 & -1/420 & 1/210 & 1/210 & 1/210 & 1/420 & -1/210 & -1/210 & -1/210 & 1/420 & -1/420 \\
-1/210 & 3/70 & -1/35 & 1/105 & 1/210 & 1/210 & -2/105 & 1/105 & -1/105 & -1/210 & 3/70 & -1/35 & 1/105 & 1/210 & 1/210 & -2/105 & 1/105 & 1/105 \\
-1/210 & -1/210 & 1/42 & 1/210 & 1/210 & 1/210 & 1/210 & 1/210 & -1/210 & -1/210 & -1/210 & 1/42 & 1/210 & 1/210 & 1/210 & 1/210 & 1/210 & -1/210 \\
1/210 & -4/35 & 0 & 2/105 & -1/210 & -8/105 & 2/105 & 2/105 & -2/105 & 1/210 & -4/35 & 0 & 2/105 & -1/210 & -8/105 & 2/105 & 2/105 & -2/105 \\
0 & 0 & 0 & 1/14 & 0 & 0 & 0 & -1/14 & 0 & 0 & 0 & 0 & 1/14 & 0 & 0 & 0 & -1/14 & 0 \\
-1/210 & -1/210 & -1/210 & -1/420 & -1/210 & -1/210 & -1/210 & -1/420 & -1/420 & 2/105 & 2/105 & 2/105 & 1/105 & 2/105 & 2/105 & 2/105 & 1/105 & 1/105 \\
-1/210 & -8/105 & 2/105 & -2/35 & -1/210 & -8/35 & 2/105 & -2/105 & -2/105 & 2/105 & 32/105 & -8/105 & 8/105 & 2/105 & 32/105 & -8/105 & 8/105 & 8/105 \\
1/210 & 1/210 & -2/105 & -1/105 & 1/210 & 1/210 & -2/105 & -1/105 & -1/105 & -2/105 & -2/105 & 8/105 & 4/105 & -2/105 & -2/105 & 8/105 & 4/105 & 4/105 \\
1/210 & 1/210 & 1/210 & -1/210 & 1/210 & 1/210 & 1/210 & -1/210 & -1/210 & -2/105 & -2/105 & -2/105 & 2/105 & -2/105 & -2/105 & -2/105 & 2/105 & 2/105 \\
-1/210 & -1/210 & -1/210 & -1/420 & 1/210 & 1/210 & 1/210 & -1/420 & 1/420 & 2/105 & 2/105 & 2/105 & 1/105 & -2/105 & -2/105 & -2/105 & 1/105 & -1/105 \\
1/210 & -3/70 & 1/35 & -1/35 & -1/210 & -1/210 & 2/105 & -1/105 & 1/105 & -2/105 & 6/35 & -4/35 & 4/105 & 2/105 & 2/105 & -8/105 & 4/105 & -4/105 \\
1/210 & 1/210 & -1/42 & -1/210 & -1/210 & -1/210 & -1/210 & -1/210 & 1/210 & -2/105 & -2/105 & 2/21 & 2/105 & 2/105 & 2/105 & 2/105 & 2/105 & -2/105 \\
-1/210 & 4/35 & 0 & -2/105 & 1/210 & 8/105 & -2/105 & -2/105 & 2/105 & 2/105 & -16/35 & 0 & 8/105 & -2/105 & -32/105 & 8/105 & 8/105 & -8/105 \\
0 & 0 & 0 & -1/14 & 0 & 0 & 0 & 1/14 & 0 & 0 & 0 & 0 & 2/7 & 0 & 0 & 0 & -2/7 & 0
\end{bmatrix}$$

where 4(1+1).(1+1)+2=18 Which is 18× 18 square matrix.

Then $Ar(Q_{2\cdot3\cdot5}\times C_5)\cdot(\equiv * (Q_{2\cdot3\cdot5}\times C_5))^{-1}=$

$$\left[\begin{array}{ccccccccc|ccccccccc}
2&2&2&1&2&2&2&1&1&2&2&2&1&2&2&2&1&1\\
0&2&0&1&0&2&0&1&1&0&2&0&1&0&2&0&1&1\\
0&0&2&1&0&0&2&1&1&0&0&2&1&0&0&2&1&1\\
0&0&0&1&0&0&0&1&1&0&0&0&1&0&0&0&1&1\\
2&2&2&1&0&0&0&1&0&2&2&2&1&0&0&0&1&0\\
0&2&0&1&0&0&0&1&0&0&2&0&1&0&0&0&1&0\\
0&0&2&1&0&0&0&1&0&0&0&2&1&0&0&0&1&0\\
0&0&0&1&0&0&0&1&0&0&0&0&1&0&0&0&1&0\\
1&1&1&1&0&0&0&0&0&1&1&1&1&0&0&0&0&0\\
\hline
0&0&0&0&0&0&0&0&0&2&2&2&1&2&2&2&1&1\\
0&0&0&0&0&0&0&0&0&0&2&0&1&0&2&0&1&1\\
0&0&0&0&0&0&0&0&0&0&0&2&1&0&0&2&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&1&1\\
0&0&0&0&0&0&0&0&0&2&2&2&1&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&2&0&1&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&2&1&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0&0
\end{array}\right]$$

where 4(1+1).(1+1)+2=18 Which is 18× 18 square matrix.

Second: *By proposition(4.20) , then $U(Q_{30})$*

$$U(Q_{30})=\begin{bmatrix}
2&2&2&1&2&2&2&1&1\\
0&2&0&1&0&2&0&1&1\\
0&0&2&1&0&0&2&1&1\\
0&0&0&1&0&0&0&1&1\\
2&2&2&1&0&0&0&1&1\\
0&2&0&1&0&0&0&1&0\\
0&0&2&1&0&0&0&1&0\\
0&0&0&1&0&0&0&1&0\\
1&1&1&1&0&0&0&0&0
\end{bmatrix}$$

Then by Theorem (5.1):

$$U(Q_{30}\times C_7)=\left[\begin{array}{c|c} U(Q_{30}) & U(Q_{30}) \\ \hline 0 & U(Q_{30}) \end{array}\right]=$$

$$\left[\begin{array}{ccccccccc|ccccccccc}
2&2&2&1&2&2&2&1&1&2&2&2&1&2&2&2&1&1\\
0&2&0&1&0&2&0&1&1&0&2&0&1&0&2&0&1&1\\
0&0&2&1&0&0&2&1&1&0&0&2&1&0&0&2&1&1\\
0&0&0&1&0&0&0&1&1&0&0&0&1&0&0&0&1&1\\
2&2&2&1&0&0&0&1&0&2&2&2&1&0&0&0&1&0\\
0&2&0&1&0&0&0&1&0&0&2&0&1&0&0&0&1&0\\
0&0&2&1&0&0&0&1&0&0&0&2&1&0&0&0&1&0\\
0&0&0&1&0&0&0&1&0&0&0&0&1&0&0&0&1&0\\
1&1&1&1&0&0&0&0&0&1&1&1&1&0&0&0&0&0\\
\hline
0&0&0&0&0&0&0&0&0&2&2&2&1&2&2&2&1&1\\
0&0&0&0&0&0&0&0&0&0&2&0&1&0&2&0&1&1\\
0&0&0&0&0&0&0&0&0&0&0&2&1&0&0&2&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&1&1\\
0&0&0&0&0&0&0&0&0&2&2&2&1&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&2&0&1&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&2&1&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0&0
\end{array}\right]$$

where 4(1+1).(1+1)+2=18 Which is 18× 18 square matrix.

Proposition (5.3)

If $m=r_1 . r_2$, $r_1,r_2 \neq 2$ and r_1,r_2 are primes number then the matrix

$B(Q_{2m}\times C_7)$ and the matrix $E(Q_{2m}\times C_7)$ of the group $Q_{2m}\times C_7$ are :

$$B(Q_{2m}\times C_7)=S\ .\left[\begin{array}{c|c} 0 & B(Q_{2m}) \\ \hline B(Q_{2m}) & -B(Q_{2m}) \end{array}\right]$$

18×18square matrix.

and

$$E(Q_{2m}\times C_7)=\left[\begin{array}{c|c} \mathbf{0} & E(Q_{2m}) \\ \hline E(Q_{2m}) & \mathbf{0} \end{array}\right].S$$

18×18square matrix.

$$\textit{Where}\quad S=\begin{bmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{bmatrix}$$

18×18 square matrix.

Proof:

Using theorem (5.1) we get in the form of $U(Q_{2m}\times C_7)$ and in the above model form $B(Q_{2m}\times C_7)$ and $E(Q_{2m}\times C_7)$ then:

$B(Q_{2m}\times C_7)\cdot U(Q_{2m}\times C_7)\cdot E(Q_{2m}\times C_7)=diag\{2,2,2,2,2,2,2,2,2,1,1,1,1,1,1,1,1,1\}=D(Q_{2m}\times C_7)$

Which is 18×18 square matrix.

Example(5.4)

To get the matrices $B(Q_{2.3.5}\times C_7)$ and $E(Q_{2.3.5}\times C_7)$ by proposition (4.21) to find $B(Q_{2.3.5})$ and $E(Q_{2.3.5})$:

$$B(Q_{2.3.5}) = \begin{bmatrix} 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E(Q_{2.3.5}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Then by proposition (5.3)

$$B(Q_{2.3.5}\times C_7) = S \;.\; \left[\begin{array}{c|c} 0 & B(Q_{2.3.5}) \\ \hline B(Q_{2.3.5}) & -B(Q_{2.3.5}) \end{array}\right] =$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 0 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$E(Q_{2.3.5}\times C_7) = \left[\begin{array}{c|c} 0 & E(Q_{2.3.5}) \\ \hline E(Q_{2.3.5}) & 0 \end{array}\right] . S =$$

$$
\begin{bmatrix}
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&-1&0&0&0&-1&-1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&1&0&0&0&1&1&0&0&0&0&0&0&0&1&-1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&-1&-1&-1&0&0&0&0&0&0&0&0&1&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&1&1&0&0&0&0&0&0&0&1&-1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0
\end{bmatrix}
$$

Example(5.5)

To find $D(Q_{2.3.5} \times C_7)$ and the cyclic decomposition of the factor group

$AC(D(Q_{2.3.5} \times C_7))$. by proposition (4.20)we get in the form of $U(Q_{2.3.5})$

$$
U(Q_{2.3.5}) = \begin{bmatrix}
2&2&2&1&2&2&2&1&1\\
0&2&0&1&0&2&0&1&1\\
0&0&2&1&0&0&2&1&1\\
0&0&0&1&0&0&0&1&1\\
2&2&2&1&0&0&0&1&0\\
0&2&0&1&0&0&0&1&0\\
0&0&2&1&0&0&0&1&0\\
0&0&0&1&0&0&0&1&0\\
1&1&1&1&0&0&0&0&0
\end{bmatrix}
$$

By theorem (5.1) we get in the form of $U(Q_{2.3.5} \times C_7)$

$$
U(Q_{2.3.5} \times C_7) = \left[\begin{array}{ccccccccc|ccccccccc}
2&2&2&1&2&2&2&1&1&2&2&2&1&2&2&2&1&1\\
0&2&0&1&0&2&0&1&1&0&2&0&1&0&2&0&1&1\\
0&0&2&1&0&0&2&1&1&0&0&2&1&0&0&2&1&1\\
0&0&0&1&0&0&0&1&1&0&0&0&1&0&0&0&1&1\\
2&2&2&1&0&0&0&1&0&2&2&2&1&0&0&0&1&0\\
0&2&0&1&0&0&0&1&0&0&2&0&1&0&0&0&1&0\\
0&0&2&1&0&0&0&1&0&0&0&2&1&0&0&0&1&0\\
0&0&0&1&0&0&0&1&0&0&0&0&1&0&0&0&1&0\\
1&1&1&1&0&0&0&0&0&1&1&1&1&0&0&0&0&0\\
\hline
0&0&0&0&0&0&0&0&0&2&2&2&1&2&2&2&1&1\\
0&0&0&0&0&0&0&0&0&0&2&0&1&0&2&0&1&1\\
0&0&0&0&0&0&0&0&0&0&0&2&1&0&0&2&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&1&1\\
0&0&0&0&0&0&0&0&0&2&2&2&1&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&2&0&1&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&2&1&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0&0
\end{array}\right]
$$

And We find the matrices $B(Q_{2.3.5}\times C_7)$ and $E(Q_{2.3.5}\times C_7)$ as in example (5.4),then

$B(Q_{2.3.5}\times C_7)\cdot U(Q_{2.3.5}\times C_7)\cdot E(Q_{2.3.5}\times C_7)=diag\{2,2,2,2,2,2,2,2,2,2,2,2,1,1,1,1,1,1\}=D(Q_{2.3.5}\times C_7)$

Then by theorem(4.12), we have $AC(D(Q_{2.3.5}\times C_7))=\bigoplus_{i=1}^{12} C_2$

Theorem(5.6):

If $m=r_1.r_2$, $r_1, r_2 > 2$ Where greatest common denominator $(r_1,r_2)=1$ and r_1, r_2 are primes numbers

$$AC(Q_{2m}\times C_7)=\bigoplus_{i=1}^{12} C_2$$

Proof:

By Theorem (5.1),we find matrix $U(Q_{2m}\times C_7)$ and by Proposition (5.3) we find matrixs $B(Q_{2m}\times C_7)$ and $E(Q_{2m}\times C_7)$:

$B(Q_{2m}\times C_7)\cdot U(Q_{2m}\times C_7)\cdot E(Q_{2m}\times C_7)=diag\{2,2,2,2,2,2,2,2,2,2,2,2,2,2,1,1,1,1\}$ Then by Theorem (4.12) we have:

$$AC(Q_{2r_1.r_2}\times C_7)=\bigoplus_{i=1}^{12} C_2$$

Example (5.12):

Let's take the two groups $Q_{66}\times c_7$ and $Q_{70}\times c_7$ then :

$$1-AC(Q_{66}\times c_7)=AC(Q_{2.3.11}\times c_7)=\bigoplus_{i=1}^{12} C_2$$

$$2-AC(Q_{70}\times c_7)=AC(Q_{2.5.7}\times c_7)=\bigoplus_{i=1}^{12} C_2$$

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