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Contraction Type Mapping on 2-Metric Spaces

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Abstract: *The superimposition of infinite number of intervals $[a_1, b_1]$, $[a_2, b_2]$, $[a_3, b_3]$,......., $[a_n, b_n]$ follows two laws of randomness if*

(a) $a_i \neq a_j$; $i,j= 1, 2,......, n$,
(b) $b_i \neq b_j$; $i,j= 1, 2,......, n$,
(c) $\max(a_i) \leq \min(b_i)$; $i= 1, 2,..., n$, where $n \to \infty$

Keywords: *Superimposition of sets, Probability distribution function, Glivenko – Cantelli Lemma.*

I. Introduction

Construction of normal fuzzy number has been discussed in ([1], [2]) based on the randomness – fuzziness consistency principle deduced by Baruah ([3], [4], [5]). Based on this aforesaid principle by including two more conditions which are not mentioned by Baruah, we have shown that if we superimpose infinite number of intervals $[a_1, b_1]$, $[a_2, b_2]$, $[a_3, b_3]$,......., $[a_n, b_n]$, then the values $a_{(1)}, a_{(2)}, ... , a_{(n)}$ follows an uniform probability distribution function and the values $b_{(1)}, b_{(2)},.... , b_{(n)}$ follows an another complementary uniform probability distribution function where $a_{(1)}, a_{(2)}, ... , a_{(n)}$ and $b_{(1)}, b_{(2)}, ... , b_{(n)}$ are arranged in increasing order of magnitude of $a_1, a_2, a_3, ..., a_n$ and $b_1, b_2, b_3, ..., b_n$ respectively. If $\alpha = \min(a_i)$, $\beta = \max(a_i)$, $\mu = \min(b_i)$, $\gamma = \max(b_i)$, by satisfying the condition $a_i \neq a_j$, $b_i \neq b_j$ and $\max(a_i) < \min(b_i)$; $i,j= 1, 2,......, n$, then we can define the function $\psi(x)$ as

$$\begin{aligned}\psi(x) &= \psi_1(x) && \text{if } \alpha \leq x \leq \beta,\\ &= 1-\psi_2(x) && \text{if } \mu \leq x \leq \gamma,\\ &= 1 && \text{if } \beta \leq x \leq \mu,\\ &= 0 && \text{otherwise}.\end{aligned}$$

Where $\Psi_1(x)$ being a continuous distribution function in the interval $[\alpha, \beta]$, and $(1- \Psi_2(x))$ being a continuous distribution function in the interval $[\mu, \gamma]$, with $\Psi_1(\alpha) = \Psi_2(\gamma) = 0$ and $\Psi_1(\beta) = \Psi_2(\mu) = 1$.

II. The Operation Of Set Superimposition

The operation of set superimposition of two real intervals $[a_1, b_1]$ and $[a_2, b_2]$ as

$$[a_1, b_1](S)[a_2, b_2] = [a_{(1)}, a_{(2)}] \cup [a_{(2)}, b_{(1)}]^{(2)} \cup [b_{(1)}, b_{(2)}]$$

Where $a_{(1)} = \min(a_1, a_2), a_{(2)} = \max(a_1, a_2), b_{(1)} = \min(b_1, b_2)$ and $b_{(2)} = \max(b_1, b_2)$.Here we have assumed without any loss of generality that $a_1 \neq a_2$, $b_1 \neq b_2$ and $[a_1, b_1] \cap [a_2, b_2]$ is not void or in other words that $\max(a_i) < \min(b_i), i = 1,2$

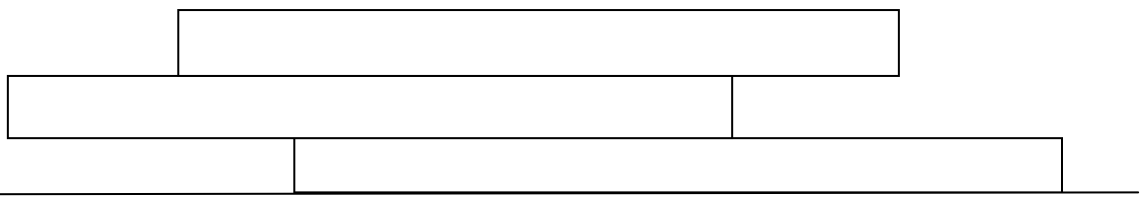

Figure1: Superimposition of $[x_1, y_1]^{\left(\frac{1}{3}\right)}, [x_2, y_2]^{\left(\frac{1}{3}\right)}$ and $[x_3, y_3]^{\left(\frac{1}{3}\right)}$

Figure2: Cumulative and complementary cumulative distribution functions

For the three intervals $[x_1, y_1]^{(1/3)}, [x_2, y_2]^{(1/3)} \, and \, [x_3, y_3]^{(1/3)}$ all with elements with a constant probability equal to 1/3 everywhere, we shall have

$$[x_1, y_1]^{(1/3)}(S)[x_2 y_2]^{(1/3)}(S)[x_3, y_3]^{(1/3)} = [x_{(1)}, x_{(2)}]^{(1/3)} \cup [x_{(2)}, x_{(3)}]^{(2/3)} \cup [x_{(3)}, y_{(1)}]^{(1)} \cup [y_{(1)}, y_{(2)}]^{(2/3)} \cup [y_{(2)}, y_{(3)}]^{(1/3)}$$

where, for example $[y_{(1)}, y_{(2)}]^{(2/3)}$ represents the interval $[y_{(1)}, y_{(2)}]$ with probability 2/3 for all elements in the entire interval, $x_{(1)}, x_{(2)}, x_{(3)}$ being values of x_1, x_2, x_3 arranged in increasing order of magnitude, and similarly $y_{(1)}, y_{(2)}, y_{(3)}$ being values of y_1, y_2, y_3 arranged in increasing order of magnitude again. We here presumed that $[x_1, y_1] \cap [x_2, y_2] \cap [x_3, y_3]$ is not void and $x_1 \neq x_2 \neq x_3$ and $y_1 \neq y_2 \neq y_3$.

It can be seen that for n intervals $[a_1, b_1]^{1/n}$, $[a_2, b_2]^{1/n}$, ..., , $[a_n, b_n]^{1/n}$ all with probability equal to 1/n everywhere, we shall have

$$[a_1, b_1]^{\frac{1}{n}}(S)[a_2, b_2]^{\frac{1}{n}}(S).........[a_n, b_n]^{\frac{1}{n}} = [a_{(1)}, a_{(2)}]^{\frac{1}{n}} \cup [a_{(2)}, a_{(3)}]^{\frac{2}{n}} \cup .... \cup [a_{(n-1)}, a_{(n)}]^{\frac{n-1}{n}}$$

$$\cup [a_{(n)}, b_{(1)}]^{(1)} \cup [b_{(1)}, b_{(2)}]^{\frac{n-1}{n}} \cup .... \cup [b_{(n-2)}, b_{(n-1)}]^{\frac{2}{n}} \cup [b_{(n-1)}, b_{(n)}]^{\frac{1}{n}},$$

Where, for example, $[b_{(1)}, b_{(2)}]^{\frac{n-1}{n}}$ represents the interval $[b_{(1)}, b_{(2)}]$ with probability $\frac{n-1}{n}$ in the entire interval, $a_{(1)}$, $a_{(2)}$,......., $a_{(n)}$ being values of a_1, a_2,.......a_n arranged in increasing order of magnitude, and $b_{(1)}$, $b_{(2)}$,......., $b_{(n)}$ being values of b_1, b_2,.......b_n arranged in increasing order of magnitude. Thus for the intervals $[a_1, b_1]^{\frac{1}{n}}, [a_2, b_2]^{\frac{1}{n}}, ...[a_n, b_n]^{\frac{1}{n}}$, all with uniform probability $\frac{1}{n}$, the probabilities of the superimposed intervals are $\frac{1}{n}, \frac{2}{n}, ..., \frac{n-1}{n}, 1, \frac{n-1}{n}, ..., \frac{2}{n}$ and $\frac{1}{n}$. These probabilities considered in two halves as

$$\left(0, \frac{1}{n}, \frac{2}{n}, ..., \frac{n-1}{n}, 1\right)$$

and

$$\left(1, \frac{n-1}{n}, ..., \frac{2}{n}, \frac{1}{n}, 0\right)$$

would suggest that they can define an empirical distribution and a complementary empirical distribution on $a_1, a_2, ..., a_n$ and $b_1, b_2, ..., b_n$ respectively. In other words, for realizations of the values of $a_{(1)}$, $a_{(2)}$,......., $a_{(n)}$ in increasing order and of $b_{(1)}$, $b_{(2)}$,......., $b_{(n)}$ again in increasing order, we can see that if we define

$$\psi_1(x) = \begin{cases} 0 & if \ x < a_{(1)} \\ \frac{r-1}{n} & if \ a_{(r-1)} \leq x \leq a_{(r)}, r = 2, 3, ..., n \\ 1 & if \ x \geq a_{(n)}, \end{cases}$$

$$\psi_2(x)=\begin{cases}1 & \text{if } x<b_{(1)}\\ 1-\dfrac{r-1}{n} & \text{if } b_{(r-1)}\le x\le b_{(r)},\ r=2,3,\ldots,n\\ 0 & \text{if } x\ge b_{(n)},\end{cases}$$

Then the Glivenko – Cantelli Lemma on Order Statistics assures that

$$\psi_1(x)\rightarrow\prod{}_1[\alpha,x],\qquad \alpha\le x\le\beta,$$
$$\psi_2(x)\rightarrow 1-\prod{}_2[\mu,x],\quad \mu\le x\le\gamma,$$

where $\prod_1[\alpha,x],\ \alpha\le x\le\beta$ and $\psi_2(x),\ \mu\le x\le\gamma$ are two probability distributions. Here in this case we have considered that max(a_i) □ min(b_i), but for large number of observation when max(a_i) = min(b_i) that is β = μ then $a_{(n)} = b_{(1)}$ and we can write

$$\psi_1(x)\rightarrow\prod{}_1[\alpha,x],\qquad \alpha\le x\le\beta,$$
$$\psi_2(x)\rightarrow 1-\prod{}_2[\beta,x],\quad \beta\le x\le\gamma,$$

where $\prod_1[\alpha,x],\ \alpha\le x\le\beta$ and $\psi_2(x),\ \beta\le x\le\gamma$ are two probability distributions.

III. CONCLUSION

The superimposition of an infinite number of intervals [a_1, b_1], [a_2, b_2], [a_3, b_3],......., [a_n, b_n] by satisfying the conditions $a_i \neq a_j$, $b_i \neq b_j$ and max(a_i) ≤ min(b_i); i,j= 1, 2,......, n, follows two laws of randomness, one of which is $a_{(1)}$, $a_{(2)}$, ... , $a_{(n)}$ follows an uniform probability distribution function and the other one is $b_{(1)}$, $b_{(2)}$,.... , $b_{(n)}$ follows an another complementary uniform probability distribution function where $a_{(1)}$, $a_{(2)}$, ... , $a_{(n)}$ and $b_{(1)}$, $b_{(2)}$, ... , $b_{(n)}$ are arranged in increasing order of magnitude of a_1, a_2, a_3, ..., a_n and b_1, b_2, b_3, ..., b_n respectively. If α = min(a_i), β = max(a_i), μ = min(b_i), γ = max(b_i) and max(a_i) □ min(b_i), then we can define the function $\psi(x)$ as

$$\begin{aligned}\psi(x)&=\psi_1(x) && \text{if } \alpha\le x\le\beta,\\ &=1-\psi_2(x) && \text{if } \mu\le x\le\gamma,\\ &=1 && \text{if } \beta\le x\le\mu,\\ &=0 && \text{otherwise}.\end{aligned}$$

Where Ψ_1(x) being a continuous distribution function in the interval [α, β], and (1- Ψ_2(x)) being a continuous distribution function in the interval [μ, γ], with Ψ_1 (α) = Ψ_2 (γ) = 0 and Ψ_1 (β) = Ψ_2 (μ) = 1.

Again if max(a_i) = min(b_i), then β = μ and we can define the function $\psi(x)$ as

$$\begin{aligned}\psi(x)&=\psi_1(x) && \text{if } \alpha\le x\le\beta,\\ &=1-\psi_2(x) && \text{if } \beta\le x\le\gamma,\\ &=0 && \text{otherwise}.\end{aligned}$$

Where Ψ_1(x) being a continuous distribution function in the interval [α, β], and (1- Ψ_2(x)) being a continuous distribution function in the interval [β, γ], with Ψ_1 (α) = Ψ_2 (γ) = 0 and Ψ_1 (β) = Ψ_2 (β) = 1.

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