

Dynamic Modeling and Experimental Analysis of Solar Light Attenuation: A Didactic Approach

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Abstract :

Differential equations are fundamental tools for describing physical and engineering phenomena, yet their teaching is often dominated by symbolic procedures that can obscure the connection between a mathematical model and an observable process. This paper presents a didactic application of a first-order linear differential equation to model solar-light attenuation with water depth in an outdoor swimming pool. Experimental information was obtained in a private swimming pool in Mexico, using a digital lux meter with a remote sensor and bar-graph display. Measurements were taken at depths from 0 to 2.0 m in 0.25 m increments. Each reported experimental value is the average of 20 measurements collected at approximately six-minute intervals from 7:00 to 8:54 a.m. Illuminance was normalized with respect to the surface value. A simple classroom hypothesis of a 5% reduction for every 0.25 m of depth leads to the discrete model $R(z) = 0.95^{4z}$, which is equivalent to the continuous exponential model $R(z) = e^{-0.20517z}$ and to the first-order differential equation $dR/dz = -0.20517R$. Comparison with the experimental averages produced $R^2=0.9932$, $RMSE=0.00870$, and a mean relative error of approximately 1.11% for nonzero depths. Linearization of the experimental data yielded an effective attenuation coefficient $k_{exp} = 0.1961m^{-1}$. The activity connects physical measurement, exponential functions, differential equations, parameter interpretation, mathematical software, and model validation, providing an accessible case study for undergraduate differential-equations courses.

Keywords: Differential equations; mathematical modeling; light attenuation; swimming pool; illuminance; educational technology; engineering mathematics.

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I. INTRODUCTION

Differential equations are widely employed to represent physical, biological, economic, and engineering systems in which variables change according to their current state or other system variables. Within engineering education, first-order ordinary differential equations (ODEs) serve as a critical link between calculus and dynamic-system modeling. However, introductory courses often prioritize classification and analytical solution techniques over model construction, interpretation, and experimental validation. Recent educational research highlights mathematical modeling as an effective approach for linking ODE concepts to real-world contexts. Lozada et al. [1] introduced a modeling activity designed to facilitate the learning of ordinary differential equation concepts, while Spooner [2] described contextual modeling experiences for differential equations instruction. In engineering education, modeling units and authentic design cases have been implemented to enhance students' ability to translate real-world scenarios into mathematical representations [3]-[6].

Digital technologies further reinforce this connection by enabling students to transition among symbolic, numerical, graphical, and tabular representations. GeoGebra and similar platforms have been examined as resources for mathematics education [7]-[10], including applications in teaching linear ODEs [11]. Concurrently, low-cost or accessible optical instrumentation has been utilized in science education to link measurement with mathematical laws governing light attenuation [12]-[15]. In water is especially suitable for a didactic application because a simple proportional-loss assumption leads directly to an exponential solution and a first-order linear differential equation. The purpose of this paper is therefore to model the attenuation of solar illuminance with depth in swimming-pool water, compare the model with experimental averages obtained using a digital lux meter, and discuss the activity as a teaching resource for undergraduate differential-equations courses. The objective is not to construct a complete radiative-transfer model, but to demonstrate how a familiar physical phenomenon can connect measurement, mathematical abstraction, analytical solution, software visualization, and quantitative validation.

II. THEORETICAL BACKGROUND

A first-order ordinary differential equation can generally be written as indicated in Eq. (1)

$$\frac{dy}{dx} = f(x, y) \quad (1)$$

When the rate of change of a quantity is proportional to the amount currently present, a simple decay model is:

$$\frac{dy}{dx} = ky, \quad \forall k < 0 \quad (2)$$

For the present application, let $I(z)$ denote the measured illuminance at water depth z and let $I(0) = I_0$ denote the surface value. A simple attenuation assumption states that the infinitesimal loss in illuminance is proportional to the illuminance present and to the incremental distance traveled. Thus,

$$\frac{dI}{dz} = kI \quad (3)$$

Solving Eq. (3) leads to Eq. (4)

$$\begin{aligned} \frac{dI}{I} &= k dz \\ \int \frac{dI}{I} &= \int k dz \\ \ln(I) + \ln(c) &= kz \\ \ln(cI) &= kz \\ e^{\ln(cI)} &= e^{kz} \\ cI &= e^{kz} \\ I &= \frac{1}{c} e^{kz} \\ I(z) &= C e^{kz} \end{aligned} \quad (4)$$

Substituting the initial condition $I(0) = I_0$, then:

$$I_0 = C e^{k(0)} \Rightarrow C = I_0 \therefore I(z) = I_0 e^{kz} \quad (5)$$

Normalizing the measured value with respect to the surface value defines:

$$R(z) = \frac{I(z)}{I_0} = e^{kz} \quad (6)$$

The exponential structure is analogous to Beer-Lambert-type attenuation relationships. Recent work has revisited intuitive derivations of the Beer-Lambert law for classroom use [12], and simulation-based activities have been used to connect light absorption with mathematical models [13], [14]. However, a lux meter measures illuminance, a photometric quantity weighted according to human visual sensitivity. Consequently, the coefficient identified in this work is interpreted as an effective photometric attenuation coefficient under the experimental conditions rather than as a universal spectral absorption coefficient of pure water. Underwater illumination is wavelength dependent, and different spectral components can attenuate at different rates [15].

From the educational perspective, the activity follows a modeling cycle in which a physical phenomenon is observed, variables and assumptions are selected, a mathematical model is formulated, the model is solved, and its predictions are compared with measurements. Such modeling experiences are relevant in engineering

mathematics because they require students to interpret parameters and assumptions rather than treating equations only as symbolic exercises [3]-[6]. Digital mathematical environments can support this process through interactive visualization and comparison of representations [7]-[11], [16].

III. DEVELOPMENT

The experimental data were obtained in a private outdoor swimming pool located in a city in northern Veracruz, Mexico. A digital lux meter equipped with a remote sensor and graphical bar display was used to measure solar illuminance. The study uses nine water depths:

$$z = [0, 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00] \quad (7)$$

For each depth, the value used in this article is the average of 20 measurements. The first measurements corresponding to the different depths were obtained at approximately 7:00 a.m.; subsequent measurements were taken at intervals of about six minutes, and the final measurements were obtained at approximately 8:54 a.m. The present paper deliberately works with the average values only because its principal objective is didactic: to contrast a simple first-order model with experimental information rather than to characterize the temporal variation of solar irradiance.

To eliminate the absolute scale of the lux-meter readings and focus on attenuation with depth, the experimental values were normalized as:

$$R_{exp}(z) = \frac{I_{exp}(z)}{I_{exp}(0)} \quad (8)$$

An intuitive classroom hypothesis was then introduced: relative illumination decreases by approximately 5% for every 0.25 m increase in water depth. In discrete form,

$$R(z + 0.25) = 0.95R(z) \quad (9)$$

After n increments,

$$R_n = 0.95^n = 0.95^{\frac{z}{0.25}} = 0.95^{4z} \quad (10)$$

Ec. (10), can also be written as:

$$R(z) = e^{\ln(0.95^{4z})} = e^{4z \ln(0.95)} \approx e^{-0.20517z} \quad (11)$$

Thus, the discrete percentage-loss hypothesis is equivalent to the continuous differential equation:

$$\frac{dR}{dz} = -0.20517 \quad (12)$$

The calculated values were compared with the experimental averages. The absolute and percentage relative errors were evaluated using:

$$E_a = |R_{exp} - R_{model}| \quad (13)$$

$$E_r(\%) = \frac{|R_{exp} - R_{model}|}{R_{exp}} \times 100 \quad (14)$$

Overall model performance was characterized through the root-mean-square error and coefficient of determination:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (R_{exp}(i) - R_{model}(i))^2} \quad (15)$$

Mathematical software was used to calculate the curves, errors, goodness-of-fit indicators, and graphical representations. This computational stage is intended as an educational complement to, rather than a replacement for, the analytical derivation.

IV. RESULTS

Table 1 compares the normalized experimental averages with the values calculated from the 5%-per-0.25m model.

Table 1. Experimental and calculated relative illuminance

Depth z (m)	Experimental R(z)	Calculated R(z)	Absolute error	Relative error (%)
0.00	1.000	1.00000	0.00000	0.000
0.25	0.950	0.95000	0.00000	0.000
0.50	0.912	0.90250	0.00950	1.042
0.75	0.867	0.85737	0.00963	1.111
1.00	0.825	0.81451	0.01049	1.272
1.25	0.784	0.77378	0.01022	1.304
1.50	0.745	0.73509	0.00991	1.330
1.75	0.708	0.69834	0.00966	1.364
2.00	0.673	0.66342	0.00958	1.423

The experimental results show a monotonic decrease in normalized illuminance with increasing depth. At 0.50 m, 91.2% of the normalized surface value remained; at 1.00 m, the value was 82.5%; and at 2.00 m, the measured relative illuminance was 0.673. The didactic model predicts $R(2) = 0.66342$, corresponding to an absolute difference of only 0.00958 at the greatest depth considered. For the complete set of nine averaged observations, the didactic model produced:

$$RMSE = 0.00870, \quad R^2 = 0.9932 \quad (16)$$

and the mean percentage relative error for the eight nonzero depths was approximately:

$$mean(E_r) = 1.11\% \quad (17)$$

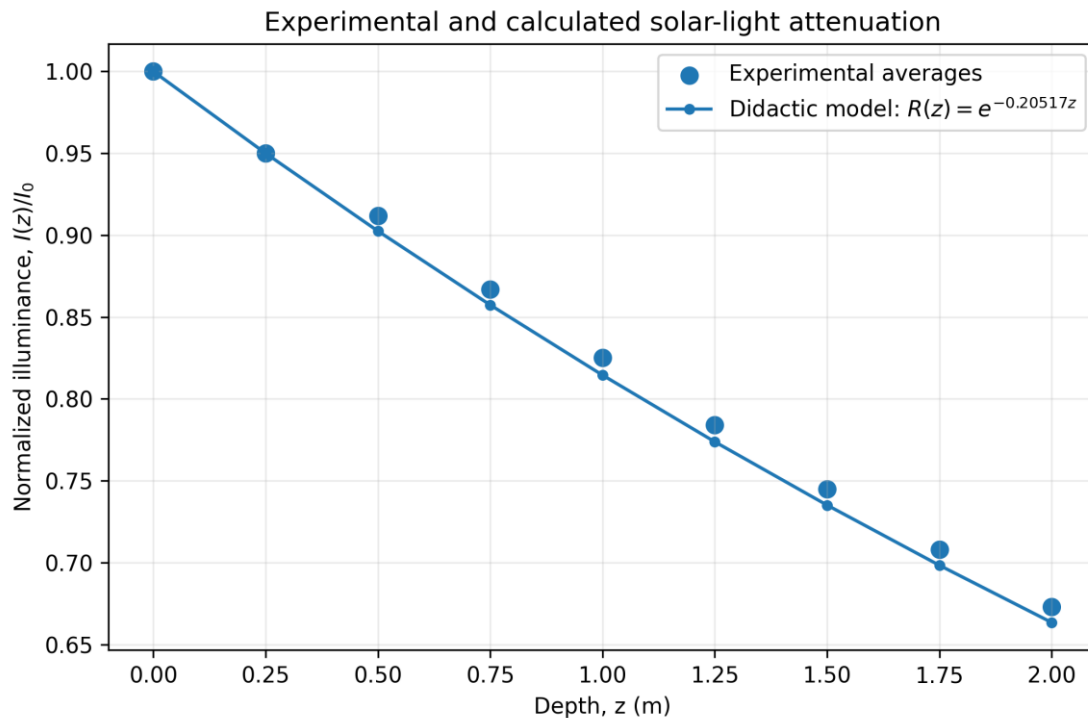


Figure 1. Comparison between normalized experimental illuminance averages and the first order exponential attenuation model.

Figure 1 shows that the calculated exponential curve closely follows the experimental averages throughout the 0-2 m interval. The small and systematic separation observed after 0.50 m indicates that the 5% discrete-loss assumption slightly overestimates attenuation, but the discrepancy remains small for the intended classroom application. Linearization of the experimental averages provides an independent estimate of the effective attenuation coefficient. Fitting $\ln[R(z)]$ versus z while imposing $\ln[R(0)] = 0$ yields:

$$k_{exp} = 0.1961 \text{ m}^{-1} \quad (18)$$

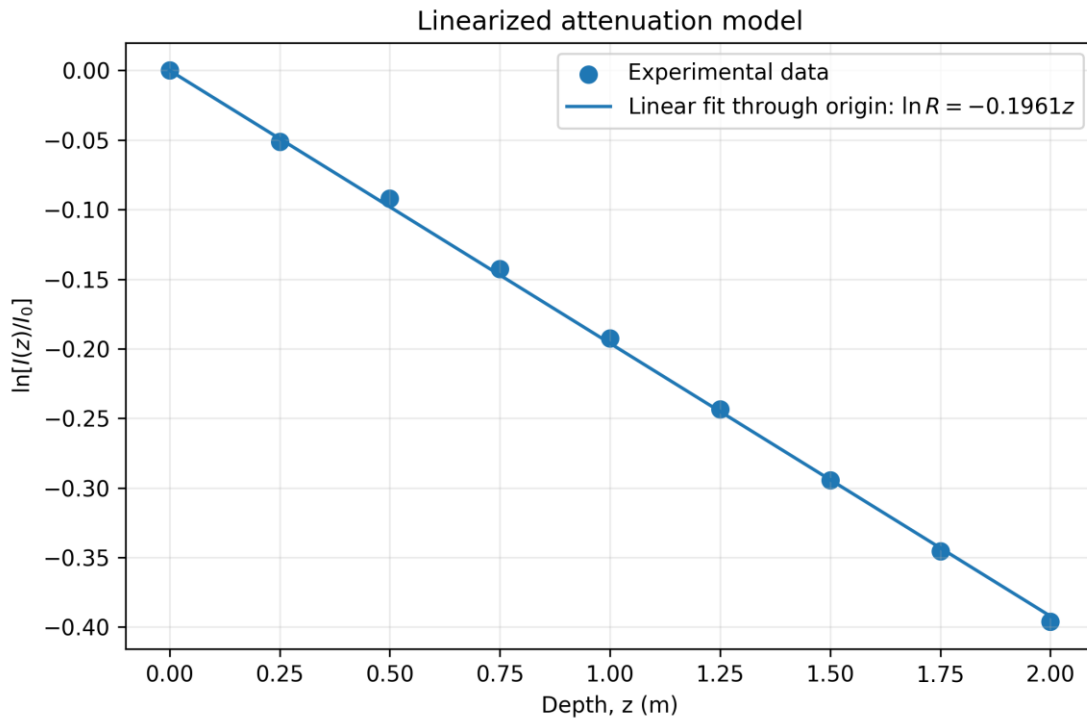


Figure 2. Linearized representation of the experimental attenuation data.

The coefficient obtained from the experimental averages is close to the coefficient implied by the didactic hypothesis, $k_{didactic} = 0.2052 \text{ m}^{-1}$. Their difference is approximately 4.6% relative to k_{exp} . Thus, the simple assumption introduced before the continuous derivation leads to a parameter of the same magnitude as that inferred from the measurements.

V. FINDINGS AND COMMENTS

The primary mathematical result demonstrates that the measured attenuation, within the examined depth range, is accurately described by a first-order proportional-decay model. The differential equation $dI/dz = kI$ possesses a clear physical interpretation: at each depth, the local rate of decrease is proportional to the remaining illuminance. Similarly, $(1/I)(dI/dz) = k$ indicates that the fractional attenuation rate per unit depth remains constant under the model's assumptions.

The close correspondence between $k_{didactic} = 0.2052 \text{ m}^{-1}$ and $k_{exp} = 0.1961 \text{ m}^{-1}$ offers significant pedagogical value. Students may start with a repeated-percentage rule, reformulate it as an exponential function, identify the corresponding differential equation, and utilize experimental data to evaluate the validity of the underlying assumption. This process integrates sequences, exponential and logarithmic functions, derivatives, separation of variables, initial conditions, regression, error analysis, and parameter interpretation within a single accessible problem.

The exponential model aligns with the mathematical structure of Beer-Lambert-type attenuation [12]-[14]. However, this experiment should not be regarded as a spectroscopic determination of water's intrinsic absorption coefficient. The swimming pool environment introduces multiple optical factors, including absorption, scattering, reflection at the air-water interface, solar incidence angle, wall and bottom reflections, water composition, and sensor response, all of which may influence the measurement. Additionally, a lux meter measures photometrically weighted illuminance rather than spectral irradiance. Underwater-lighting studies

indicate that attenuation is highly wavelength-dependent [15]. Therefore, in this context, k is most appropriately described as an effective illuminance attenuation coefficient specific to the experimental setup.

This limitation holds educational value. Mathematical models serve as simplified representations rather than exact replications of reality, and their utility depends on the underlying assumptions, variables, instrumentation, and applicable range. A high R^2 value does not indicate that all optical mechanisms have been captured. Rather, it suggests that the chosen first-order model offers a suitable approximation to the averaged data within the studied interval.

Recent research on contextual mathematical modeling further supports this didactic approach. Engineering students benefit from engaging with authentic mathematical problems and interpreting the resulting parameters and representations [3]-[6]. Digital tools such as MATLAB or GeoGebra facilitate visualization and transitions among representations [7]-[11], and recent studies highlight the positive impact of digital tasks in engineering mathematics [16]. In this activity, the lux meter serves as measurement technology, while mathematical software provides visualization and analysis capabilities.

A recommended classroom sequence includes: (1) observing and discussing the physical phenomenon; (2) providing or reproducing normalized measurements; (3) formulating a percentage-loss hypothesis; (4) constructing the discrete model; (5) converting it to exponential form; (6) deriving and solving the corresponding first-order ordinary differential equation; (7) plotting experimental and theoretical values; (8) calculating errors and goodness-of-fit indicators; (9) linearizing the experimental data to estimate k ; and (10) discussing the physical and mathematical limitations of the model. This sequence prioritizes modeling and interpretation without necessitating advanced instrumentation.

The use of accessible instrumentation aligns with educational spectroscopy studies where students construct or utilize simple devices to make absorption and attenuation both visible and quantifiable [17], [18]. Similarly, simulation-based research demonstrates that visual-conceptual tools assist students in connecting macroscopic observations with symbolic absorption models [13], [14]. While these studies do not directly validate the current swimming-pool experiment, they offer pedagogical support for integrating measurement, visualization, and mathematical modeling.

This study does not claim a measured improvement in student learning, as no educational intervention or pretest-posttest design was implemented. Its primary contribution is the presentation of a didactic case and a reproducible mathematical application. Future classroom research could assess learning outcomes associated with this activity, as demonstrated in studies employing GeoGebra for linear ordinary differential equations [11] and in broader technology-supported mathematics education [7]-[10], [19], [20].

VI. CONCLUSIONS

This study demonstrates a didactic application of a first-order linear differential equation to model solar-light attenuation with increasing depth in swimming-pool water. Experimental averages, measured using a digital lux meter at depths ranging from 0 to 2.0 meters, were compared with a simple exponential attenuation model.

The primary contribution of this work is educational rather than optical. It connects a familiar physical phenomenon with measurement, proportional rates of change, exponential functions, initial-value problems, first-order differential equations, mathematical software, and quantitative model validation. This activity also demonstrates a key principle of applied mathematics: a model can be effective if its assumptions and limitations are clearly understood.

Integrating a digital lux meter with mathematical software offers an accessible example of educational technology that links physical experimentation with mathematical abstraction. This case study can be incorporated into undergraduate differential equations courses, particularly within engineering and applied science programs, to encourage students to progress from the question 'How is this equation solved?' to broader inquiries such as 'Where does the equation originate?', 'What does its parameter represent?', and 'How accurately does its solution reflect measured reality?'

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