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Stochastic Behavior of Standby System with Two Types of Workload and Three Types of Repair

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Abstract: In the present paper, we analyses a tow-identical unit deteriorating standby system model having two types of workload i.e. normal and fast. The system works under varying workload. When workload is more the standby also starts operation and the system becomes parallel until the workload decreases. The unit may be repairable under normal and fast repair mode. Using regenerative point's technique different measures of reliability are obtained.

Key words: Deteriorating, Reliability, Standby, Repair.

I. Introduction

Gupta and Sammerwar (1) have studied a standby system model with varying rates of failure, repair, inspection and post repair. Recently Gupta and Deshmukh (2) have analyzed a two unit redundant reparable system model with varying workload. Generally, it is observed that repairs are not perfect i.e. after repair the unit may not be as good as new and some post repair is required.

In the present paper, we analyses a tow-identical unit deteriorating standby system model having two types of workload i.e. normal and fast. The system works under varying workload. When workload is more the standby also starts operation and the system becomes parallel until the workload decreases. The unit may be repairable under normal and fast repair mode. As the workload increases the fast repair facility is called for fast repair and the time spent in earlier repair go waste. The fast repair facility is very fast as compared to normal repair. When a unit completes normal repair, it goes under post repair. Selecting suitable regeneration time points with Markov renewal process, several important measures of reliability are obtained i.e.

- Steady state transition probabilities and Mean Sojourns times.
- Mean time to System Failure (***MTSF***)
- Point wise and steady state availability of the system.
- Probability that the repairman is busy.
- Expected number of visits by the repairman.
- Expected profit earned by the system.

II. System Description And Assumptions

(a) There is a two-identical –unit deteriorating standby system. Initially one unit is operative and other is kept as standby.
(b) When workload increases the standby unit becomes operative and vice-versa.
(c) Failure time distributions of operative and standby units are exponential with different parameters.
(d) If a unit fails during standby state, it requires minor repair. After minor repair or fast repair there is no need of post repair.
(e) There are two-repair facilities normal and fast. The normal repair of the failed unit starts instantaneously upon failure. Repair time distributions are arbitrary, i.e. time-dependent. Repair policy first come first served.
(f) Post repair of the just repaired unit starts immediately (within no time) and after the post repair the unit becomes as good as new.
(g) When workload increases during post repair the fast repair facility is called.
(h) Once the fast repair facility is called it is allowed to return only when the system is completely repaired.

III. Notations And States Of The System

α , β	: Constant failure rates of operative, standby unit
θ , η	: Constant rates of increase, decrease in workload.
f (.)	: *pdf* of repair time of the unit failed during operation.
g (.)	: *pdf* of post repair time of the just repaired unit.
k(.), h(.)	: *pdf* of repair/ fast repair time of unit failed from standby state.

The system can be in any one of the following states:

$S_0 \equiv (O,S)$: One unit is operative and kept as standby,

$S_1 \equiv (O,O)$: Both the units operating parallely,

$S_2 \equiv (O,F_r)$: One unit is operative and other failed under repair,

$S_3 \equiv (O,F_{sr})$: One unit is operative and other failed during standby state,

$S_4 \equiv (O,F_{pr})$: One unit is operative and other unit is under post repair,

$S_5 \equiv (O,F_{fr})$: One unit is operative and other unit is under fast repair,

$S_6 \equiv (F,F_{fR})$: Both the units are failed and the fast repair continued from earlier state,

$S_7 \equiv (F,F_{pr})$: Both the units are failed and one is under post repair.

Regenerative states: S_0 *to* S_5 ; Non-regenerative: S_6 , S_7

The epochs at which the system enters the states S_0 *to* S_5 are regenerative points and the entrance time instants at S_6 , S_7 are non-regenerative. Let *E* be the set of regenerative states $\{S_1, S_2, \ldots, S_5\}$. The transition between the possible states is shown by figure 1.

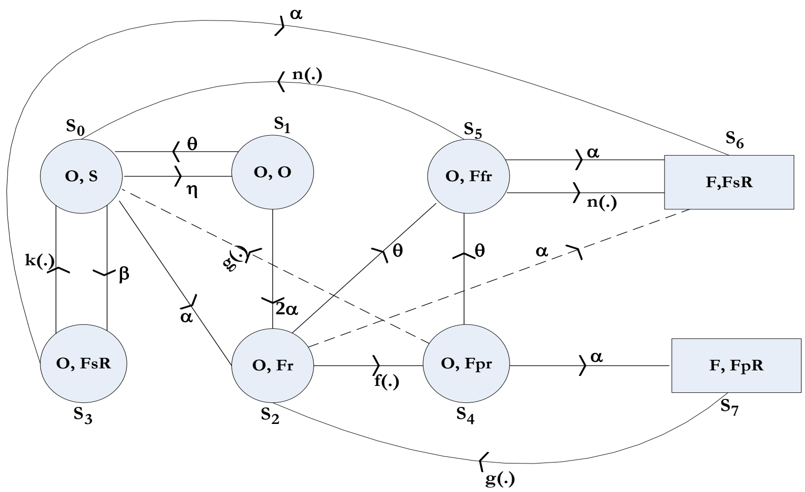

Fig. 1

IV. Transition Probabilities And Mean Sojourn Times

Let $T_0, T_1, \ldots, T_n$ be the epochs at which the system enters any state $S_j \in E$. Let X_n denotes the state visited at epoch T_n . Then $\{X_n, T_n\}$ is a Markov-renewal process with state-space *E* and

$$Q_{ij}(t) = P\left[X_{n+1} = j, T_{n+1} - T_n \le t \mid X_n = i\right]$$

Is the semi-Markov kernel over *E*. The (***i,j***)th element of the transition probability matrix of the embedded Markov chain is

$$P_{ij} = Q_{ij}(\infty) = \lim Q_{ij}(t);$$ (limit ***t*** tends to ***infinity***)

Expressed as integrals by simple probabilistic consideration the non-zero elements of $Q_{ij}(t)$ are:

$$Q_{01}(t) = \theta \int_0^t e^{-(\alpha+\beta+\theta)u} du = \theta\left[1 - e^{-(\alpha+\beta+\theta)t}\right] / (\alpha+\beta+\theta),$$

$$Q_{02}(t) = \alpha\left[1 - e^{-(\alpha+\beta+\theta)t}\right] / (\alpha+\beta+\theta), \quad Q_{03}(t) = \beta\left[1 - e^{-(\alpha+\beta+\theta)t}\right] / (\alpha+\beta+\theta),$$

$$Q_{10}(t) = \eta\left[1 - e^{-(2\alpha+\eta)t}\right] / (2\alpha+\eta), \quad Q_{12}(t) = 2\alpha\left[1 - e^{-(2\alpha+\eta)t}\right] / (2\alpha+\eta),$$

$$Q_{25}(t) = \int_0^t \theta\, e^{-(\alpha+\theta)u}\overline{F}(u)\,du = Q_{25}^{(6)}(t),$$

$$Q_{30}(t) = \int_0^t e^{-\alpha u}\, dK(u)\,, \qquad Q_{36}(t) = \int_0^t \alpha\, e^{-\alpha u}\,\overline{K}(u)\,du \qquad Q_{40}(t) = \int_0^t e^{-(\alpha+\theta)u}\, dG(u),$$

$$Q_{47}(t) = \int_0^t \alpha\, e^{-(\alpha+\theta)u}\,\overline{G}(u)\,du,$$

$$Q_{50}(t) = \int_0^t e^{-\alpha u}\, dH(u)\,, \quad Q_{56}(t) = \int_0^t \alpha\, e^{-\alpha u}\,\overline{H}(u)\,du = Q_{55}^{6}(t),$$

$$Q_{65}(t) = \int_0^t h(u)\,du\,, \qquad Q_{72}(t) = \int_0^t g(u)\,du\,.$$

To write expression for $Q_{35}^{6}(t)$, we let that the system passes from S_3 to S_6 during $(u,u+du)$, $u<t$. Further we assume that the system passes from S_6 to S_5 during $(v, v+du)$,(u,t). Thus the probability of this contingency is,

$$Q_{35}^{6}(t) = \int_0^t \alpha\, e^{-\alpha u}\,\overline{K}(u)\,du \quad ; \qquad Q_{42}^{7}(t) = \int_0^t \alpha\, e^{-(\alpha+\theta)u}\,\overline{G}(u)\,du\,.$$

Since, $\lim_{t\to\infty} Q_{ij}(t) = p_{ij}$, the non-zero transition probabilities p_{ij} are given below:

$$p_{01} = \theta/(\alpha+\beta+\theta), \qquad p_{03} = \beta/(\alpha+\beta+\theta), \quad p_{02} = \alpha/(\alpha+\beta+\theta)$$

$$p_{10} = \eta/(2\alpha+\eta), \qquad p_{10} = 2\alpha/(2\alpha+\eta), \quad p_{24} = \tilde{F}(\alpha+\theta),$$

$$p_{25} = \theta\left[1-\tilde{F}(\alpha+\theta)\right]/(\alpha+\theta) = p_{25}^{6}, \qquad p_{26} = \alpha\left[1-\tilde{F}(\alpha+\theta)\right]/(\alpha+\theta),$$

$$p_{30} = \tilde{K}(\alpha, \qquad p_{36} = \left[1-\tilde{K}(\alpha\right], \qquad p_{45} = \theta\left[1-\tilde{G}(\alpha+\theta)\right]/(\alpha+\theta),$$

$$p_{47} = \alpha\left[1-\tilde{G}(\alpha+\theta)\right]/(\alpha+\theta), \qquad p_{50} = \tilde{H}(\alpha), \quad p_{56} = \left[1-\tilde{H}(\alpha\right] = p_{55}^{(6)},$$

$$p_{65} = p_{72} = 1.$$

The two-step steady state transition probabilities obtained are as follows:

$$p_{35}^{(6)} = \left[1-\tilde{K}(\alpha)\right] = p_{36}, \qquad p_{42}^{(7)} = \alpha\left[1-\tilde{G}(\alpha+\theta)\right]/(\alpha+\theta) = p_{47}$$

It can be easily verified that,

$$p_{01} + p_{02} + p_{03} = p_{10} + p_{12} = p_{25} + p_{24} + p_{26} = p_{30} + p_{35}^{(6)} = p_{40} + p_{45} + p_{42}^{(7)} = 1$$

$$p_{50} + p_{56} = p_{65} = p_{72} = 1.$$

Mean sojourn time μ_i in state S_i is defined as the time that the system continues in state S_i before transiting to any other state. If T denotes the sojourn time in S_i then μ_i in state S_i is,

$$\mu_i = E(T) = \int_0^\infty P(T > t)\,dt\,.$$

Using this we can obtain the following expressions for μ_i ; $i = 0, 1, 2, \ldots, 5$.

$$\mu_0 = 1/(\alpha+\beta+\theta), \quad \mu_1 = 1/(2\alpha+\eta), \qquad \mu_2 = \int_0^\infty e^{-(\alpha+\theta)t}\overline{F}(t)\,dt,$$

$$\mu_3 = \int_0^\infty e^{-\alpha t}\,\overline{K}(t)\,dt, \quad \mu_4 = \int_0^\infty e^{-(\alpha+\theta)t}\overline{G}(t)\,dt, \quad \mu_5 = \int_0^\infty e^{-\alpha t}\,\overline{H}(t)\,dt\,.$$

m_{ij} is defined as the contribution to sojourn time in state S_i before transiting to state S_j . To calculate m_{ij} , we note that, mathematically,

$$m_{ij} = -\lim_{s\to 0} \frac{d}{ds}\tilde{Q}_{ij}(s) = -\tilde{Q}'_{ij}(0)$$

Taking Laplace-Stieltjies transform of $Q_{ij}(t)$, we obtain,

$m_{01} = \theta/(\alpha+\beta+\theta)^2$, $\quad m_{03} = \beta/(\alpha+\beta+\theta)^2$, $\quad m_{02} = \alpha/(\alpha+\beta+\theta)^2$.

$m_{10} = \eta/(2\alpha+\eta)^2$, $\quad m_{10} = 2\alpha/(2\alpha+\eta)^2$,

$m_{24} = \int_0^\infty t\, e^{-(\alpha+\theta)u}\, dF(t)$, $m_{26} = \int_0^\infty t\, \alpha\, e^{-(\alpha+\theta)u}\, \overline{F}(t)\, dt$, $\quad m_{25} = \int_0^\infty t\, \theta\, e^{-(\alpha+\theta)u}\, \overline{F}(t)\, dt$,

$m_{30} = \int_0^\infty t\, e^{-\alpha u}\, dK(u)$, $\; m_{36} = \int_0^\infty t\, \alpha\, e^{-\alpha u}\, \overline{K}(u)\, du \quad m_{40} = \int_0^\infty t\, e^{-(\alpha+\theta)u}\, dG(u)$,

$m_{47} = \int_0^\infty t\, \alpha\, e^{-(\alpha+\theta)u}\, \overline{G}(u)\, du$, $\quad m_{30} = \int_0^\infty t\, e^{-\alpha u}\, dK(u)$, $m_{56} = \int_0^\infty t\, \alpha\, e^{-\alpha u}\, \overline{H}(u)\, du$,

$m_{65} = \int_0^\infty t\, h(t)\, dt$, $\quad m_{72} = \int_0^\infty t\; g(t)\, dt = m_2$ (*say*).

It can easily be seen that,

$$m_{01} + m_{02} + m_{03} = \mu_0,\ m_{10} + m_{12} = \mu_1,\ m_{25} + m_{24} + m_{26} = \mu_2,\ m_{30} + m_{35}^{(6)} = \mu_3$$

$$m_{40} + m_{45} + m_{42}^{(7)} = \mu_4,\ m_{50} + m_{56} = \mu_5,\ m_{65} = \int_0^\infty t\, h(t)\, dt = m_1(say),\ m_{72} = \int_0^\infty t\, g(t)\, dt = m_2(say)$$

V. Mean Time To System Failure

Let the random variable T_i denotes the time to first failure of the system when it starts operation in state S_i . To obtain the distribution of time to system failure (*TSF*), we suppose that the failed states S_6 and S_7 as absorbing. By simple probabilistic arguments, the following recursive relations for $\pi_i(t)$ may be obtained,

$$\pi_0(t) = Q_{01}(t)\,(s)\,\pi_1(t) + Q_{02}(t)(s)\pi_2(t) + Q_{03}(t)(s)\pi_3(t)$$
$$\pi_1(t) = Q_{10}(t)\,(s)\,\pi_0(t) + Q_{12}(t)(s)\pi_2(t)$$
$$\pi_2(t) = Q_{25}(t)\,(s)\,\pi_5(t) + Q_{24}(t)(s)\pi_4(t) + Q_{26}(t)$$
$$\pi_3(t) = Q_{30}(t)\,(s)\,\pi_0(t) + Q_{36}(t)$$
$$\pi_4(t) = Q_{40}(t)\,(s)\,\pi_0(t) + Q_{45}(t)(s)\pi_5(t) + Q_{46}(t)$$
$$\pi_5(t) = Q_{50}(t)\,(s)\,\pi_0(t) + Q_{56}(t)$$

Taking Laplace-Stieltjies transform of above equations and solving for $\tilde{\pi}_0(s)$, we have,

$$\tilde{\pi}_0(s) = N_1(s)/D_1(s)$$

Where omitting the argument's from $\tilde{Q}_{ij}(s)$ for brevity,

$$N_1(s) = \left[\left(\tilde{Q}_{25}\tilde{Q}_{56} + \tilde{Q}_{24}\tilde{Q}_{45}\tilde{Q}_{56} + \tilde{Q}_{24}\tilde{Q}_{47} + \tilde{Q}_{26}\right)\left(\tilde{Q}_{01}\tilde{Q}_{12} + \tilde{Q}_{02}\right) + \tilde{Q}_{03}\tilde{Q}_{36}\right]$$
$$D_1(s) = \left[\left(1 - \tilde{Q}_{01}\tilde{Q}_{10} - \tilde{Q}_{03}\tilde{Q}_{30}\right) - \left(\tilde{Q}_{01}\tilde{Q}_{12} + \tilde{Q}_{02}\right)\left(\tilde{Q}_{25}\tilde{Q}_{50} + \tilde{Q}_{24}\tilde{Q}_{40} + \tilde{Q}_{24}\tilde{Q}_{45}\tilde{Q}_{50}\right)\right]$$

The mean time to system failure (***MTSF***) when the system starts from state S_0 is,

$$MTSF = \left[\frac{\left(\mu_0 + p_{01}\mu_1 + p_{03}\mu_3\right) + \left[\mu_2 + p_{24}\mu_4 + \mu_5\left(p_{25} + p_{24}p_{45}\right)\right]\left(p_{01}p_{12} + p_{02}\right)}{\left(1 - p_{01}p_{10} - p_{03}p_{30}\right) - \left(p_{01}p_{12} + p_{02}\right)\left[p_{24}p_{40} + p_{50}\left(p_{25} + p_{24}p_{45}\right)\right]}\right]$$

Where $m_1 = \int_0^\infty t\, k(t)\, dt$ = expected repair time of a failed (from standby) unit,

$m_5 = \int_0^\infty t\, f(t)\, dt$ = mean time of post repair.

VI. Availability Analysis

Let $M_i(t)$ be the probability that the system up initially in regenerative states S_i is up at epoch t without passing through any other regenerative state, we have,

$M_0(t) = e^{-(\alpha+\beta+\theta+\eta)t}$; $M_1(t) = e^{-(2\alpha+\eta)t}$; $M_2(t) = e^{-(\alpha+\theta)t}\overline{H}(t)$,

$M_3(t) = e^{-\alpha t}\overline{K}(t)$, $M_4(t) = e^{-(\alpha+\theta)t}\overline{G}(t)$, $M_5(t) = e^{-\theta t}$.

Further, let $q_{ij}(t) = \frac{d}{dt}Q_{ij}(t)$ and $q_{ij}^{(k)}(t) = \frac{d}{dt}Q_{ij}^{(k)}(t)$.

Starting from state S_i the probability that the system is available for operation at the time instant t is denoted by $A_i(t)$. By simple probabilistic arguments, the following recursive relations are obtained,

$$A_0(t) = M_0(t) + q_{01}(t)(c)A_1(t) + q_{02}(t)(c)A_2(t) + q_{03}(t)(c)A_3(t)$$
$$A_1(t) = M_1(t) + q_{10}(t)(c)A_0(t) + q_{12}(t)(c)A_2(t)$$
$$A_2(t) = M_2(t) + q_{25}(t)(c)A_5(t) + q_{25}^{(6)}(t)(c)A_5(t) + q_{24}(t)(c)A_4(t)$$
$$A_3(t) = M_3(t) + q_{30}(t)(c)A_0(t) + q_{35}^{(6)}(t)(c)A_5(t)$$
$$A_4(t) = M_4(t) + q_{40}(t)(c)A_0(t) + q_{42}^{(7)}(t)(c)A_2(t) + q_{45}(t)(c)A_5(t)$$
$$A_5(t) = M_5(t) + q_{50}(t)(c)A_0(t) + q_{55}^{(6)}(t)(c)A_5(t)$$

Taking Laplace transform of above equations and solving for $A_0(s)$. we have,

$$A_0(s) = N_2(s) / D_2(s)$$

Where

$$N_2(s) = \Big[\left(M_0^* + q_{01}^*M_1^* + q_{03}^*M_3^*\right)\left(1 - q_{42}^{(7)*}q_{24}^*\right)\left(1 - q_{55}^{(6)*}\right) + \left(1 - q_{55}^{(6)*}\right)\left(q_{01}^*q_{12}^* + q_{02}^*\right)\left(M_2^* + q_{24}^*M_4^*\right) + \left(q_{25}^*q_{55}^{(6)*}M_5^* + q_{24}^*q_{45}^*M_5^*\right)\left(q_{01}^*q_{12}^* + q_{02}^*\right) + M_5^*q_{35}^{(6)*}q_{03}^*\left(1 - q_{24}^*q_{42}^{(7)*}\right)\Big]$$

$$D_2(s) = \Big\{\left(1 - q_{01}^*q_{10}^*\right)\left(1 - q_{55}^{(6)*}\right)\left(1 - q_{24}^*q_{42}^{(7)*}\right) - \left(q_{01}^*q_{12}^* + q_{02}^*\right)\Big[\left(q_{25}^* + q_{25}^{(6)*}\right)q_{50} + q_{26}^*q_{65}^*q_{50}^* + q_{24}^*q_{40}^*\left(1 - q_{55}^{(6)*}q_{65}^*\right) + q_{24}^*q_{45}^*q_{50}^*\Big] - q_{03}^*q_{30}^*\left(1 - q_{55}^{(6)*}\right)\left(1 - q_{24}^*q_{42}^{(7)*}\right) - q_{03}^*q_{35}^{(6)*}q_{50}^*\left(1 - q_{24}^*q_{42}^{(7)*}\right)$$

Here, for brevity we have omitted the argument‘s’ from $q_{ij}(s), q_{ij}^{(k)*(s)}$ and $M_i(s)$. The steady state availability of the system is,

$$A_0 = \lim_{s\to 0} s\, A_0(s) = N_2(0) / D_2'(s),$$

Where in view of

$$M_0(0) = \mu_0 \;, M_1(0) = \mu_1 \;, M_2(0) = \mu_2 \;, , M_3(0) = \mu_3 \;, M_4(0) = \mu_4, M_5(0) = \mu_5$$

$$N_2 = \Big\{\left(1 - p_{55}^{(6)}\right)\left(1 - p_{24}p_{42}^{(7)}\right)\left(\mu_0 + p_{01}\mu_1 + p_{03}\mu_3\right) + \left(1 - p_{55}^{(6)}\right)\left(p_{01}p_{12} + p_{02}\right)\left(\mu_2 + p_{24}\mu_4\right) \Big\} + \left(p_{01}p_{12} + p_{02}\right)\mu_5\left(p_{25} + p_{26} + p_{24}p_{45}\right) + p_{35}^{(6)}p_{03}\mu_5\left(1 - p_{24}p_{42}^{(7)}\right)$$

$$D_2' = \left\{\left(1 - p_{24}p_{42}^{(7)}\right)\left[\mu_0 p_{50} + p_{01}\mu_1\left(1 - p_5^{(6)}\right) + \mu_5 p_{03}p_{35}^{(6)}\right] + \left(p_{01}p_{12} + p_{02}\right)\right.$$
$$\times\left[p_{50}\mu_2 + \mu\left(p_{25}^{(6)} + p_{24}p_{45}\right) + \mu_4 p_{24}\left(1 - p_{55}^{(6)}\right) + m_1\left(p_{25}^{(6)}p_{50} + p_{24}p_{40}p_{56}\right)\right]$$
$$+ m_1\left[p_{55}^{(6)}\left(1 - p_{24}p_{42}^{(7)}\right)\left(1 - p_{01}p_{10} - p_{03}p_{30}\right)\right]$$

Where m_1 is define earlier.

VII. Busy Period Analysis

Let $B_i(t)$ be the probability that starting from state S_i the system is under normal repair at epoch *t* and $W_i(t)$ is the probability that the system initially under repair in regenerative state $S_i \in E$ continues to be under repair in states for at least time *t* without passing through any other regenerative state or returning to itself through one or more states. By probabilistic arguments, the following recursive relations are,

$$B_0(t) = q_{01}(t)(c)B_1(t) + q_{02}(t)(c)B_2(t) + q_{03}(t)(c)B_3(t)$$
$$B_1(t) = q_{10}(t)(c)B_0(t) + q_{12}(t)(c)B_2(t)$$
$$B_2(t) = W_2(t) + q_{25}(t)(c)B_5(t) + q_{25}^{(6)}(t)(c)B_5(t) + q_{24}(t)(c)B_4(t)$$
$$B_3(t) = W_3(t) + q_{30}(t)(c)B_0(t) + q_{35}^{(6)}(t)(c)B_5(t)$$
$$B_4(t) = q_{40}(t)(c)B_0(t) + q_{42}^{(7)}(t)(c)B_2(t) + q_{45}(t)(c)B_5(t)$$
$$B_5(t) = q_{50}(t)(c)B_0(t) + q_{55}^{(6)}(t)(c)B_5(t)$$

Where $W_2(t) = \mu_2 = \overline{F}(t)\ ;\ W_3(t) = \mu_3 = \overline{K}(t)$

Taking Laplace transform of above equations and solving for $B_0(s)$**.** we have,

$$B_0(s) = N_3(s) / D_2(s)$$

Where omitting the argument's' for brevity,

$$N_3(s) = \left(1 - q_{55}^{(6)*}\right)\left[W_2^*\left(q_{01}^* q_{12}^* + q_{02}^*\right) + q_{03}^* W_3^*\left(1 - q_{24}^* q_{42}^{(7)*}\right)\right]$$

and $D_2(s)$ is same as in availability.

The fraction of time for which the system is under repair is given by,

$$B_0 = \lim_{s\to 0} s\, B_0(s) = N_3(0) / D_2'(0).$$

Where in terms of $W_2(0) = \mu_2$, $W_3(0) = \mu_3$

$$N_3 = \left(1 - p_{55}^{(6)}\right)\left[\mu_2\left(p_{01}p_{12} + p_{02}\right) + \mu_3 p_{03}\left(1 - p_{24}q_{42}^{(7)}\right)\right]$$

And D_2' is same as defined earlier.

VIII. Expected Number Of Visits By The Repairman

Let $V_i(t)$ be the number of visits by the repairman in $(0,t)$. By probabilistic arguments the following recursive relations are obtained,

$$V_0(t) = Q_{01}(t)(c)V_1(t) + Q_{02}(t)(c)\left[1 + V_2(t)\right] + Q_{03}(t)(c)V_3(t)$$
$$V_1(t) = Q_{10}(t)(c)V_0(t) + Q_{12}(t)(c)\left[1 + V_2(t)\right]$$
$$V_2(t) = Q_{25}(t)(c)V_5(t) + Q_{25}^{(6)}(t)(c)V_5(t) + Q_{24}(t)(c)V_4(t)$$
$$V_3(t) = Q_{30}(t)(c)V_0(t) + Q_{35}^{(6)}(t)(c)V_5(t)$$
$$V_4(t) = Q_{40}(t)(c)V_0(t) + Q_{42}^{(7)}(t)(c)\left[1 + V_2(t)\right] + Q_{45}(t)(c)V_5(t)$$
$$V_5(t) = Q_{50}(t)(c)V_0(t) + Q_{55}^{(6)}(t)(c)V_5(t)$$

Taking Laplace transform of above equations and solving for $\widetilde{V}_0(s)$**.** we have,

$$\tilde{V}_0(s) = N_4(s) / D_3(s),$$

Where omitting the argument's' for brevity

$$N_4(s) = \Big\{\Big(\tilde{Q}_{01}\tilde{Q}_{12} + \tilde{Q}_0\Big)\Big(1-\tilde{Q}_{55}^{(6)}\Big)\Big[\tilde{Q}_{25} + \tilde{Q}_{24} + \tilde{Q}_{25}^{(6)}\Big] + \Big(\tilde{Q}_{01}\tilde{Q}_{12} + \tilde{Q}_{02}\Big)\tilde{Q}_{45}\tilde{Q}_{24}\tilde{Q}_{55}^{(6)} + \Big\}\tilde{Q}_{03}\tilde{Q}_{35}^{(6)}\Big(1-\tilde{Q}_{55}^{(6)}\Big)\Big(1-\tilde{Q}_{24}\tilde{Q}_{42}^{(7)}\Big)$$

and

$$D_2(s) = \Big\{\Big(1-\tilde{Q}_{01}\tilde{Q}_{10}\Big)\Big(1-\tilde{Q}_{55}^{(6)}\Big)\Big(1-\tilde{Q}_{24}\tilde{Q}_{42}^{(7)}\Big) - \Big(\tilde{Q}_{01}\tilde{Q}_{12} + \tilde{Q}_{02}\Big)\Big[\tilde{Q}_{25}^{(6)}\tilde{Q}_{50} + \tilde{Q}_{25}^{(6)}\tilde{Q}_{50} + \tilde{Q}_{24}\tilde{Q}_{40}\Big(1-\tilde{Q}_{55}^{(6)}\Big) + \tilde{Q}_{24}\tilde{Q}_{45}\tilde{Q}_{50}\Big] - \tilde{Q}_{03}\tilde{Q}_{30}\Big(1-\tilde{Q}_{55}^{(6)}\Big)\Big(1-\tilde{Q}_{24}\tilde{Q}_{42}^{(7)}\Big) - \tilde{Q}_{03}\tilde{Q}_{35}^{(6)}\tilde{Q}_{50}\Big(1-\tilde{Q}_{24}\tilde{Q}_{42}^{(7)}\Big)\Big\}$$

In steady state the number of visits per unit time, when the system starts at the entrance into S_0 is,

$$V_0 = \lim_{s\to 0} s\,\tilde{V}_0(s) = N_4(0) / D_2'(0)$$

Where,

$N_4 = \Big\{\Big(1-p_{55}^{(6)}\Big)\Big[p_{01}p_{12} + p_{02} + \Big(1-p_{24}p_{42}^{(7)}\Big)p_{03}p_{35}^{(6)}\Big] + p_{45}p_{24}p_{55}^{(6)}\Big(p_{01}p_{12} + p_{02}\Big)\Big\}$ and D_2' is defined earlier.

IX. Cost Analysis

Let K_0 = revenue per unit up time earned by the system

K_1 = repair cost per unit time

K_2 = cost per visit by the repairman.

The expected total cost in (0,t)

$$C(t) = K_0\mu_{up}(t) - K_1\mu_b(t) - K_2V_0(t)$$

The expected cost per unit time in the steady state is,

$$C(t) = K_0A_0(t) - K_1B_0(t) - K_2V_0$$

X. Estimation With Exponential Distribution

Let the repair times follows the exponential distribution,

$$f(t) = ae^{-at}; a > 0, k(t) = de^{-dt}; d > 0, g(t) = ce^{-ct}; c > 0; h(t) = be^{-bt}; b > 0$$

Hence the transition probabilities are,

$p_{01} = \theta/(\alpha+\beta+\theta)$, $p_{03} = \beta/(\alpha+\beta+\theta)$, $p_{02} = \alpha/(\alpha+\beta+\theta)$

$p_{10} = \eta/(2\alpha+\eta)$, $p_{12} = 2\alpha/(2\alpha+\eta)$, $p_{24} = a/(\alpha+\theta+a)$,

$p_{25} = \theta[1-p_{24}]/(\alpha+\theta)$, $p_{26} = \alpha[1-p_{24}]/(\alpha+\theta) = p_{25}^{(6)}$,

$p_{30} = d/(\alpha+a)$, $p_{36} = (1-p_{30})$, $p_{40} = c/(\alpha+\theta+c)$

$p_{45} = \theta[1-p_{40}]/(\alpha+\theta)$,

$p_{47} = \alpha[1-p_{40}]/(\alpha+\theta) = p_{42}^{(7)}$, $p_{50} = b/(\alpha+b)$, $p_{56} = [1-p_{50}] = p_{55}^{(6)}$,

$p_{65} = p_{72} = 1.$

Now the mean sojourns times are,

$$\mu_0 = 1/(\alpha+\beta+\theta), \mu_1 = 1/(2\alpha+\eta), \mu_2 = a/(\alpha+\theta+a), \mu_3 = d/(\alpha+d)$$

$$\mu_4 = c/(\alpha+\theta+c), \mu_5 = b/(\alpha+b), m_1 = 1/b, m_5 = 1/c$$

Now, substituting the arbitrary values in the above equation we can estimate the mean time to system failure, availability, busy period, expected number of visit by the repairman and cost earned by the system.

$\beta=\theta=\eta=a=b=c=d=5.0$					
α	**10**	**20**	**30**	**40**	**50**
MTSF	**0.58**	**0.29**	**0.19**	**0.14**	**0.11**
A_0	**0.88**	**0.62**	**0.48**	**0.40**	**0.34**
B_0	**0.20**	**0.09**	**0.05**	**0.03**	**0.02**
V_0	**0.77**	**0.61**	**0.49**	**0.41**	**0.35**
$\beta=\alpha=\eta=a=b=c=d=5.0$					
θ	**10**	**20**	**30**	**40**	**50**
MTSF	**1.17**	**1.17**	**1.18**	**1.18**	**1.18**
A_0	**1.31**	**1.54**	**1.73**	**1.89**	**2.02**
B_0	**0.34**	**0.30**	**0.27**	**0.24**	**0.22**
V_0	**1.01**	**1.25**	**1.45**	**1.61**	**1.74**
$\alpha=\theta=\eta=a=b=c=d=5.0$					
β	**10**	**20**	**30**	**40**	**50**
MTSF	**1.12**	**1.08**	**1.06**	**1.05**	**1.04**
A_0	**1.25**	**1.35**	**1.41**	**1.45**	**1.48**
B_0	**0.44**	**0.54**	**0.60**	**0.63**	**0.66**
V_0	**0.84**	**0.84**	**0.84**	**0.84**	**0.83**
$\alpha=\beta=\theta=a=b=c=d=5.0$					
η	**10**	**20**	**30**	**40**	**50**
MTSF	**1.16**	**1.16**	**1.16**	**1.16**	**1.16**
A_0	**1.18**	**1.19**	**1.20**	**1.20**	**1.20**
B_0	**0.37**	**0.38**	**0.39**	**0.39**	**0.40**
V_0	**0.84**	**0.84**	**0.84**	**0.84**	**0.83**

Table 1- Estimation of Reliability Measures

XI. Conclusions And Interpretations

From the above table we perceive that,

(a) As the value of the failure rate on operative unit increases, mean time to system failure, availability, busy period and expected number of visits by the repairman increases.

(b) As the value of the failure rate on standby unit increases, mean time to system failure decreases. Availability, busy period and expected number of visits by the repairman increases.

(c) As the value of the workload increases, mean time to system failure, availability and expected number of visits by the repairman increases. Busy period by the repairman decreases

(d) As the value of the workload decreases, mean time to system failure remains constant. Availability and busy period by the repairman increases. Expected number of visits by the repairman decreases.

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